



Research on Weight Determination Method of Cable Damage Index Based on Game Theory

Guanxu Long^{1,2,a}, Yerong Hu^{3,b*}, Guohua Liu^{1,2,c}, Zhang Rui^{3,d} and Wenliang Zhang^{1,2,e}

¹Shandong High-speed Group Co., LTD., Jinan, ShanDong, 250101, China

²Shandong Provincial Key Laboratory of Highway Technology and Safety Assessment, Jinan , ShanDong, 250101, China

³College of Highway Engineering, Chang'an University, Xi'an, Shaanxi, 710064, China

^alonggx5@163.com, ^b*1106262734@qq.com, ^c975072883@qq.com
^d1448875868@qq.com, ^ewenliangzhang@cumt.edu.cn

Abstract. To address the issue of subjectivity or objectivity bias in determining the weights of bridge performance indicators using a single weighting method, this study employs the uncertain analytic hierarchy process (UAHP) and entropy weight method to calculate the subjective and objective weights of evaluation indicators. Furthermore, a game theory model is utilized to obtain the optimal comprehensive weights of the evaluation indicators. The UAHP incorporates interval numbers instead of precise values in judgment matrices to reflect the uncertainty and fuzziness of expert cognition. The entropy weight method uses the correlation matrix of evaluation indicators as the judgment matrix and conducts data standardization to obtain objective weights. The game theory approach minimizes the deviation between the subjective weights from UAHP and the objective weights from the entropy weight method. The effectiveness of this approach is verified by taking the cable sheath damage index of cable-stayed bridges as an example in evaluating the reasonableness of calculating comprehensive indicator rankings using this method.

Keywords: bridge performance indicators, uncertain analytic hierarchy process, entropy weight method, game theory model, cable sheath damage index of cable-stayed bridges.

1 Introduction

Currently, bridge maintenance has become a major focus in the construction of bridges in China. However, there is still limited research on the development of comprehensive evaluation indicator systems, which are necessary to standardize bridge maintenance work. In the United States, where bridge construction began earlier, comprehensive bridge maintenance practices have been implemented. The Federal Highway Administration (FHWA) in the United States divides bridges into four main

components: deck system, superstructure, substructure, and channel with channel protection^[1]. Each component is further divided into various elements, encompassing almost all bridge elements^[2]. In order to improve the classification rules for bridge elements, they are divided according to three levels: national bridge elements, bridge management elements, and user-defined elements. For bridge condition rating, a scale ranging from 0 to 9 is used, where 9 represents the most ideal condition, and each component's condition is also classified into 10 levels ranging from 0 to 9^[3, 4]. The American bridge indicator system not only considers technical condition indicators but also includes applicability indicators such as bridge width, traffic volume, and clearance under the bridge. Currently, the bridge performance indicator system established in China mainly follows the guidelines outlined in Evaluation Standard for Technical Condition of Highway Bridges JTG/T H21 2011^[5]. This evaluation standard uses a hierarchical synthesis method to establish a five-level performance indicator system, namely the whole-bridge level, part level, component level, element level, and defect level. When assessing the technical condition of bridges that do not belong to any of the five categories, a hierarchical synthesis assessment method is used. Firstly, the deduction values of the evaluation indicators are determined based on manually collected data. Then, the scores of the elements are calculated, followed by the determination of the scores for component and subsequently for the whole bridge. This bridge indicator system has clear levels and a straightforward and objective calculation method, providing a solid basis for evaluating the operational performance and making maintenance decisions for bridges in China.

After extensive research by scholars both domestically and internationally, the methods for determining the weights of bridge performance indicators can generally be divided into three categories: subjective weighting, objective weighting, and combined weighting. Subjective weighting is a method that relies on the subjective experience of bridge experts to determine indicator weights. However, this method is heavily influenced by subjective factors, resulting in low utilization of the true values of the indicators and an inability to fully reflect the importance of each indicator. Objective weighting, on the other hand, determines weights based on actual data, enhancing objectivity. However, this method often ignores expert knowledge and relies heavily on the availability of samples, which can limit its applicability. Whether using subjective or objective weighting methods, relying solely on one approach can lead to imbalanced weights, thereby affecting the accuracy of the evaluation results. The Analytic Hierarchy Process (AHP) is a method used for multi-criteria decision-making, which involves the process of subjective weight assignment. In AHP, decision-makers need to make subjective judgments on the relative importance of different factors and criteria, and assign weights to them. The AHP has achieved numerous accomplishments in the research of bridge performance evaluation. The Analytic Hierarchy Process (AHP) has achieved numerous accomplishments in the research of bridge performance evaluation. Xiao Xin^[6] established a hierarchical and graded evaluation system for railway bridges based on the AHP method. Xu Xiang^[7] applied the group AHP method to determine the weights of indicators in the comprehensive technical condition assessment model of suspension bridges. Liang Li^[8] collected subjective weights from experts through the AHP and introduced the minimum Euclidean distance to adjust the

subjective weights. The adjusted weights were then used as positive base points for multiple iterations until the results converged to optimal weights. Xu^[9] presents a cloud-based analytic hierarchical process (C-AHP) rating system to determine inspection intervals of key structural components of suspension bridges. Yang^[10] proposed a new comprehensive state assessment method for long-span PSC continuous box girder bridges that takes into account the interval uncertainty of measured data and the impact of conflicting measured data, by combining the Analytic Hierarchy Process (AHP) with the improved interval evidence theory (IET). The entropy weight method is a multi-criteria decision-making approach used to determine the relative importance of various indicators in decision-making. Unlike traditional weighting methods, the entropy weight method uses entropy theory to calculate the weights of indicators. It has also been widely applied in bridge performance assessment. S.A.Moufti^[11] established and evaluated the state evaluation model of concrete bridge, based on fuzzy membership degree. Yang Zhang^[12] proposed a evaluation method of bridge inspection indexes based on entropy-weight extension matter-element model, which calculate the weight of secondary inspection index, according to the measured value of the inspection index. Peng Zhang^[13] proposed a method for assessing the technical condition of stone arch bridges based on the entropy method-cloud model, which based on the existing assessment standards for stone arch bridges. Firstly, the method converted the comment set and evaluation data into a cloud model. Then it used the entropy method to adjust the weights of the stone arch bridge components and introduced the combined fuzzy pasting schedule method calculating the similarity to obtain the evaluation results. Huifeng Su^[14] conducted a safety condition assessment of the Channel 1 Bridge using the Analytic Hierarchy Process (AHP).

The use of Analytic Hierarchy Process (AHP) and Entropy Weight Method has made the evaluation results more objective and accurate, which is helpful for the maintenance and management of bridges.

To avoid the problem of excessive subjectivity or objectivity caused by using a single weighting method, this study first calculates the subjective and objective weights of each evaluation indicator using the uncertain analytic hierarchy process(UAHP) and the entropy weight method, respectively. Then, a game theory model is employed to determine the optimal comprehensive weights, making the weight allocation more comprehensive. This lays the foundation for accurate bridge assessments in future studies.

2 Determination Method of Weighted Evaluation for Sheath Damaged Index of Stay Cable

2.1 Determining the Subjective Weights of Indicators Based on Uncertain Analytic Hierarchy Process (UAHP)

The Analytic Hierarchy Process (AHP) is a method that decomposes a complex problem into a hierarchical structure and enables the ranking of decision alternatives based on judgments. It allows for the unified treatment of qualitative and quantitative

factors in decision-making, making it practical and systematic^[15]. However, the traditional AHP method constructs judgment matrices that lack flexibility and do not consider the uncertainty and fuzziness of expert cognition, which may not align with real-world scenarios.

To address this limitation, an interval number can be used instead of a single value when constructing the judgment matrix, which greatly reflects the uncertainty and fuzziness of expert cognition. In the case of bridge evaluation indicator systems, there are numerous factors influencing the importance between indicators, making it difficult for experts to accurately grasp their relative importance. To better reflect the actual state of things, this study adopts the uncertain analytic hierarchy process to determine indicator weights.

2.1.1. Establishing a Hierarchical Structure Model.

Based on the complexity and level of analysis required for the problem at hand, a complex problem is decomposed into a hierarchical structure with "goal layer-criterion layer-sub-criterion layer-...-alternative layer". Each layer typically consists of no more than 9 elements. The hierarchical structure model is illustrated in Figure 1.

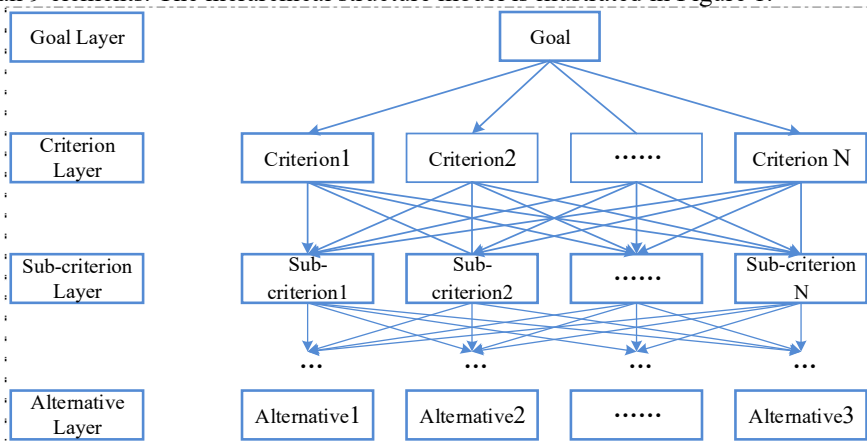


Fig. 1. Hierarchical Structure Model

2.1.2. Constructing Interval Judgment Matrices.

In the UAHP, the construction of judgment matrices is similar to the traditional AHP approach. However, instead of using specific values, interval numbers are used. Interval AHP constructs the judgment matrix by pairwise comparisons, determining the relative importance between two indicators. When determining the relative importance between two indicators, interval numbers are scaled using a 1/9 to 9 proportion scale, which reflects the judgment capability of most individuals^[16]. The meaning of the 1 to 9 scale is explained in Table 1. The constructed interval judgment matrix is shown in Table 2

Table 1. Description of Scale Meaning (1-9)

Scale	Meaning
1	Indicates equal importance between the two factors.
3	Indicates slightly greater importance of the first factor compared to the second.
5	Indicates significantly greater importance of the first factor compared to the second.
7	Indicates strongly greater importance of the first factor compared to the second.
9	Indicates extremely greater importance of the first factor compared to the second.
2,4,6,8	Represents intermediate values between the adjacent descriptions mentioned above.
Count backwards	The judgment of the comparison between factor i and j is denoted as a_{ij} . The judgments for comparing the factors are as follows: $a_{ji} = \frac{1}{a_{ij}}$

Table 2. Interval Judgment Matrix

A	A_1	A_2	...	A_n
A_1	[1, 1]	[a_{12} , b_{12}]	...	[a_{1n} , b_{1n}]
A_2	...	[1, 1]	...	[a_{2n} , b_{12}]
...	[1, 1]	...

2.1.3. Calculation of Subjective weight.

When multiple experts provide interval value evaluations for the bottom-level indicators of a bridge, the method of set-valued statistics can be used to determine the weight of each expert's opinion. Assuming there are n experts who provide evaluations for a certain indicator, resulting in n interval values, we can form a set-valued statistical sequence: $[u_1^{(1)}, u_2^{(1)}], [u_1^{(2)}, u_2^{(2)}], \dots, [u_1^{(n)}, u_2^{(n)}]$. By utilizing the random set center, the weights can be calculated as follows:

$$\omega = \frac{1}{2} \frac{\sum_{k=1}^n \omega_k [(u_2^{(k)})^2 - (u_1^{(k)})^2]}{\sum_{k=1}^n \omega_k [u_2^{(k)} - u_1^{(k)}]} \tag{1}$$

2.2 Determining the Objective Weights of Indicators Based on Entropy Weight Method

Entropy is a measure of uncertainty. The greater the uncertainty, the higher the entropy, indicating a larger amount of information contained. On the other hand, the smaller the

uncertainty, the lower the entropy, indicating a smaller amount of information contained. Based on the characteristics of entropy, it can be used to assess the degree of dispersion or variability of an indicator. If an indicator has a higher level of dispersion, it will have a greater impact on the overall evaluation and therefore a higher weight. Conversely, if an indicator has a lower level of dispersion, it will have a smaller impact and a lower weight in the overall evaluation.

Use the correlation matrix of evaluation criteria as the judgment matrix, where m represents the rating level of the criteria and n represents the number of evaluation criteria. Perform data normalization on the judgment matrix:

$$R = (r_{ij})_{m \times n} \quad (i = 1, 2, \dots, m; j = 1, 2, \dots, n) \tag{2}$$

$$r'_{ij} = \frac{r_{ij} - r_i^{\min}}{r_i^{\max} - r_i^{\min}} \tag{3}$$

$$R' = (r'_{ij})_{m \times n} \quad (i = 1, 2, \dots, m; j = 1, 2, \dots, n) \tag{4}$$

$$H_j = -\frac{1}{\ln(n)} \sum_{i=1}^m f_{ij} \ln(f_{ij}) \tag{5}$$

$$f_{ij} = \frac{1 + r_{ij}}{\sum_{i=1}^m (1 + r_{ij})} \tag{6}$$

$$\omega_j = \frac{1 - H_j}{n - \sum_{j=1}^n H_j} \tag{7}$$

Where $R' = (r'_{ij})_{m \times n} \quad (i = 1, 2, \dots, m; j = 1, 2, \dots, n)$ is the judgment matrix after normalization. ω_j is the final determined index entropy weight.

2.3 Determining the Optimal Comprehensive Weights of Indicators Based on Game Theory Model

Both subjective weighting based on Analytic Hierarchy Process (AHP) and objective weighting based on Entropy Weight Method have their advantages and limitations. Therefore, it is necessary to combine the subjective and objective weights to achieve

an optimal weight value. Currently, a comprehensive method based on game theory can effectively integrate the two approaches to find an optimal weight value. The basic principle is to minimize the deviation between the subjective and objective weights, allowing them to compete with each other while maintaining consistency. In this study, the optimal comprehensive weight values are determined using the game theory approach, following these main steps:

2.3.1. Establishing the Basic Weight Vector Set.

If there are m methods to determine the weights of n indicators, we can obtain m sets of weight vectors for the indicators: $\omega_i = (\omega_{i1}, \omega_{i2}, \dots, \omega_{in}), i = 1, 2, \dots, m$. From this, we obtain a set of weights:

$$\omega = (\omega_{1j}^T, \omega_{2j}^T, \dots, \omega_{mj}^T), j = 1, 2, \dots, n \tag{8}$$

2.3.2. Determination of Optimal Comprehensive Weights.

Perform arbitrary linear combinations of the m sets of weight vectors, $\omega = \sum_{i=1}^m \alpha_i \omega_i^T (\alpha_i > 0)$, and optimize α_i and to minimize their discrepancy of ω and ω_j , i.e., $\min \left\| \sum_{i=1}^m \alpha_i \omega_i^T \right\|_2, i = 1, 2, \dots, m$. The optimal first-order derivative condition for the above equation can be derived from the differential properties of matrices, which is:

$$\begin{pmatrix} \omega_1 \omega_1^T & \omega_1 \omega_2^T & \dots & \omega_1 \omega_m^T \\ \omega_2 \omega_1^T & \omega_2 \omega_2^T & \dots & \omega_2 \omega_m^T \\ \dots & \dots & \dots & \dots \\ \omega_m \omega_1^T & \omega_m \omega_2^T & \dots & \omega_m \omega_m^T \end{pmatrix} \begin{pmatrix} \alpha_1 \\ \alpha_2 \\ \dots \\ \alpha_m \end{pmatrix} = \begin{pmatrix} \omega_1 \omega_1^T \\ \omega_2 \omega_2^T \\ \dots \\ \omega_m \omega_m^T \end{pmatrix} \tag{9}$$

By solving the equation above, we obtain $(\alpha_1, \alpha_2, \dots, \alpha_m)$. After normalizing using, we obtain the optimal weight allocation ratio coefficients α_i^* :

$$\alpha_i^* = \frac{\alpha_i}{\sum_{i=1}^m \alpha_i} \tag{10}$$

Then the optimal comprehensive weight ω^* is obtained.

$$\omega^* = \sum_{i=1}^m \alpha_i^* \omega_i^T \tag{11}$$

3 Case Study

3.1 Project Overview

The overall layout of a double-tower double-cable-plane steel box girder cable-stayed bridge is shown in Figure 2. The semi-floating system is adopted, and the bridge span arrangement is (110+236+458+236+110=1150m). The stay cables are arranged in the middle span and the secondary side span. The main girder adopts streamlined flat steel box girder. The stay cables are arranged in fan form, and the space double cable plane is anchored outside the steel box girder. In a certain inspection of the cable sheath of a cable-stayed bridge, it was found that the protective layer of the sheath was damaged. Among them, there were obvious scratches and scratches on the surface of the sheath, local warping, obvious extrusion marks, slight holes in the local area, shallow depth, and slight fire marks in individual areas of the bridge deck, which did not affect the normal use.

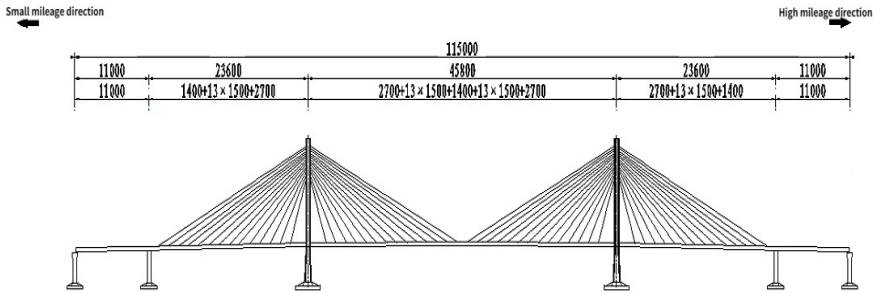


Fig. 2. Overall layout of cable-stayed bridge

3.2 Determination of Cable Sheath Protection Layer Damage Indicators

Matter element to be evaluated refers to the scale level of the cable sheath damage indicators. According to the Evaluation Standard for Technical Condition of Highway Bridges, JTG/T H21-2011^[5], cable sheath protection layer damage indicators are divided into four scale levels. For the sub-evaluation indicators after subdivision, since they are all qualitative indicators, interval evaluation values are given using adjacent scale levels. The optimal intervals for the subdivided cable sheath damage indicators are shown in Table 3. The actual measured values in the table are determined based on the actual condition of the defects combined with the grading standard table.

Table 3. Classical domain, joint domain and measured value of damage index of sheath protective layer

Sub-evaluation index c_i	classical domain				joint domain	measured value
	I	II	III	IV		
Scratch c_1	[0,1)	[1,2)	[2,3)	[3,4]	[0,4]	2.8
Scratch mark c_2	[0,1)	[1,2)	[2,3)	[3,4]	[0,4]	2.5
Compression deformation c_3	[0,1)	[1,2)	[2,3)	[3,4]	[0,4]	2.2
Cavern c_4	[0,1)	[1,2)	[2,3)	[3,4]	[0,4]	1.5
Special damage c_5	[0,1)	[1,2)	[2,3)	[3,4]	[0,4]	1.2

The expressions of classical domain, nodal domain and matter element in the damage index of cable protective layer are as follows :

$$R_1 = \begin{bmatrix} N_1 & c_1 & (0,1) \\ & c_2 & (0,1) \\ & c_3 & (0,1) \\ & c_4 & (0,1) \\ & c_5 & (0,1) \end{bmatrix} \quad R_2 = \begin{bmatrix} N_2 & c_1 & (1,2) \\ & c_2 & (1,2) \\ & c_3 & (1,2) \\ & c_4 & (1,2) \\ & c_5 & (1,2) \end{bmatrix} \quad R_3 = \begin{bmatrix} N_3 & c_1 & (2,3) \\ & c_2 & (2,3) \\ & c_3 & (2,3) \\ & c_4 & (2,3) \\ & c_5 & (2,3) \end{bmatrix}$$

$$R_4 = \begin{bmatrix} N_4 & c_1 & (3,4) \\ & c_2 & (3,4) \\ & c_3 & (3,4) \\ & c_4 & (3,4) \\ & c_5 & (3,4) \end{bmatrix} \quad R_p = \begin{bmatrix} P & c_1 & (0,4) \\ & c_2 & (0,4) \\ & c_3 & (0,4) \\ & c_4 & (0,4) \\ & c_5 & (0,4) \end{bmatrix} \quad R_T = \begin{bmatrix} T & c_1 & (2.8) \\ & c_2 & (2.5) \\ & c_3 & (2.2) \\ & c_4 & (1.5) \\ & c_5 & (1.2) \end{bmatrix}$$

3.3 Calculation of Damage Indicator Correlation

According to the classical domain and the nodal domain determined above, the correlation degree of each sub-evaluation index with respect to each scale is calculated. The larger the correlation value, the closer the representative sub-evaluation index is to the scale. The scale corresponding to the maximum correlation value is the level of the sub-evaluation index. The calculation results are shown in Table 4.

Table 4. Evaluation index correlation degree

Sub-evaluation index c_i	I	II	III	IV	grade
Scratch c_1	-0.6	-0.4	0.2	-0.1429	III
Scratch mark c_2	-0.5	-0.25	0.5	-0.25	III
Compressional deformation c_3	-0.4	-0.1	0.2	-0.3077	III
Cavern c_4	-0.25	0.5	-0.25	-0.5	II
Special damage c_5	-0.1429	0.2	-0.4	-0.6	II

Through the maximum correlation degree of each sub-evaluation index in Table 4, it can be seen that the scales of the five sub-evaluation indexes are 3,3,3,2,2, respectively, which is consistent with the previous scale determined by the sub-evaluation index grading evaluation table. It can be seen that it is reasonable to determine the scale of the evaluation index by the extension matter-element model.

3.4 Determination of Comprehensive Rating Scale for Element Evaluation Indicators

The subjective weight of the sub-evaluation index relative to the evaluation index can be obtained by the above-mentioned uncertain analytic hierarchy process, that is

$$\alpha_1 = (0.0382, 0.0958, 0.1729, 0.3180, 0.3751) \tag{12}$$

According to the correlation degree of Table 4, the objective weight obtained by entropy weight method is :

$$\alpha_2 = (0.2124, 0.2034, 0.1992, 0.1934, 0.1916) \tag{13}$$

Obtained by formula (6), normalized by formula (7), the optimal distribution ratio coefficient of subjective and objective weight can be obtained:

$$\alpha_1^* = 0.9268, \alpha_2^* = 0.0732 \tag{14}$$

Finally, according to formula (8), the optimal comprehensive weight of each index can be obtained:

$$\omega^* = (0.0509, 0.1037, 0.1748, 0.3089, 0.3617) \tag{15}$$

Using formula (9) to calculate the comprehensive correlation degree:

$$\begin{aligned}
 K_j(P_0) &= \sum_{i=1}^n \omega_i K_j(x_i) \\
 &= (0.0509, 0.1037, 0.1748, 0.3089, 0.3617) \begin{pmatrix} -0.6 & -0.4 & 0.2 & -0.1429 \\ -0.5 & -0.25 & 0.5 & -0.25 \\ -0.4 & -0.1 & 0.2 & -0.3077 \\ -0.25 & 0.5 & -0.25 & -0.5 \\ -0.1429 & 0.2 & -0.4 & -0.6 \end{pmatrix} \\
 &= (-0.2812, 0.1630, -0.1249, -0.4585)
 \end{aligned} \tag{16}$$

Therefore $\max_{j \in \{1,2,\dots,m\}} K_j(x_i) = 0.1630$, the damage index scale of the protective layer of the cable sheath member of the cable-stayed bridge is 2.

4 Conclusion

(1) This study introduces the uncertain analytic hierarchy process(UAHP) to determine subjective weights of indicators and utilizes the entropy weight method to determine objective weights of indicators. By avoiding the use of a single weighting method that may result in excessive subjectivity or objectivity, the proposed approach provides a more balanced weight allocation. Furthermore, by incorporating the physical element extension model, the method presents a way to determine the scale levels from sub-evaluation indicators to overall evaluation indicators.

(2) Taking the cable sheath protection layer damage indicators as an example, the maximum correlation values obtained from the calculations indicate the scale levels for the five sub-evaluation indicators to be 3, 3, 3, 2, 2, respectively. This aligns with the scale levels determined through the sub-evaluation indicator grading table. Hence, it can be concluded that using the physical element extension model to determine the scale levels of evaluation indicators is reasonable, thus validating the rationality of this method in calculating comprehensive indicator rankings.

Acknowledgments

This is supported by The Science and Technology Project of Shandong Department of Transportation (2021B51).

Reference

1. ADMINISTRATION F H, PART C. National Bridge Inspection Standards[J]. 2004.
2. X Y Ma. Integration and development of bridge technical condition assessment method, degradation model and inspection maintenance management system [D]. Chang'an University, 2016.

3. G Y Zhang. Legislation and implementation of bridge inspection in the United States [J]. Shanghai Highways, 2011.
4. Y Q Zhang. On the fuzzy grading standard of bridge evaluation at home and abroad [J]. Communications Standardization, 2005(10): 58-61.
5. Research Institute of Highway Ministry of Transport. Standards for Technical Condition Evaluation of Highway Bridges [M]. Standards for Technical Condition Evaluation of Highway Bridges, 2011.
6. X Xiao. Technical condition assessment of railway bridge based on analytic hierarchy process [J]. Railway Engineering, 2020(10): 46-50.
7. X Xu, Q Huang, Y Ren, et al. Weight determination of suspension bridge state evaluation index based on group AHP [J]. Journal of Hunan University(Natural Sciences), 2018(03): 122-128.
8. [8] L Liang, G H Xing, F Y Wu. Application of double base point iterative weighting method in bridge engineering [J]. Journal of Northeastern University (Natural Science), 2019(05): 740-744.
9. X Xu, X Y Lin, Z G Qing. C-AHP rating system for routine general inspection of long-span suspension bridges[J]. Structure and Infrastructure Engineering, 2023,19(5).
10. Y Yang, J Peng, C S Cai, et al. Improved Interval Evidence Theory-Based Fuzzy AHP Approach for Comprehensive Condition Assessment of Long-Span PSC Continuous Box-Girder Bridges[J]. Journal of Bridge Engineering, 2019,24(12).
11. Moufti S A, Zayed T, Dabous S A. Fuzzy defect based condition assessment of concrete bridges: Ifsa World Congress & Nafips Meeting, 2013[C].
12. Y Zhang, J Liu, P Liang, et al. Comprehensive evaluation of bridge detection index based on entropy weight extension matter-element model [J]. Journal of Chang'an University(Natural Science Edition), 2022(06): 42-52.
13. P Zhang, ZHONG S, ZHU R, et al. Technical status assessment of stone arch bridge based on entropy weight method and cloud model [J]. Journal of Zhengzhou University(Engineering Science), 2022(01): 69-75.
14. Su H, Guo C, Han T, et al. Research on Safety State Evaluation of Cable-Stayed Bridge Structures across the Sea [J]. Journal of Marine Science and Engineering, 2023, 11 (11).
15. S B XU. Introduction to Analytic hierarchy process [M]. 1990.
16. G C Li. Fuzzy comprehensive evaluation system of mine ventilation index in Dongqu mine [D]. Taiyuan: Taiyuan University of Technology, 2004.

Open Access This chapter is licensed under the terms of the Creative Commons Attribution-NonCommercial 4.0 International License (<http://creativecommons.org/licenses/by-nc/4.0/>), which permits any noncommercial use, sharing, adaptation, distribution and reproduction in any medium or format, as long as you give appropriate credit to the original author(s) and the source, provide a link to the Creative Commons license and indicate if changes were made.

The images or other third party material in this chapter are included in the chapter's Creative Commons license, unless indicated otherwise in a credit line to the material. If material is not included in the chapter's Creative Commons license and your intended use is not permitted by statutory regulation or exceeds the permitted use, you will need to obtain permission directly from the copyright holder.

