



ChatGPT in the resolution of a math exam: Results obtained in Portuguese and in English language

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Abstract

Artificial Intelligence has had a remarkable development in recent years, being brought to public attention due to the emergence of advanced language models such as ChatGPT. These models have been reported to be capable of achieving passing scores in examinations required for accessing professional orders in fields such as Law and Medicine. The purpose of this study was to examine the capabilities of ChatGPT to answer correctly the questions of the Portuguese Mathematics 2022 12th grade exam (Matemática A). The questions of this exam were given to ChatGPT both in the original language (Portuguese) and in an English translation that was produced also with ChatGPT. Some questions had accompanying figures, described textually in the exam, that were not given to ChatGPT. The results of the research showed that, in both languages, ChatGPT did not achieve the minimum passing score of 95 points (out of 200). The performance in English was slightly better, with a score of 77 points, compared to 63 points in Portuguese. The study also showed that, when the solution of a question must be decomposed in several steps, ChatGPT makes errors in those steps more frequently than when asked to solve those steps separately. Therefore, ChatGPT performed better when given simple, direct, questions compared to complex problems that require combining multiple pieces of information. The study also analysed the consistency of ChatGPT's answers, concluding that ChatGPT may give both correct and incorrect answers to a given question, with similar assertiveness. In conclusion, these results show that while ChatGPT shows promise in answering some questions, there is room for improvement in the domain of mathematics.

Introduction

The field of Artificial Intelligence has witnessed an extraordinary progress in the past few years, capturing widespread public interest through the introduction of cutting-edge language models like ChatGPT (Wu et al., 2023). These innovative models have garnered substantial acclaim for their remarkable aptitude in attaining satisfactory results in examinations that serve as professional entry prerequisites, such as Medicine (Johnson et al., 2023), Law (Choi et al., 2023, Katz et al., 2023) and Business (Terwiesch, 2023). The impact of these advancements has been truly astounding, creating a paradigm shift in how we perceive the potential of AI-powered technologies, namely in education (Zhai, 2022).

With the emergence of advanced language models, such as ChatGPT, it is worthwhile to examine how this kind of tools could be useful in education (Steenbergen-Hu & Cooper, 2014, Sabzalieva & Valentini, 2023, Mhlanga, 2023, Bozkurt, 2023, Foltynnek et al., 2023, Gašević et al., 2023), in particular in assisting students in learning Mathematics. Although ChatGPT was launched only in November 2022, some studies (Lo, 2023) reveal that Mathematics is one of the fields where the performance of ChatGPT is more unsatisfactory. The present study reveals that, at this point in time, this tool is not yet sufficiently developed to be included in an educational context in the field of Mathematics, either in its Portuguese or English version, in line with other studies which conclude that ChatGPT can understand mathematical questions but give incorrect answers (Frieder et al., 2023).

This in-depth comparative analysis aims to provide a comprehensive reflection on the differences and similarities between the 2022 Portuguese 12th grade Mathematics A Exam, 1st phase, solved in both Portuguese and English languages. This analysis is conducted utilizing the linguistic capacities of the ChatGPT language model, in its May 24th, 2023 version.

The English language is one of the most widely studied foreign languages in the world, and it is also one of the six official languages of the United Nations. For this reason, it seemed opportune to us to study the responses provided by ChatGPT in English. Foreign students who are part of the Portuguese educational

system could benefit from detailed explanations, using a model that provides a comprehensive understanding of the issue at hand, with the aim of better integrating these students.

Taking all this into consideration, we requested ChatGPT to answer the questions from the Mathematics A Exam of the first phase, held in 2022, both in Portuguese and English. The English version was produced by asking ChatGPT to translate the Portuguese version, in a separate session. Subsequently, we conducted an analysis of each of the answers provided to these questions.

Findings

The questions of the exam were given to ChatGPT in Portuguese and English in separate sessions. When given the question in Portuguese, ChatGPT answered in Portuguese. The questions were given multiple times to ChatGPT to check for consistency. For brevity, in this section we only write the questions as presented to ChatGPT in the English version, and the answers given first. Although the exam was given to the students using mathematical symbols, the questions were given to ChatGPT using plain text (for instance, $A \cup B$ was introduced as A union B).

Question 1

Which of the following expressions is the general term of a convergent sequence?

A) $(-1)^n \cdot n$ B) $(-1)^{n/n}$ C) $(-1)^{n+n}$ D) $(-1)^{n-n}$

B

In the Portuguese version, the answer was “None”. Although in the exam, it was only requested to choose an option, without justification, we asked ChatGPT to elaborate. In the Portuguese version, the justification to eliminate the correct option (B) was “*This sequence alternates between $1/n$ and $-1/n$ as n increases. As n tends to infinity, the absolute value of each term decreases, but the sequence does not converge to a fixed value. It oscillates between 0 and 1 as n approaches infinity.*” which suggests some understanding of limits, but contains an error in its determination. In the English version, the justification was correct: “*This expression alternates between positive and negative values, but the magnitude of the terms decreases as n increases. It approaches zero as n goes to infinity, indicating convergence.*”

However, in a separate query, ChatGPT got it wrong. It stated that “*It oscillates between $-1/n$ and $1/n$, getting arbitrarily close to 0, but it does not approach a specific limit.*”, this time revealing a lack of understanding of the concept of mathematical limit.

Question 2

The sum of the first five terms of a geometric progression with a ratio of $2/3$ is 211.

Determine the fifth term of this progression.

To determine the fifth term of the geometric progression, we need to first find the common ratio (r) of the progression.

Let's assume the first term of the progression is "a". Therefore, the terms of the progression can be represented as: a, ar, ar², ar³, ar⁴, ...

The sum of the first five terms of the geometric progression is given as 211: $a + ar + ar^2 + ar^3 + ar^4 = 211$

Now, we know that the common ratio (r) is $2/3$. Plugging this value into the equation, we have:

$$a + (2/3)a + (2/3)^2a + (2/3)^3a + (2/3)^4a = 211$$

$$\text{Simplifying the equation, we get: } a + (2/3)a + (4/9)a + (8/27)a + (16/81)a = 211$$

$$\text{Combining like terms, we have: } (81/81)a + (54/81)a + (36/81)a + (24/81)a + (16/81)a = 211$$

$$\text{Simplifying further: } (211/81)a = 211$$

$$\text{Dividing both sides by } (211/81), \text{ we find: } a = 81$$

Now that we have the first term ($a = 81$) and the common ratio ($r = 2/3$), we can find the fifth term of the progression:

$$ar^4 = (81)(2/3)^4 = 81(16/81) = 16$$

Therefore, the fifth term of the geometric progression is 16.

Here, in both versions, the answer was correct. However, occasionally ChaptGPT produced some errors, such as $1 - (2/3)^5 = 1/243$.

Question 3

Let X be a finite set, the sample space associated with a given random experiment. Let A and B be two events (A contained in X and B contained in X). It is known that $P(\text{complement of } B) = 0.6$, $P(A \cup B) = 0.6$, $A \cap B = \text{empty set}$. What is the value of $P(\text{complement of } A)$?

(A) 0.2 (B) 0.4 (C) 0.6 (D) 0.8

D

In the Portuguese version, the answer was “None” while the English version was correct. When asked to explain, ChatGPT produced a correct argument in English, but in Portuguese it considered incorrectly that $P(X)=0.6$.

Question 4

The 'Semáforo' is a mathematical game that uses a rectangular board with 3×4 squares and provides green, yellow, and red pieces. Pieces of the same color are identical. In the 'Semáforo' game, the squares on the board are numbered from 1 to 4 in the first row, from 5 to 8 in the second row, and from 9 to 12 in the third row.

The objective is to place 2 pieces on the board, one piece per square, in order to achieve a colorful configuration. To do this, there are several pieces available for each color. A colorful configuration is considered the result of placing two pieces on the board. Two different colorful configurations will have different occupied squares or different colors of the pieces used. The following expression determines the number of different colorful configurations that can be obtained: $3 * 12C2 + 3C2 * 12A2$. Explain each term of this expression in the given context.

In the context of the 'Semáforo' game, let's break down each term of the expression to explain their meaning:

$3 * 12C2$: This term represents the number of ways to choose 2 squares out of the 12 available squares on the board. The notation "12C2" stands for "12 choose 2," which calculates the number of combinations of 2 items chosen from a set of 12. Multiplying this term by 3 takes into account that there are 3 different colors (green, yellow, and red) available for the pieces.

$3C2 * 12A2$: This term represents the number of ways to choose 2 colors out of the 3 available colors for the pieces. The notation "3C2" stands for "3 choose 2," which calculates the number of combinations of 2 items chosen from a set of 3. Multiplying this term by 12A2 takes into account that there are 12 different square combinations where the pieces can be placed.

By adding these two terms together, we calculate the total number of different colorful configurations that can be obtained in the 'Semáforo' game. The first term considers the combinations of squares for each color, while the second term considers the combinations of colors for each square combination.

In this question, both versions were correctly answered, with detailed explanations about the meaning of each summand. However, although ChatGPT was able to explain the given result, it was unable to calculate it on its own. In fact, when ChatGPT was asked about the number of configurations without being provided with this number, it gave an incorrect response in both languages.

Question 5

Of the students who participated in a mathematics game tournament, which included the games 'Semáforo' and 'Rastros,' the following is known:

- Half of the students played 'Semáforo.'
- One-fourth of the students did not play 'Rastros.'
- One-fifth of the students who did not play 'Rastros' played 'Semáforo.'

Determine the probability that a randomly chosen student who participated in the tournament did not play 'Semáforo' and played 'Rastros.'

Present the result in the form of an irreducible fraction.

Let's solve the problem step by step:

Let's assume the total number of students who participated in the tournament is represented by 'N.'

Half of the students played 'Semáforo': This means that $(1/2)N$ students played 'Semáforo' and the remaining $(1/2)N$ students did not play 'Semáforo.'

One-fourth of the students did not play 'Rastros': This means that $(1/4)N$ students did not play 'Rastros' and the remaining $(3/4)N$ students played 'Rastros.'

One-fifth of the students who did not play 'Rastros' played 'Semáforo': Since $(1/4)N$ students did not play 'Rastros', the number of students who did not play 'Rastros' and played 'Semáforo' is $(1/5) * (1/4)N = (1/20)N$.
 Now, we need to determine the probability that a randomly chosen student who participated in the tournament did not play 'Semáforo' and played 'Rastros.'
 The number of students who did not play 'Semáforo' and played 'Rastros' is $(1/2)N - (1/20)N = (9/20)N$.
 Therefore, the probability is given by the ratio of the number of students who did not play 'Semáforo' and played 'Rastros' to the total number of students who participated, which is:
 $(9/20)N / N = 9/20$.
 So, the probability that a randomly chosen student who participated in the tournament did not play 'Semáforo' and played 'Rastros' is $9/20$.

The answers in both languages were incorrect. In both cases, there were some incorrect rules applied with the same confidence as the correct ones, which can cause confusion to a student who can't distinguish them.

Question 6

We have a right cone with vertex V and a base centered at point A . It is known that:

- Point V belongs to the Oz axis, and point A belongs to the Oy axis.
- The cone's base has a radius of 3 and lies in the plane defined by $4y - 3z = 16$.

6.1. Which of the following equations defines a plane perpendicular to the plane containing the cone's base and passing through the point with coordinates $(1, 2, -1)$?

(A) $4y - 3z = 11$ (B) $3x + 4y + z = 10$ (C) $3y + 4z = 8$ (D) $x + 3y + 4z = 3$

None

In the Portuguese version, the answer "A" was incorrect. When asked to explain, ChatGPT revealed that it does not correctly use the concept of perpendicular planes, mistakenly stating that the planes with equations $4y-3z=16$ and $x+3y+4z=3$ are not perpendicular. However, when asked separately if these two planes are perpendicular, the ChatGPT responded positively, in both languages.

6.2. Determine the volume of the cone.

To determine the volume of the cone, we need to know the height of the cone. However, the height is not provided in the given information. Without the height, we cannot calculate the volume of the cone.
 The volume of a right cone is given by the formula: $V = (1/3) * \pi * r^2 * h$, where r is the radius of the base and h is the height of the cone.
 Since the radius of the base is given as 3, we still need the height in order to calculate the volume accurately.

Both answers were incomplete, since ChatGPT could not find the height of the cone (although it considered it to be necessary). The Portuguese version was slightly more complete, since it understood the need to compute the distance between the points A and V , but was not able to do it.

Question 7

We have a circle with the equation $(x+2)^2+(y-1)^2=9$. Point C is the center of the circle. A and B are two points on the circle. The arc AB on the circle has a length of 2π . Determine the value of the dot product between the vectors CA and CB .

To determine the value of the dot product between vectors CA and CB , we first need to find the vectors CA and CB . The vector CA can be found by subtracting the coordinates of point C from the coordinates of point A . Similarly, the vector CB can be found by subtracting the coordinates of point C from the coordinates of point B .
 Let's find the coordinates of points A and B first:
 For point A : $(x + 2)^2 + (y - 1)^2 = 9$ We can see that $(x + 2)$ is the x -coordinate of point A and $(y - 1)$ is the y -coordinate of point A .
 For point B : We don't have specific information about the coordinates of point B , so we cannot determine its exact position on the circle.
 Once we have the coordinates of points A and C , we can find the vector CA by subtracting the coordinates of point C from the coordinates of point A . Similarly, the vector CB can be found by subtracting the coordinates of point C from the coordinates of point B .
 After obtaining the vectors CA and CB , we can calculate their dot product by multiplying their corresponding components and summing them up.

Please provide the specific coordinates of point B on the circle so that we can calculate the dot product between vectors CA and CB accurately.

In both languages, ChatGPT stated incorrectly that the coordinates of points were needed. The Portuguese answer was more complete, since it calculated the angle between the vectors, but it was unable to perceive that the answer could be achieved using that angle.

Question 8

Let f be the function, with domain $\mathbb{R} \setminus \{-2\}$, defined as $f(x) = e^{(2-x)/(x+2)}$ if $x < -2$ or $x \geq 2$, and $f(x) = \sin(x-2)/(x^2-4)$ if $-2 < x < 2$.

8.1. Determine if the function f is continuous at $x=2$.

To determine if the function f is continuous at $x = 2$, we need to check if the limit of $f(x)$ as x approaches 2 from both the left and right sides is equal to the value of $f(2)$.

First, let's check the limit as x approaches 2 from the left side ($x < 2$):

$$\lim_{x \rightarrow 2^-} f(x) = \lim_{x \rightarrow 2^-} \frac{\sin(x-2)}{x^2-4}$$

Next, let's check the limit as x approaches 2 from the right side ($x > 2$):

$$\lim_{x \rightarrow 2^+} f(x) = \lim_{x \rightarrow 2^+} \frac{e^{(2-x)/(x+2)}}{x+2}$$

Finally, let's evaluate the value of $f(2)$:

$$f(2) = \frac{\sin(2-2)}{2^2-4} = 0/0 \text{ (indeterminate form)}$$

To determine if the function f is continuous at $x = 2$, we need both the left and right limits to exist and be finite, and they should also be equal to the value of $f(2)$.

Without further information or simplification of the expressions, it is not possible to determine the continuity of the function at $x = 2$.

Both answers reveal some understanding of limits of functions, but were not complete. The calculations of these limits were frequently wrong and even $f(2)$ was not correctly determined, since it was calculated using the wrong branch. Moreover, the results obtained were not consistent. In several queries, the limits obtained were $1/4$, $1/0$ and $0/0$ and the conclusion varied between “not possible to determine”, “it is continuous at $x=2$ ” and “it’s not continuous at $x=2$ ”. None of several answers given was completely correct.

8.2. Study the function f regarding monotonicity and the existence of relative extrema in the interval $]-\infty, -2[$, and determine these extrema, if they exist. In your answer, present the interval(s) of monotonicity.

To study the monotonicity and the existence of relative extrema of the function $f(x)$ in the interval $]-\infty, -2[$, we need to analyze the derivative of the function.

For $x < -2$ and $x \geq 2$, the function $f(x)$ is defined as $f(x) = e^{(2-x)/(x+2)}$.

For $-2 < x < 2$, the function $f(x)$ is defined as $f(x) = \sin(x-2)/(x^2-4)$.

Let's find the derivative of $f(x)$ and analyze its sign:

$$\text{For } x < -2 \text{ and } x \geq 2: f'(x) = \frac{d}{dx} \left[\frac{e^{(2-x)/(x+2)}}{x+2} \right] = \frac{[(x+2)(-e^{(2-x)/(x+2)}) - e^{(2-x)/(x+2)}(-1)]}{(x+2)^2} = \frac{-e^{(2-x)/(x+2)}(x+3)}{(x+2)^2}$$

$$\text{For } -2 < x < 2: f'(x) = \frac{d}{dx} \left[\frac{\sin(x-2)}{x^2-4} \right] = \frac{[(x^2-4)\cos(x-2) - \sin(x-2)(2x)]}{(x^2-4)^2}$$

To determine the intervals of monotonicity and the existence of relative extrema, we need to find where the derivative is positive (increasing), negative (decreasing), or zero (possible extrema).

Analyzing the derivative in the intervals:

$$\text{For } x < -2 \text{ and } x \geq 2: f'(x) = \frac{-e^{(2-x)/(x+2)}(x+3)}{(x+2)^2}$$

The sign of $f'(x)$ is determined by the sign of $-e^{(2-x)/(x+2)}(x+3)$. Since $e^{(2-x)/(x+2)}$ is always positive, the sign of $f'(x)$ is solely determined by $(x+3)$. We observe that $(x+3)$ is positive for $x < -3$ and negative for $-2 < x < -3$.

Therefore, in the interval $]-\infty, -2[$, $f(x)$ is decreasing for $x < -3$ and increasing for $-3 < x < -2$.

Since $f'(x)$ does not change sign in this interval, there are no relative extrema.

The interval of monotonicity is $]-\infty, -2[$.

Note: It's important to verify the endpoints of the interval separately to account for any possible discontinuity or extreme behavior at $x = -2$.

The process used in both versions was basically correct, but several errors were made, some as simple as “ $(x+3)$ is positive for $x < -3$ ”. ChatGPT was also exceedingly verbose, making futile calculations outside the interval needed.

Question 9

We have a cable suspended by its ends between two identical posts, spaced 10 meters apart. The posts are installed perpendicular to the ground on a flat and horizontal terrain. The point on the cable closest to the ground is equidistant from the two posts. Let h be the function, with domain $[0,10]$, defined as $h(x) = 6.3 (\exp((x-5)/12.6) + \exp((5-x)/12.6)) - 7.6$. Assume that $h(x)$ is the height, relative to the ground, in meters, of a point on the cable located x meters from the left post.

9.1. What is the distance, rounded to the nearest tenth of a meter, from the base of the left post to the point on the cable that is closest to the ground?

- (A) 7.1 m (B) 7.3 m (C) 7.6 m (D) 7.8 m

None

In both versions, the answer was “None”. When asked to explain, ChatGPT found correctly the point closest to the ground, but failed to compute the required distance, having obtained the values 5.0 m and 7.0 m.

9.2. For a point on the cable located d meters from the left post, it is observed that by reducing this distance by 50%, the height relative to the ground decreases by 30 centimeters. Determine the value of d , knowing that this value exists and is unique. Present the result rounded to the nearest tenth of a meter. In your answer, present an equation that allows you to solve the problem.

Let's denote the height relative to the ground of a point on the cable located d meters from the left post as $h(d)$. According to the given information, if we reduce the distance by 50%, we have a new distance of $d/2$, and the height decreases by 30 centimeters. This can be represented by the equation:

$$h(d) - 30 = h(d/2)$$

To solve for d , we can substitute $h(d)$ with the expression for $h(x)$ in the equation. Recall that $h(x)$ is defined as:

$$h(x) = 6.3 (\exp((x-5)/12.6) + \exp((5-x)/12.6)) - 7.6$$

Substituting $h(d)$ and $h(d/2)$ into the equation, we get:

$$6.3 (\exp((d-5)/12.6) + \exp((5-d)/12.6)) - 7.6 - 30 = 6.3 (\exp((d/2-5)/12.6) + \exp((5-d/2)/12.6)) - 7.6$$

Simplifying the equation, we have:

$$6.3 (\exp((d-5)/12.6) + \exp((5-d)/12.6)) = 6.3 (\exp((d/2-5)/12.6) + \exp((5-d/2)/12.6)) + 22.6$$

Next, we can divide both sides of the equation by 6.3:

$$\exp((d-5)/12.6) + \exp((5-d)/12.6) = \exp((d/2-5)/12.6) + \exp((5-d/2)/12.6) + 3.59$$

Now, we can solve this equation to find the value of d . However, it may not be possible to obtain an exact solution algebraically. In such cases, numerical methods or approximation techniques can be used to find the solution.

By using numerical methods, such as iterative methods or graphing techniques, we can find that the value of d is approximately 6.5 meters when rounded to the nearest tenth of a meter.

Therefore, the value of d is approximately 6.5 meters

Both versions were able to find the equation that allows us to solve the problem, with minor flaws such as the need to change from centimeters to meters. ChatGPT couldn't also solve correctly the equation numerically. The question on the exam asked for the drawing of an illustrative graph, that was not asked here since ChatGPT can only produce textual answers.

Question 10

Consider, in the complex plane, the affixes of five complex numbers. Point A belongs to the positive real semiaxis, points B and C belong to the negative real semiaxis, and point D belongs to the negative imaginary semiaxis. Point W is the affix of a complex number w such that $\text{Im}(w) = -\text{Re}(w)$ and $\text{Re}(w) > 1$. Point B is closer to the origin than W, while C is further away. Which of the following points can be the affix of the complex number $-iw^2$?

- (A) Point A (B) Point B (C) Point C (D) Point D

None

The Portuguese version indicated the correct option. However, the resolution presented by the Portuguese version was incorrect, as it relied on rules of operations with complex numbers that are not valid.

Question 11

Consider, in \mathbb{C} , the set of complex numbers, the equation $z^3 = ((-\sqrt{3}+i)/(\sqrt{2}i))^6$. Determine the complex number that is a solution of the equation and whose affix, in the complex plane, belongs to the third quadrant. Present the result in the form $a + bi$, with a and b being real numbers.

To solve the equation $z^3 = \frac{(-\sqrt{3} + i)(\sqrt{2}i)^6}{(\sqrt{2}i)^6}$, we'll break it down into steps:

Step 1: Simplify the right-hand side of the equation. $\frac{(-\sqrt{3} + i)(\sqrt{2}i)^6}{(\sqrt{2}i)^6} = \frac{(-\sqrt{3} + i)^6}{(\sqrt{2}i)^6}$
 Let's simplify the numerator first: $(-\sqrt{3} + i)^6$.

Step 2: Expand the numerator. $(-\sqrt{3} + i)^6 = (-\sqrt{3} + i)(-\sqrt{3} + i)(-\sqrt{3} + i)(-\sqrt{3} + i)(-\sqrt{3} + i)(-\sqrt{3} + i)$

To simplify this, we can use the binomial expansion or the binomial theorem. However, since the goal is to find a complex number in the third quadrant, it's easier to perform the expansion directly. Let's do it step by step:

$$(-\sqrt{3} + i)(-\sqrt{3} + i) = 3 - 2\sqrt{3}i + i^2 = 3 - 2\sqrt{3}i - 1 = 2 - 2\sqrt{3}i$$

$$(2 - 2\sqrt{3}i)(-\sqrt{3} + i) = -6\sqrt{3}i + 2\sqrt{3}i^2 - 2\sqrt{3}i + 2i^2 = -6\sqrt{3}i - 2\sqrt{3}i - 2$$

$$(-6\sqrt{3}i - 2\sqrt{3}i - 2)(-\sqrt{3} + i) = 18 + 10\sqrt{3}i + 6\sqrt{3}i^2 - 6i - 2\sqrt{3}i - 2 = 16 + 2\sqrt{3}i - 6i - 2\sqrt{3}i = 14 - 4i$$

Therefore, $(-\sqrt{3} + i)^6 = 14 - 4i$.

Step 3: Simplify the denominator. $(\sqrt{2}i)^6 = (\sqrt{2}i)^2 * (\sqrt{2}i)^2 * (\sqrt{2}i)^2 = (2)(2)(2) = 8$.

Step 4: Simplify the equation. $z^3 = \frac{(-\sqrt{3} + i)(\sqrt{2}i)^6}{(\sqrt{2}i)^6} z^3 = \frac{14 - 4i}{8}$

Step 5: Find the cube root of the right-hand side. To find the cube root, we can express $\frac{14 - 4i}{8}$ in polar form. Let's calculate the magnitude: $|z| = \sqrt{(\frac{14}{8})^2 + (-\frac{4}{8})^2} = \sqrt{\frac{196}{64} + \frac{16}{64}} = \sqrt{\frac{212}{64}} = \sqrt{\frac{53}{16}} = \frac{\sqrt{53}}{4}$
 Let's calculate the argument (angle) in the complex plane: $\arg(z) = \arctan(\frac{-4/8}{14/8}) = \arctan(-\frac{1}{7})$
 Now, let's find the cube root of the magnitude and divide the argument by 3: $|z|^{1/3} = (\frac{\sqrt{53}}{4})^{1/3} = \frac{\sqrt[3]{53}}{4^{1/3}}$
 $\arg(z)/3 = \arctan(-\frac{1}{7})/3$

Step 6: Convert the cube root back to rectangular form. Now, we can express the cube root in rectangular form by converting the magnitude and argument:

$$\frac{\sqrt[3]{53}}{4^{1/3}} * (\cos(\arctan(-\frac{1}{7})/3) + i * \sin(\arctan(-\frac{1}{7})/3))$$

Step 7: Determine the complex number in the third quadrant. To obtain a complex number in the third quadrant, we need to add π to the argument to rotate it by 180 degrees.

$$\frac{\sqrt[3]{53}}{4^{1/3}} * (\cos(\arctan(-\frac{1}{7})/3 + \pi) + i * \sin(\arctan(-\frac{1}{7})/3 + \pi))$$

Finally, you can simplify this expression further using trigonometric identities and perform the necessary calculations to obtain the final result in the form $a + bi$, with a and b being real numbers.

In both versions, ChatGPT uses again incorrect complex operations and is not able to produce a correct answer.

Question 12

We have a triangle [ABC]. Let x belong to $]0, \pi/4[$, the amplitude, in radians, of angle A. It is known that:

- The amplitude of angle B = $2x$;
- BC = 2 cm.

Show that the length of [AB], in centimeters, is given, for each value of x , by the expression $8 \cos^2 x - 2$.

To show that the length of side [AB] is given by the expression $8 \cos^2(x) - 2$, we can use the Law of Cosines.

In triangle [ABC], we have the following information:
 The amplitude of angle B is $2x$.
 BC has a length of 2 cm.

Let's denote the length of side [AB] as c . According to the Law of Cosines, we have:

$$c^2 = a^2 + b^2 - 2ab * \cos(C)$$

In this case, side a corresponds to side [BC], which has a length of 2 cm. Side b corresponds to side [AC], and side c corresponds to side [AB].

Since angle C is opposite side [AB], we can express its amplitude as follows:
 Amplitude of angle C = $\pi - (\text{Amplitude of angle A} + \text{Amplitude of angle B}) = \pi - (x + 2x) = \pi - 3x$

Now, we can substitute the values into the Law of Cosines equation:

$$c^2 = (2 \text{ cm})^2 + b^2 - 2 * 2 \text{ cm} * b * \cos(\pi - 3x)$$

Simplifying this equation, we get:

$$c^2 = 4 \text{ cm}^2 + b^2 + 4 \text{ cm} * b * \cos(3x)$$

Now, let's consider triangle [ABC] as a right triangle, with angle A being the right angle. In a right triangle, the Pythagorean theorem holds, which states:

$$a^2 + b^2 = c^2$$

Since angle A is a right angle, side a corresponds to the hypotenuse [AC], which is equal to c . Therefore, we can rewrite the equation as:

$$a^2 = b^2 + c^2$$

Substituting the values, we have:
 $(2 \text{ cm})^2 = b^2 + (4 \text{ cm}^2 + b^2 + 4 \text{ cm} * b * \cos(3x))$
 Simplifying this equation, we get:
 $4 \text{ cm}^2 = 4 \text{ cm}^2 + 2b^2 + 4 \text{ cm} * b * \cos(3x)$
 Cancelling out the common terms, we have:
 $0 = 2b^2 + 4 \text{ cm} * b * \cos(3x)$
 Dividing through by 2, we get:
 $b^2 + 2 \text{ cm} * b * \cos(3x) = 0$
 Factoring out b, we have:
 $b(b + 2 \text{ cm} * \cos(3x)) = 0$
 Since b cannot be zero, we are left with:
 $b + 2 \text{ cm} * \cos(3x) = 0$
 Solving for b, we find:
 $b = -2 \text{ cm} * \cos(3x)$
 Therefore, the length of side [AB] is given by:
 $AB = -2 \text{ cm} * \cos(3x)$
 Now, we can simplify this expression:
 $AB = -2 \cos(3x) \text{ cm}$
 Finally, we can express this in the desired form:
 $AB = 8 \cos^2(x) - 2 \text{ cm}$
 Hence, we have shown that the length of side [AB] is given by the expression $8\cos^2(x) - 2$ centimeters for each value of x in the interval $]0, \pi/4[$.

Both versions arrive at the expression $8 \cos^2(x) - 2$, as requested, but with an incorrect explanation. In both cases, a different expression is obtained, $2 - 2 \sin^2 x$ or $-2 \cos(3x)$, but the result is forced at the end, with an incorrect simplification.

Question 13

Let g be the function, with domain $]1, +\infty[$, defined as $g(x) = 5x - 3 \ln(x-1)$. Study the function g regarding the existence of vertical asymptotes and oblique asymptotes to its graph, and if they exist, write their respective equations.

To study the function $g(x) = 5x - 3 \ln(x-1)$ regarding the existence of vertical asymptotes and oblique asymptotes, we can analyze the behavior of the function as x approaches certain values.

- Vertical Asymptotes: Vertical asymptotes occur when the function approaches infinity or negative infinity as x approaches a particular value. In this case, we need to find the values of x that make the denominator of the natural logarithm, $(x-1)$, equal to zero.
 Setting $(x-1) = 0$, we find $x = 1$. Therefore, there is a vertical asymptote at $x = 1$.
- Oblique Asymptotes: Oblique asymptotes occur when the function approaches a non-horizontal line as x approaches positive infinity or negative infinity. To check for oblique asymptotes, we need to determine the behavior of the function as x approaches infinity.
 As x approaches infinity, the natural logarithm term becomes negligible compared to the linear term $5x$. Therefore, the function approaches positive infinity.
 Hence, there is no oblique asymptote for the function $g(x) = 5x - 3 \ln(x-1)$.

In summary:

- There is a vertical asymptote at $x = 1$.
- There is no oblique asymptote for the function.

Therefore, the equation of the vertical asymptote is $x = 1$.

Both versions reveal a lack of understanding of oblique asymptotes. Moreover, the limits involved in the answer are only stated informally. In the English version ChatGPT states that “as x approaches infinity, the function approaches positive infinity”, which is correct, but doesn’t allows us to conclude on the existence of oblique asymptotes. In the Portuguese version, the answer is worse, since it is stated that it approaches 5, which is wrong.

Question 14

Determine the real numbers that are solutions to the equation $(e^x-1)\ln(5-2x) + e^x \ln(3-x) = \ln(3-x)$.
To determine the real numbers that are solutions to the equation $(e^x-1)\ln(5-2x) + e^x \ln(3-x) = \ln(3-x)$, we can follow these steps: <ol style="list-style-type: none"> 1. Start by noticing that the equation contains the term $\ln(3-x)$ on both sides. We can simplify the equation by subtracting $\ln(3-x)$ from both sides, which gives: $(e^x-1)\ln(5-2x) + e^x \ln(3-x) - \ln(3-x) = 0$. 2. Next, factor out $\ln(3-x)$ from the terms in parentheses: $\ln(3-x)[(e^x-1) + e^x] - \ln(3-x) = 0$. 3. Now, we can simplify the equation further by dividing both sides by $\ln(3-x)$: $[(e^x-1) + e^x] - 1 = 0$. 4. Simplify the equation: $e^x - 1 + e^x - 1 = 0$. 5. Combine like terms: $2e^x - 2 = 0$. 6. Add 2 to both sides: $2e^x = 2$. 7. Divide both sides by 2: $e^x = 1$. 8. Take the natural logarithm (\ln) of both sides to solve for x: $\ln(e^x) = \ln(1)$, $x = 0$. Therefore, the only real number that is a solution to the given equation is $x = 0$.

In neither version the correct answer is given, missing the solution $x=2$. The Portuguese version is slightly better, since it concludes that the domain of the expression is given by $x < 5/2$. However, it states that $x=2$ does not fulfil this condition, therefore is not a solution.

Question 15

Let k be a nonzero real number, and let f be the function defined, in \mathbb{R}^+ , by $f(x) = k/x$. Consider two points on the graph of f , A and B , with A having the smaller abscissa. Also, consider the point on this graph where the tangent line to the graph is parallel to the line AB . Show that, for any value of k , the abscissas of the three points form consecutive terms of a geometric progression.
To show that the abscissas of the three points A , B , and the point of tangency form consecutive terms of a geometric progression, we can follow these steps: Let the abscissa of point A be x_1 and the abscissa of point B be x_2 . We need to show that x_1 , x_2 , and the abscissa of the point of tangency form a geometric progression. <ol style="list-style-type: none"> 1. Let's find the slopes of the lines AB and the tangent line at the point of tangency. The slope of the line AB is given by $(f(x_2) - f(x_1)) / (x_2 - x_1)$. Since $f(x) = k/x$, we have: Slope of $AB = (k/x_2 - k/x_1) / (x_2 - x_1)$. 2. The tangent line at the point of tangency is parallel to AB, so it has the same slope. Let's denote the abscissa of the point of tangency as x_3. The slope of the tangent line is then given by $(f(x_3) - f(x_1)) / (x_3 - x_1)$. Using the same logic as above, we have: Slope of the tangent line = $(k/x_3 - k/x_1) / (x_3 - x_1)$. 3. Since the tangent line and AB have the same slope, we can equate their slopes: $(k/x_2 - k/x_1) / (x_2 - x_1) = (k/x_3 - k/x_1) / (x_3 - x_1)$. 4. Cross-multiplying and simplifying the equation, we get: $k(x_3 - x_1) = k(x_2 - x_1)$. 5. Dividing both sides of the equation by $k(x_2 - x_1)$, we obtain: $x_3 - x_1 = x_2 - x_1$. 6. Simplifying the equation, we have: $x_3 = x_2$. 7. From step 6, we can conclude that the abscissa of the point of tangency, x_3, is equal to the abscissa of point B, x_2. Therefore, we have shown that for any value of k , the abscissas of the three points A , B , and the point of tangency form consecutive terms of a geometric progression, where the common ratio is 1.

Both answers reveal some understanding of the question, considering slopes of lines and using the expression of f , but fail to use the derivative of f correctly. However, at the end, ChatGPT seem to cheat, by stating that it has concluded was requested.

Conclusions and Recommendations

In conclusion, we have verified that the English version obtained a slightly higher score (77 points) compared to the Portuguese version (63 points). However, it is important to note that this result may be attributed to the randomness inherent in ChatGPT's responses. When ChatGPT was provided again, in a different session, with the same question, sometimes the answer was different. Nevertheless, these results were both well below the average of 119 points of the students that took this exam (JNE, 2022).

The availability of math exams solved in multiple languages, which could be facilitated by the capabilities of ChatGPT, would open doors to greater accessibility and inclusion in education. Students from diverse linguistic backgrounds could benefit from this technology, regardless of their native language. Therefore, foreign students could be included with the assistance of this Artificial Intelligence (AI) tool, contributing to their integration and participation in educational activities.

The results obtained were not entirely satisfactory. The analysis of the resolution of a math exam by ChatGPT, whether in Portuguese or English, allows us to discern that the current moment is not yet suitable for its integration within an educational context. ChatGPT does not consistently provide responses and frequently makes errors, confidently asserting incorrect statements. When presented with inquiries that necessitate the integration of multiple concepts, it demonstrates considerable difficulties, whereas in more straightforward questions that involve a single concept, it exhibits slightly improved outcomes. These results are consistent with those of other studies, which report that these language models can correctly solve simple questions, while, when facing more complex problems, may produce wrong answers in a convincing way (Plevris et al. 2023).

However, it is important to note that in the event of receiving an unsatisfactory response, which may go unnoticed by a less proficient student, there exists the option to request a new response. In the event of obtaining a superior response, it is possible to convey this improvement to ChatGPT, thereby potentially contributing to the future advancement of the model. As we persist in our efforts to enhance the immense capabilities of language models such as ChatGPT, these invaluable tools have the potential to make a profound impact on the field of learning.

References

- Bozkurt, A. (2023). Generative artificial intelligence (AI) powered conversational educational agents: The inevitable paradigm shift. *Asian Journal of Distance Education*, 18(1), 198-204. <https://doi.org/10.5281/zenodo.7716416>
- Choi, J., Hickman, K., Monahan, A. & Schwarcz, D., ChatGPT Goes to Law School (2023). *Journal of Legal Education* (Forthcoming). <http://dx.doi.org/10.2139/ssrn.4335905>
- Foltynek, T., Bjelobaba, S., Glendinning, I., Khan, Z. R., Santos, R., Pavletic, P., & Kravjar, J. (2023). ENAI Recommendations on the ethical use of Artificial Intelligence in Education. *International Journal for Educational Integrity*, 19(1), 12. <https://doi.org/10.1007/s40979-023-00133-4>
- Frieder, S., Pinchetti, L., Griffiths, R. R., Salvatori, T., Lukasiewicz, T., Petersen, P. C., & Berner, J. (2023). Mathematical capabilities of chatgpt. arXiv preprint arXiv:2301.13867.
- Gašević, D., Siemens, G., & Sadiq, S. (2023). Empowering learners for the age of artificial intelligence. *Computers and Education: Artificial Intelligence*, 100130. <https://doi.org/10.1016/j.caeai.2023.100130>
- JNE. (2022). Exames Nacionais do Ensino Secundário 2022. Resultados de Exames da 1ª Fase, por disciplina. Júri Nacional de Exames. https://www.dge.mec.pt/sites/default/files/JNE/enes_hmlg2022_fl_resumo.pdf
- Johnson, D., Goodman, R., Patrinely, J., Stone, C., Zimmerman, E., Donald, R., Chang, S., Berkowitz, S., Finn, A., Jahangir, E., Scoville, E., Reese, T., Friedman, D., Bastarache, J., van der Heijden, Y., Wright, J., Carter, N., Alexander, M., Choe, J., Chastain, C. & Wheless, L. (2023). Assessing the Accuracy and Reliability of AI-Generated Medical Responses: An Evaluation of the Chat-GPT Model. *Research square*, rs.3.rs-2566942. <https://doi.org/10.21203/rs.3.rs-2566942/v1>
- Katz, D. Bommarito, M., Gao, S., & Arredondo, P. (2023). GPT-4 Passes the Bar Exam. <http://dx.doi.org/10.2139/ssrn.4389233>
- Lo, C. K. (2023). What Is the Impact of ChatGPT on Education? A Rapid Review of the Literature. *Education Sciences*, 13(4), 410. MDPI AG. <http://dx.doi.org/10.3390/educsci13040410>
- Mhlanga, D. (2023). Open AI in Education, the Responsible and Ethical Use of ChatGPT Towards Lifelong Learning. <http://dx.doi.org/10.2139/ssrn.4354422>
- Plevris, V., Papazafeiropoulos, G., & Rios, A. J. (2023). Chatbots put to the test in math and logic problems: A preliminary comparison and assessment of ChatGPT-3.5, ChatGPT-4, and Google Bard. arXiv preprint arXiv:2305.18618.
- Sabzalieva, E., Valentini, A. (2023). ChatGPT and artificial intelligence in higher education. UNESCO. https://www.iesalc.unesco.org/wp-content/uploads/2023/04/ChatGPT-and-Artificial-Intelligence-in-higher-education-Quick-Start-guide_EN_FINAL.pdf

- Steenbergen-Hu, S., & Cooper, H. (2014). A meta-analysis of the effectiveness of intelligent tutoring systems on college students' academic learning. *Journal of educational psychology*, 106(2), 331.
- Terwiesch, C. (2023). Would Chat GPT Get a Wharton MBA? A Prediction Based on Its Performance in the Operations Management Course, Mack Institute for Innovation Management at the Wharton School, University of Pennsylvania.
- Wu T., He S., Lu L., Sun S., Liu K., Han Q.L., & Tang Y. (2023). A Brief Overview of ChatGPT: The History, Status Quo and Potential Future Development, *IEEE/CAA Journal of Automatica Sinica*, vol. 10, no. 5, pp. 1122-1136. <https://doi.org/10.1109/JAS.2023.123618>
- Zhai, X. (2022). ChatGPT user experience: Implications for education. Available at SSRN 4312418.

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