Towards the design and modeling of an Expert Computer System to help develop strategies for solving mathematical problems: Verbalization and modeling of solving problem networks

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Abstract.
Mathematical problem solving is one of the most important skills that students need to develop in order to be successful in their academic and professional lives. Mathematical problem solving engenders a variety of cognitive, metacognitive, affective, and emotional processes, which can be beneficial for the development of learners' skills and abilities.

In the realm of a digital pedagogy, process modeling by computer system designers to create intelligent tutorial systems can improve mathematical problem-solving skills. This study consists in visualizing the solving processes implemented by a sample of middle school students, analyzing them to deduce the interactions between the different problem data, the long-term memory (the memorization data) and the short-term memory (the transformation data).

We present in this article a methodological and analytical approach allowing to verbalize, to visualize the networks of mathematical problem solving and to represent them schematically. Furthermore, we provide a modeling framework for these networks. By visualizing the problem-solving processes and their underlying cognitive mechanisms, educators and researchers can gain insights into students' problem-solving strategies, identify potential challenges, and design effective interventions.

Keywords: Expert computing system, modeling, problem solving networks, mathematics.

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Introduction:

Mathematical problem solving is a fundamental skill at the heart of mathematics education (Mudaly, 2021). It goes beyond the simple application of algorithms and formulae, requiring students to think critically, reason logically, and apply their mathematical knowledge to real-world situations. The ability to solve mathematical problems not only improves students' mathematical skills but also develops their problem-solving skills, critical thinking abilities, and creativity (Feyfant, 2015). The process of solving mathematical problems is not limited to finding the right answer. It requires students to analyze the problem, identify relevant information, formulate a strategy, execute the strategy, and reflect on the solution (Polya, 1945). Throughout this process, students engage in a range of cognitive processes, such as comprehension, reasoning, abstraction, and decision-making. Understanding these cognitive processes is essential for educators to effectively teach and support students in developing their problem-solving abilities. Mathematical problem solving also presents a variety of challenges for students. They may have difficulty understanding the problem, selecting appropriate strategies, making connections between different mathematical concepts, or managing their mathematical emotions and anxiety. Recognizing and addressing these challenges is essential to creating a supportive learning environment that fosters students' problem-solving skills.

Given the importance of mathematical problem solving in the mathematical and cognitive development of students, it is crucial to explore effective pedagogical approaches that can foster and enhance problem-solving skills. Technological advances have revolutionized the field of mathematics education, opening up new possibilities for improving the teaching of problem solving. Digital tools, interactive software, and online resources offer dynamic and engaging learning environments that can help students visualize, explore, and solve mathematical problems. The integration of technology into education is opening up new ways of providing personalized and interactive learning experiences. Intelligent tutoring systems, with the ability to analyze student responses and provide personalized feedback, have demonstrated their potential to support students' problem-solving abilities. By accurately modeling problem-solving processes, these systems can offer targeted advice and support, helping students tackle complex mathematical challenges. As digital pedagogy continues to advance, the modeling of problem-solving processes by computer system designers to create intelligent tutoring systems has emerged as a promising approach to improving mathematical problem-solving skills.

In this article, we present a methodological and analytical approach for verbalizing and visualizing mathematical problem-solving networks. To this end, the study implemented one of the most commonly used methods in this type of research, namely the think-aloud technique. Our proposed approach enables a systematic exploration of the relationships between problem elements, memory processes, and problem-solving strategies. Through the development of these models, we aim to provide educators with valuable information about students' problem-solving strategies, potential barriers they may encounter, and effective instructional interventions that can be used to improve their problem-solving skills.

1 Literature review:

1.1 Teaching problem solving and Problem-Solving Networks:

The OECD (Organisation for Economic Co-operation and Development) has proposed the following definition for problem-solving competence: "Competence does not only refer to knowledge and skills, it also implies the ability to respond to complex demands and to be able to mobilize and exploit psychosocial resources (including skills and attitudes) in a particular context" (OECD, 2009). Therefore, solving calls upon the learner's abilities in memorization, perception, reasoning, conceptualization, language appropriation, but also upon his emotions, motivation, self-confidence. Bakar & al (2020) point out that mathematical problem solving requires metacognitive skills: the ability to plan, self-regulate, evaluate and reflect, which is confirmed by the work of Velasquez & al (2019).

Mathematical problem-solving ability was positively correlated with working memory ability, cognitive flexibility, and mathematical reasoning ability (Chen et al., 2020). Thus, several elements can impact problem solving. First, memory plays a crucial role in the creation and manipulation of such a model (Solaz-Portolés & Lopez, 2007). The availability of prior knowledge needed to solve the problem is also important, according to Brown & al. (1989). As well as Lave & Wegner (1991) emphasize the key role of cognition. Schoenfeld (1992) emphasizes the importance of representations in problem solving, including the retrieval of applicable information from long-term memory (such as facts, concepts, principles, properties, analogies, generalizations, strategies, approaches, algorithms, attitudes). Therefore, solving a mathematical problem involves two aspects: the problem domain, which includes its context, data and the question to be solved, and the cognitive process domain, which concerns the understanding of the problem, prior knowledge, strategies, approaches, algorithms and planning needed to solve it (De Jong, 2005).

A problem-solving network represents the interconnected relationships between mathematical concepts, strategies, and problem-solving techniques. It provides a holistic view of the problem-solving process, enabling learners to identify patterns, connections, and dependencies within a mathematical problem.

In summary, the problem-solving model proposed by Frensh and Funke (1995) provides a useful framework for understanding and modeling mathematical problem-solving situations. Solving a mathematical problem involves several key steps, including reading and comprehending the problem, identifying the desired goal or solution, and devising a systematic approach that utilizes appropriate operations, algorithms, and tools to overcome any obstacles or challenges encountered along the way. By applying this model, educators and researchers can gain insights into the cognitive processes involved in mathematical problem-solving and develop effective instructional strategies to support students in their problem-solving endeavors.

1.2 Computer Tutoring Systems for Mathematics : A Review of Existing Systems and their Effectiveness :

Mathematics holds significant importance in diverse fields, including science, engineering, and finance. Nonetheless, many students encounter difficulties in grasping mathematical concepts, and traditional teaching methods may not effectively address their needs. One potential solution to this challenge lies in computer tutoring systems specifically designed for mathematics. These systems offer personalized and adaptive learning experiences, aiding students in enhancing their mathematical skills and building confidence.

Over the past few years, there has been a notable surge in the interest surrounding computer tutoring systems for mathematics. These systems leverage various techniques such as machine learning, data mining, and artificial intelligence to deliver tailored and adaptive tutoring experiences. By accommodating the unique requirements and aptitudes of individual students, they can provide immediate feedback and guidance, ensuring a more effective learning process.

Our review focuses on systems that help students solve mathematical problems, including arithmetic, algebra, geometry, and calculus.

There are various computer tutoring systems for mathematics available today. Some of the most popular systems include:

ALEKS (2001) is an adaptive learning system that uses artificial intelligence to assess students' knowledge and skills and provide personalized instruction. It offers courses in arithmetic, algebra, and calculus and has been shown to improve students' performance and engagement in mathematics (Fang & al., 2019).

Knewton (Jones & Bomash, 2018) is another adaptive learning system that uses machine learning algorithms to personalize the learning experience. It offers courses in various subjects, including mathematics, and has been used by several educational institutions.

DreamBox (Grams, 2018) is a gamified learning system that offers personalized math instruction for elementary and middle school students. It uses a combination of adaptive learning, game-based learning, and feedback mechanisms to engage students and improve their math skills.

Khan Academy (Kelly & Rutherford, 2017) is a popular online learning platform that offers video lessons, interactive exercises, and personalized coaching in various subjects, including mathematics. It has been used by millions of students worldwide and has been shown to improve students' performance and confidence in mathematics.

Other studies have presented work related to the development of intelligent tutoring systems for mathematical problem solving such as: TURING (TUtoRiel INtelligent en Géométrie) (El-Khoury et al, 2005) which aims to improve students' mathematical problem solving skills. This system was designed to provide personalized feedback to students based on their performance and learning needs. It was created using an intelligent tutoring system (ITS) approach, which involves analyzing student responses to identify strengths and weaknesses and adapting instructional materials accordingly.

The study used a sample of high school students to test the effectiveness of the system. The results showed that the system was effective in improving students' problem-solving skills, with significant improvements observed in the experimental group compared to the control group.

There is also the QED-Tutrix system (Leduc, 2016) which is an intelligent tutoring system designed to help students solve proof problems in plane geometry. Developed by Nicolas Leduc in 2016, the system uses artificial intelligence and natural language processing to guide students through the problem-solving process. QED-Tutrix aims to provide personalized feedback to students based on their individual problem-solving strategies and to improve their understanding of the underlying concepts of geometry. The system was developed at the Ecole Polytechnique de Montréal, Canada.

Our review shows that computer tutoring systems can significantly improve students' performance and engagement in mathematics. Many of the systems we reviewed use adaptive learning techniques to provide personalized instruction and feedback, which can help students learn at their own pace and improve their understanding of mathematical concepts.

However, there are still some challenges that need to be addressed. One of the main challenges is the need for more personalized and adaptive tutoring systems. While many of the systems we reviewed are adaptive, they may not always provide the most effective feedback and guidance for each student. Additionally, the lack of effective feedback mechanisms in some systems may hinder students' learning.

Another challenge is the difficulty of integrating computer tutoring systems into the classroom. While some systems are designed for use in the classroom, others may be more suitable for use at home or in other settings. Additionally, some students may prefer traditional teaching methods or may have limited access to technology.

Computer tutoring systems for mathematics offer a potential solution to the challenges of teaching and learning mathematics. They can provide personalized and adaptive learning experiences that can improve students' skills and confidence in mathematics. Our review of existing systems has shown that many of them are effective in improving students' performance and engagement in mathematics.

However, there are still challenges that need to be addressed to make computer tutoring systems more effective. These include the need for more personalized and adaptive tutoring systems, the lack of effective feedback mechanisms, and the difficulty of integrating these systems into the classroom.

Overall, computer tutoring systems for mathematics have great potential to revolutionize the way we teach and learn mathematics. As technology continues to advance, we can expect to see even more innovative and effective computer tutoring systems in the future.

1.3 Visualizing and Mapping Problem Solving Processes Using Solving Aloud:

Various studies have emphasized the importance of modeling and visualizing mathematical problem-solving processes to better understand how students approach and solve mathematical problems. Suurtamm (2019) in their systematic review of the literature on modeling problem solving processes in mathematics education, identified various models and frameworks for representing problem solving processes, including the cognitive model, activity-based model, and network model. Network visualization of mathematical problem solving is a method of representing the problem-solving networks by showing the different interactions between several domains involved in solving a problem (Boero et al., 2008). This modeling helps researchers and teachers to better understand the cognitive processes involved in students' problem solving in mathematics in order to develop the most appropriate and adaptive teaching-learning devices.

Several verbalization techniques have been mentioned in the literature on the study of problem-solving processes: questionnaires (Lorenzo, 2005), process mining methods (Glodhammer & Barkow, 2017). However, solving aloud remains one of the most widely used methodological approaches in research to account for problem solving processes and the internal model hit.

Solving aloud refers to the process of verbalizing one's thoughts while trying to solve a problem. This is a common strategy used in problem solving tasks. Solving aloud can help individuals clarify their thinking, identify errors, and communicate their thought process to others. It can also help with metacognition or the ability to reflect on one's own thought processes (Alvarez et al., 1990). According to Van Someren et al. (1994), the think-aloud method is a practical approach to modeling the cognitive processes generated by a learner during problem solving.

1.4 Research focus

This research aims to investigate and document the various steps and cognitive processes employed by a group of middle school students when solving a specific type of mathematical problem, namely an algebraic problem. The primary goal is to examine and map out the students' problem-solving network, analyze their strategies, and subsequently construct a model of the problem-solving pathway. The focus of the model is primarily on understanding the interactions between the problem data, which includes the problem domain and context, the prerequisite knowledge stored in the long-term memory domain, and the data generated through the different transformations occurring in the working memory domain. By delving into these aspects, we seek to gain insights into the cognitive mechanisms involved in the resolution of mathematical problems and provide a theoretical framework for understanding and improving students' problem-solving abilities in algebra.

2 Methodology approach

2.1 Selection of the student sample

The study involved a group of 20 high school students who were in their third year during the 2021-2022 academic year. These students were chosen to participate in a think-aloud session, and they represented a diverse range of characteristics, including gender, mathematics proficiency, motivation, and language and verbal communication skills. To alleviate any potential stress related to grades, the students were informed that the assignment would not be graded. This approach aimed to create a supportive environment where students felt comfortable employing their own problem-solving methods and expressing their thoughts openly without concern for evaluation.

2.2 Choice of problem situation

" A father's age is twice that of his son. 12 years ago, a father's age was triple that of his son. How old is the father? How old is the son? "

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The test problem, which is taken from the 3rd year college textbook of the Moroccan program, is part of the numerical calculation activities and more precisely the algebraic activities. This problem, which is related to everyday life, participates with other disciplines: French (the tenses and conjugation of verbs) and the science of life and earth (fertility of a man). It develops in the student several skills and competencies: the ability to make a choice, proof, manipulation, imagination, self-criticism, clear thinking, organization and planning; mathematizing the situation by determining and analyzing linguistic and conceptual data, which leads to the solution of a first-degree equation with one unknown or a system of two first degree equations with two unknowns.

2.3 Data Collection:

During the think-aloud sessions, the students' verbalizations were recorded, and additional data were collected from their written traces. These rich sources of information were then utilized to construct a process map, also known as a network map. This map visually represents the flow of data among the three domains that are integral to problem-solving: the problem domain, the long-term memory domain, and the transformation domain. By analyzing and mapping out this data flow, we gain a deeper understanding of how information is exchanged and utilized during the problem-solving process, shedding light on the cognitive mechanisms at play.

3 Result and discussion

Through a thorough analysis of the test situation, we were able to identify three key problem domains that play a crucial role in problem-solving: problem data, memory data, and transformation data. To provide a comprehensive framework for our study, we organized and presented these domains in Table 1. This framework serves as a valuable tool for structuring and categorizing the various components involved in the problem-solving process, allowing for a systematic exploration and understanding of the interplay between these domains

Table 1: The data used in solving the problem (frame of reference for the analysis).

Problem data	Memory dada	Transformation data	Solution
Dp1 : son's age	DM1 : the present tense of the verb to be	DT1 : let x be the son's age	S1
Dp2 : father's age	DM2 : age times two	DT2 : father's age is 2x years old	S2
Dp3 : a father's age is twice that of his son	DM3 : age will decrease by 12 years	DT3 : 2x-12	
Dp4 : 12 years ago	DM4 : the verb to be in the imperfect tense	DT4 : x-12	
Dp5 : was	DM5 : triple is three times	DT5 2x-12=3(x-12)	
Dp6 : triple		DT6 : 2x-12=3x-36	
	DM6 : k(a+b)= ka+kb		
	DM7 : if ax +b=cx+d then ax-cx=d-b	DT7 : 2x-3x=-36+12	
	DM8 : the rule of the sum of two relative numbers		
		DT8 : -x=-24	
	DM9 : if ax=b then x=b/a		

This paper introduces two distinct resolution network models, namely the ETP map and the EPP map. These models were derived from the analysis of resolution process data obtained from two distinct groups of students: high performers in mathematics and low performers in mathematics. The ETP map represents the resolution process model generated by the high-performing students, while the EPP map represents the resolution process model generated by the low-performing students. By comparing and contrasting these two maps, we aim to gain insights into the differences in problem-solving approaches and strategies employed by students with varying levels of mathematical proficiency.

3.1 Novice Network Map:

Giving the problem to SHP and asking what we were going to do, she replied that he would first read the problem, determine the unknowns, look for relationships between the data, then solve the equations if they existed, then verify the results obtained. I asked her to tell me how she was going to read? she replied, "I'm going to read the text word by word, trying to understand the mathematical meaning of each piece of data and to determine the main data". When asked to specify the problem data, she replied, "The father's age is double that of his son; 12 years ago, the father's age was triple that of his son". Then he said, "To understand the text properly, I'm going to apply this problem to myself. My father's age is double my age, i.e. my age times two", continuing to read the problem, she said "so I have to calculate my age and my father's age". What SHP has done so far is enter the phase of understanding linguistic information and finding problem data. After 40 seconds of silence, I asked her to tell me why the silence, she said she didn't understand what "12 years ago" meant and how to translate it into mathematical notation, then she mentioned "I'll take x as my age, so my father's age is 2x, 12 years ago my age times 3 equals my father's age which is 2x years, so I'll have a first degree equation which I'll solve to find the solution".

In this stage, she sought to find the links and relationships between the elements of the problem and to devise a plan to help solve the problem, forgetting to translate "12 years ago", which is a key piece of data in this problem, into mathematical notation and in relation to the other elements of the problem.

SHP understood well what was asked of her, formulating a plan, however, when executing SHP plan to find a first-degree equation "3x=2x", a few gestures of uncertainty began to appear on her face as she declared "it's impossible I found that x = 0, so I've made mistakes somewhere!". During the execution of the plan, SHP detected that there was an error by realizing that age is a strictly positive number. So, she asked to come back to the problem again and try to detect her error in order to correct it.

SHP reread the text three times, repeating the sentence: "12 years ago, that is, before the age of 12", a 16-second silence, then she said "before the age of 12, I wasn't the same age, if I'm now 14, before the age of 12 I was 14 -12, that's 2 years, yes, so before the age of 12 I was x-12 and my father was and 2x-12".

Returning to the stage of understanding and translating and transforming the problem's information and data, SHP re-personalized the problem to try and understand it better, and then found that she had ignored an important piece of data, "12 years ago", which means that each age decreases by 12 years. SHP had a problem in the comprehension phase and in translating and transforming linguistic information and data into mathematical notation and equations.

She continued her process, trying to regulate her paths and find the links between these new transformation data as she found them.

Detecting and correcting errors enabled SHP to reformulate the links between the new elements and find a first-degree equation with a single unknown x. After solving this equation by putting the unknowns in one member and the numbers in the other, she found the first solution. Then she replaced x by 24 to find the father's age, and said "my age is 24 and my father's age is 48, but we need to check these results". Backtracking is a very important step in solving mathematical problems, and she didn't forget it after verification, declaring that "the son's age is 24 and the father's age is 48". When asked if there were any other methods for solving this problem, she replied that she could change the unknowns and take x to be the father's age and $x/2$ years the son's age, but she would follow the same procedure.

In analyzing SHP's problem-solving process, we can see that she has a good command of the steps in the problem-solving process and that she has a logical mind, but she has a problem in the comprehension phase and in the phase of translating and transforming linguistic information into mathematical notation, an algorithm into an equation, which influences her performance in mathematical problem-solving. So, she needs training in these two cognitive strategies. This result confirms what we said earlier in the theoretical part about the relationship between these two processes and mathematical problem-solving performance.

SHP was able to recall 66.66% of the pre-required data from the MLT and he used 90% of the data from the different transformations (MT), the remaining 33.34% of the pre-required data and 10% of the data from the different transformations are due to the fact that he took shortcuts.

The SHP resolution network model (Figure 1) consists of 21 passes after navigating between the three problem domains. This number of passes can be reduced if he has correctly determined all the problem data and if he has been able from the outset to translate and transform "12 years ago" into mathematical notation, and to avoid getting stuck and going back and forth to the first part of the resolution.

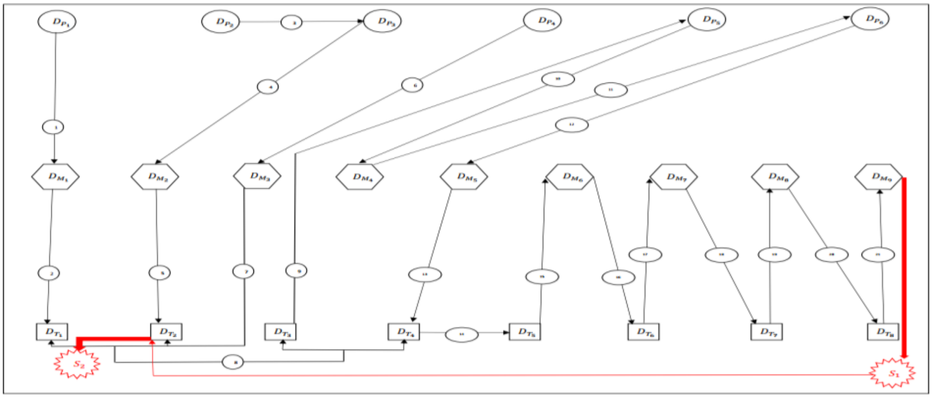

Fig 1: SHP Student Resolution Network Model

SLP began its cognitive process by reading the statement. By answering my question: why do you read? And how do you read? She replied, "The first time I did a normal reading, but the second time I imagined that someone was in front of me, so that I could understand". In this comprehension stage, SLP also personalized the problem by imagining that there was someone in front of her, and he began to read the text addressing this character, then he said to me "I have two unknowns: my age and my father's age, so I have to find the equations of the system, then solve the system and find the solution". Asking what system? And why does he say system? Answering this question, SLP told me that he had already faced this kind of problem and that for him all analysis problems are solved by solving a system, because all the problems he had encountered before were with a system of two equations. Once he'd drawn up his plan, he began the execution stage by reading the statement again, then telling me "the data for the problem are: double, 12 and triple". Then he chose the unknowns, saying "x is my age and y is my father's age.

In trying to understand the statement, SLP didn't determine many of the problem data essential for solving it, because for him, the only essential problem data are the numerical data or the numerical linguistic data, but the other information is given just to help us understand the statement. In the plan design phase, we noticed that he had many false or irrelevant pieces of knowledge, links and relationships between his erroneous knowledge and also false semantic representations.

The cognitive process continued by reformulating other problem information as follows: "The double of triple of my age after 12 years, i.e. 3×x+12, is equal to his age after 12 years, i.e. y + 12, so we'll have the second equation 3x+12=y+12".

This reformulation confirmed our first remark about the state of SLP's knowledge and know-how in his cognitive structure in memory. Then he avoided making a prediction and continued the path of resolution as he had imagined it from the start, arriving at the following system $\begin{cases} 2x = y \\ 3x + 12 = y + 12 \end{cases}$

Then he showed his uncertainties about how to solve a system by declaring that he had forgotten the process. After a minute or so he said, "I remember that we have to look for x and then replace it to find y, so I'm going to sum the two equations member by member".

When he tried to solve the system using this method, he made his calculations and then told me "it's impossible, the age must be a positive integer and I found that x=-21, so my method is wrong". SLP didn't think at all about regulating the formulation of the equations or the links between the unknowns, what he was thinking about was just finding a way of solving the system, especially as he expressed his lack of knowledge about the steps involved in solving systems, which influenced his process. Then he multiplied the first equation by 1 and the second by minus (-1). He found the new system which is $\begin{cases} 23 - 1 = x \\ 3 \times 1 + 12 = y + 12 \end{cases}$. Then he said "I'll simplify - 1 with 1 is 0, so we'll have x=19 , then I'll replace x by 19 in the second equation to find y. We'll then have 19×3+12=y+12 , so y = 57, so we've found the solution: the father's age is 57. So the solution is that the son's age is 19 and the father's age is 57.

In declaring the results, SLP was not only unsure of the outcome of his calculations, but he also failed to declare his uncertainty at any stage of his solution, forgetting the verification stage to be sure of the result obtained.

The process of solving this SLP problem has shown that he has not mastered the steps involved in solving mathematical problems, that he has several difficulties in determining the problem data, his knowledge of poorly organized, the existence of erroneous semantic representations .... Namely: 1×a=1; when he multiplied the equations by 1 or (-1), he did it right for the first member; he simplified 1 with (-1) as long as 1 is multiplied by 3, moreover 1 exists in the first equation as long as (-1) is in the second.... This has influenced its resolution and its resolution network, which is very complicated (like a spider star) as shown in figure 2. So SLP needs a lot of work in solving mathematical problems.

SLP was only able to determine 33.33% of the pre-required MLT data. This influenced its resolution and navigation and led to a lot of back and forth around 47 passages as shown in figure 2, without arriving at the results.

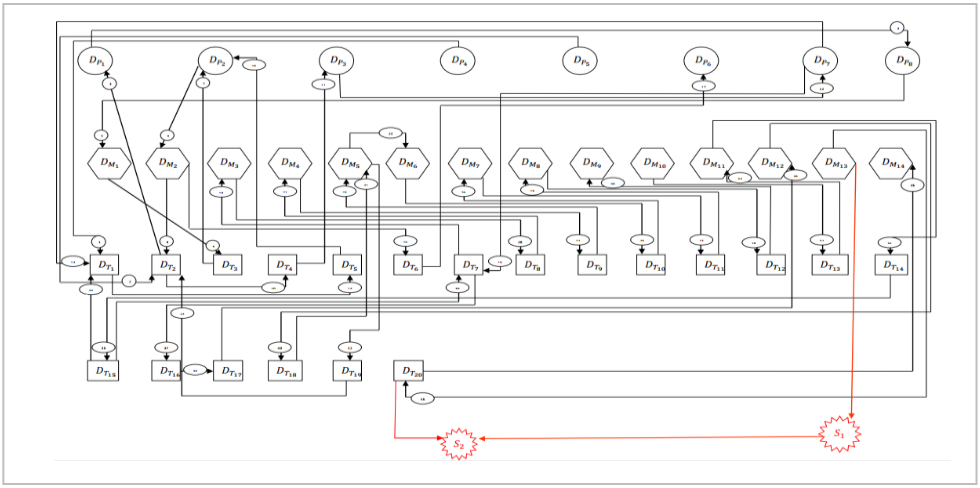

Fig 2: SLP Student Resolution Network Map Model

This study shows that students who were able to solve problems more effectively had clearer and better organized networks than those who were struggling with the problems.

By studying the example solving networks, we can see that the model (figure 3) is composed of three distinct domains (Bahbah, 2023) : the problem-situation domain, which includes the context, data, notions, concepts, definitions, etc.; the MLT-process domain, which includes the necessary data and prior knowledge; and the MT-process domain, which consists of all the transformations and applications that are going to be generated at the level of working memory. Therefore, solving a mathematical problem involves an interaction between these three domains in an interactive way.

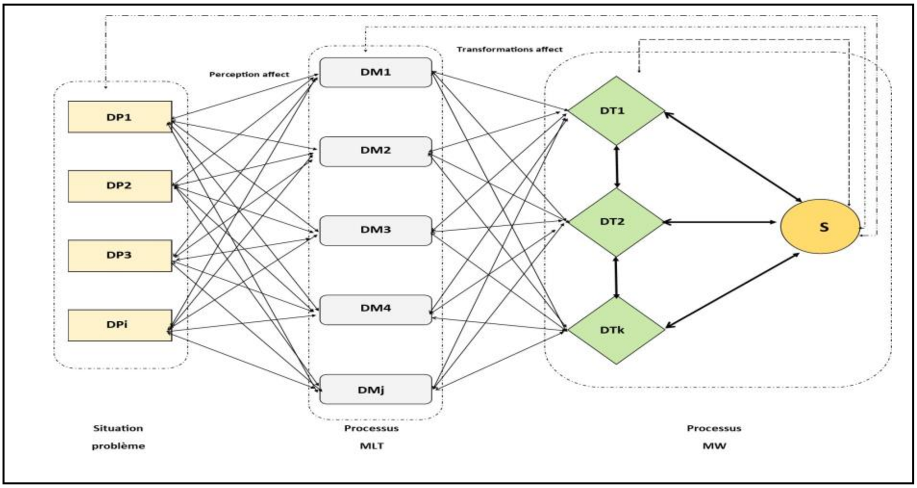

Fig 3: Modeling the network for solving a mathematics problem

Conclusion:

The analysis of learning theories and related literature has underscored the invaluable role of modeling in the teaching and learning of problem-solving. This study and its findings highlight the value of modeling in both student and teacher contexts, as it enables the tracing of the process network employed by each student when encountering a problem situation. By doing so, it provides valuable insights into how students approach and solve mathematical problems, facilitating the identification of various difficulties and stumbling blocks they may encounter.

The use of modeling holds immense potential for both students and teachers. For students, it offers a means to gain a deeper understanding of their own problem-solving strategies and cognitive processes. By mapping out their individual approaches, students can become more aware of their strengths and weaknesses, ultimately leading to improved problem-solving abilities. On the other hand, teachers benefit from this approach by gaining a clearer understanding of how their students engage with mathematical problems. It allows them to recognize the diverse challenges students face and identify specific areas where support and intervention may be required.

However, we acknowledge the challenges inherent in this arduous endeavor, particularly the substantial data collection and analysis required, involving numerous interviews, transcripts, and network maps. This process can be time-consuming and demanding. Furthermore, some students may encounter difficulties in organizing their logical thinking, especially when grappling with complex problems, and expressing their ideas verbally. This can present challenges in accurately capturing and tracking their thinking processes, particularly as the complexity and number of problem-related demands increase.

Nonetheless, despite these challenges, the insights gained from this study can significantly contribute to the field's understanding of effective teaching strategies. It sheds light on the nuanced nature of problem-solving as a cognitive activity, offering insights into the strategies and techniques that lead to successful outcomes. Moreover, it helps identify common difficulties and obstacles that students encounter, paving the way for the development of targeted instructional approaches and interventions.

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