Beliefs about the Nature of Mathematics and Its Effects on Teaching and Assessment of Learning

Mutia Fariha[1], Rahmah Johar[2*], Rini Oktavia[3], Mailizar Mailizar[2]

[1] Graduate School of Mathematics and Applied Sciences, Universitas Syiah Kuala Banda Aceh, Indonesia
[2]Department of Mathematics Education, Universitas Syiah Kuala, Banda Aceh, Indonesia
[2]Department of Mathematics, Universitas Syiah Kuala, Banda Aceh, Indonesia
rahmah.johar@usk.ac.id

Abstract. Mathematics teacher beliefs are one of the mental models that are important for teachers to have in teaching mathematics. Three important aspects of mathematics teach- er beliefs are beliefs about the nature of mathematics (BNM), beliefs about the teaching of mathematics (BTM), and beliefs about assessment in mathematics learning (BAM). This study aims to determine the effect of BNM on BTM and BAM in elementary school teachers who teach mathematics. Data was collected using a mathematics teacher beliefs questionnaire and its validity was tested. Data processing and analysis uses the PLS-SEM method. The results showed that there was a direct and significant effect of BNM on BTM. The direct effect of BNM on BAM is very low. BTM acts as a moderating variable between BTM and BAM which provides a positive and significant indirect effect of increasing BNM on BAM. 0

Keywords: Mathematics Teacher Beliefs, Teaching Mathematics, Teacher Knowledge.

1 Introduction

The practices carried out by teachers can be influenced by the teacher's mentality which is a belief system about teaching and learning mathematics [2]. This is what became known as Mathematics Teacher Beliefs (MTB). Several studies have shown that teachers' beliefs in mathematics can have an impact on the learning they manage [1], [3], [4]. MTB is considered to be a mediating effect for knowledge of mathematics content and teachers' attitudes towards learning [5], has a significant relationship between PCK and learning practices [6], has a correlation with the teacher's teaching style [1], [2], and can have an effect on students' beliefs about the mathematics they teach [7]. So that the teacher's beliefs, knowledge of what is taught, and the learning practices carried out by the teacher are considered as the Trichotomy of Mathematics Teacher Education [8].

There are three important aspects in MTB including beliefs about the nature of mathematics (BNM), beliefs about the teaching of mathematics (BTM), and beliefs about assessment in mathematics learning (BAM) [1], [3], [4]. BNM believes that Mathematics itself is a collection of facts, concepts and rules that are static and

R. Johar et al. (eds.), *Proceedings of the 2nd Annual International Conference on Mathematics, Science and Technology Education (2nd AICMSTE)*, Advances in Social Science, Education and Humanities Research 828,
https://doi.org/10.2991/978-2-38476-216-3_5

integrated, but solving mathematical problems is dynamic and open along with the process of human investigation and discovery [2], [9].

BTM describes the teacher's belief in the importance of teaching mathematics and the teacher's belief in the importance of helping students to learn mathematics correctly [10]. While BAM is the belief that in learning mathematics it is necessary to have an assessment to determine the effectiveness of the learning being carried out [11]. According to Ernest (1989) The relationship between these three aspects is described in Fig. 1.

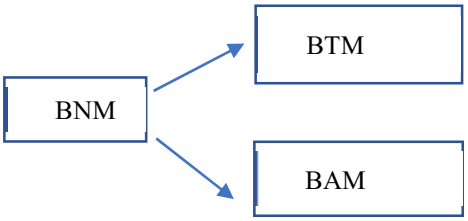

Fig. 1. Relationship between aspects of MTB

Beliefs about the nature of mathematics (BNM) is a teacher's mental in teaching mathematics and conducting assessments in learning mathematics. BNM is also an aspect of CFR that has an influence on learning practice so that it has an impact on the teacher's assessment [11], [12]. BNM can also be considered to have direct influence on BAM or BTM [13]. BTM is also considered to be able to influence BAM so that it is possible to be a moderating (intermediary) variable that explains the relationship between BNM and BAM [12]. Thus Fig. 1 can become Fig. 2.

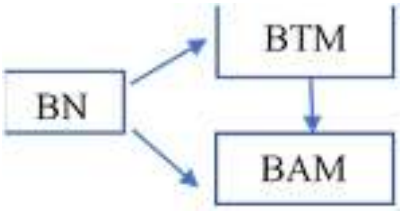

Fig. 2. The relationship BTM as a mediating variable to BNM and B

Several studies in several countries have found that MTB has an influence on the quality of teaching teachers and their attitudes towards mathematics [3], [5], [22],[27], [14], [21]. The results show that teachers who have better beliefs about what they teach tend to make the quality of learning better than teachers who have lower beliefs.

However, research related to this has never been conducted on elementary school teachers in Aceh, so there is no information about the relationship between MTB for existing elementary school teachers. This is what prompted researchers to conduct a study to find out the relationship between MTB aspects for teachers so that they can provide information about the relationship between each aspect. The information can be used as a consideration in education and training involving elementary school teachers.

2 Method

This research is quantitative research. Data were collected randomly using a MTB questionnaire that had been developed and verified by [3]. The questionnaire consisted of 30 items covering 3 aspects of MTB, namely BNM consisting of 9 items, BTM 11 items, and BAM 10 items. Data collection was carried out by visiting SD/MI (Elementary School/Islamic Elementary Madrasa) in Banda Aceh City and sharing them online. Samples that meet the requirements are teachers who teach mathematics or class teachers who also teach mathematics in their class. After collecting and selecting data, 315 samples were obtained and 171 met the requirements for further analysis. Processing and data analysis using the Partial Least Square-Structural Equation Modeling (PLS-SEM) method with SmartPLS 3.

Based on the theory that has been described previously, the model path diagram is present on Fig. 3.

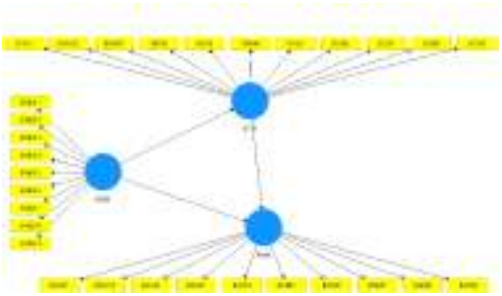

Fig. 3. Model path diagram of the relationship between BNM, BTM, and BAM

Based on the path diagram model the exogenous variables are BNM while the endogenous variables are BTM and BAM. The problems analyzed in this study are;

1. How does BNM affect BTM?
2. How does BNM affect BAM?
3. What is the effect of BTM as a moderating variable between BNM and BAM?

The criteria for data analysis use the criteria specified in the PLS-SEM, known as the PLS-SEM rule of thumb [4]. Data analysis in PLS-SEM includes two parts, namely the outer model and the inner model. The outer model is the relationship between variables (BNM, BTM, and BAM) with each indicator. This is to see whether each indicator can explain the variable. The inner model is used to see the relationship between exogenous and endogenous variables. The inner model is also known as the structural model. The test criteria are in accordance with the Ta The outer model describes the relationship between variables (BNM, BTM, and BAM) and each indicator as presented at Table 1**.**

Table 1. Test criteria

NO	Analisis	Kriteria

	Outer Model	
1	Convergent validity test This test uses the analysis Average Variance Extracted (AVE), AVE is used to measure the ability of latent variables to explain the variance of the indicators.	> 0.5 is valid If the AVE value is < 0.5, the test is still valid if composite reliability and cronbach's alpha > 0.7
2	Composite Reliability Composite reliability is used to measure the reliability of each indicator on a latent variable	> 0.7 is reliable
3	Reliability test The reliability test is used to see the reliability of each item of the instrument used. Testing uses *Cronbach's alpha*	> 0.7 is reliable
	Inner Model	
4	Analyzing path coefficients to determine direct effects between variables, calculating indirect effects from moderating variables, as well as the total effects produced due to moderating variables	> 0, the closer to 1 the better
5	Evaluation of structural models Test this by analyzing the output of R^2 R^2 is to measure the strength of the influence of latent (exogenous) variables on endogenous variables	> 0.75 substansial, $0.25 < R^2 < 0.74$ moderate < 0.25 weak
6	Measuring the effect of exogenous variables to endogenous variables. This test uses the output value f^2 (effect size),	0.02 small 0.15 medium 0.35 large

3 Results and Discussion

1.1 Outer Model Analysis

Analysis of the outer model aims to determine the relationship between latent variables and the indicators as presented in Table 2.

Table 2. Outer model test results

	AVE	**Composite Reliability**	***Cronbach's alpha***
BNM	0.589	0.934	0.922
BTM	0.446	0.839	0.790
BAM	0.444	0.891	0.861

The AVE value in Table 2 shows that the AVE value is < 0.5. However, this can be accepted by considering the composite reliability and cronbach alpha values > 0.7, so it can be said that the relations between the variables and their indicators is valid and reliable so that they can be used in analyzing the relationships between the variable in question [29], [30]

1.2 Inner Model Analysis

Inner analysis is a structural analysis to determine the influence of exogenous variables on endogenous variables. Figure 4 shows the analysis results on the path diagram carried out.

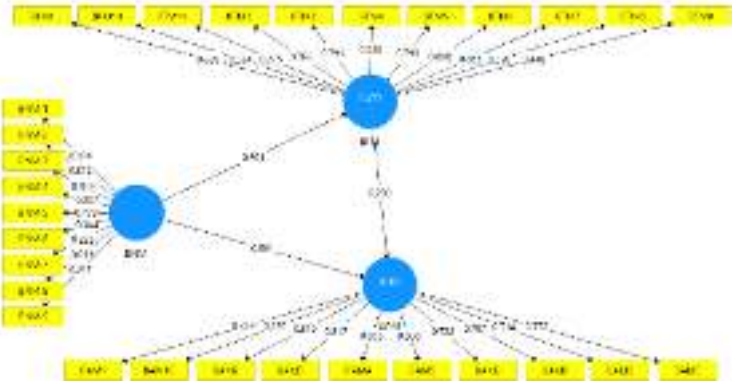

Fig. 4. Path analysis results

To analyze the inner model, the first is to evaluate the significance of the relationship between constructs/variables. This is shown by the path coefficient value which describes the strength of the relationship between constructs/variables in the model. The value of the coefficient path according to the resulting model is presented as Table 3.

Table 3. Path coefficient values

	BAM	**BNM**	**BTM**
BAM			
BNM	0.091		0.691
BTM	0.709		

The increase in BNM has a direct effect on BAM which is shown in Table 3 is 0.091 or 9.1% positively. This effect is classified as very low. On the other hand, increasing BNM has a relatively high direct positive influence of 0.691 or 69.1% on increasing BTM. This means that an increase in BNM will increase BTM by 69.1%. BTM also has a direct effect on increasing BAM by 0.709 or 70.9%. This means that every increase in BTM has an effect on increasing BAM by 70.9%. BTM which acts as a mediating variable between BNM and BAM also shows a different influence. If the direct effect of BNM on BAM is low, namely 9.1%, then the indirect effect of BNM on BAM with BTM as a moderating variable gives different results as an indirect effect as shown in Table 4.

Table 4. Indirect effect

	BAM	**BNM**	**BTM**
BAM			
BNM	0.490		
BTM			

The values in Table 7 show that BNM has an indirect effect through BTM on increasing BAM by 0.490 or 49%. This is very different from the direct effect of BNM on BAM which is shown in Table 3. Without the BTM as a moderating variable, BNM has a small influence, it's only 9.1%, while the presence of BTM as a moderating variable has a large influence (49%) on increasing BAM, so it can be concluded that BTM is important to improving BAM for teachers. The BTM as a moderating variable can strengthen the effect of BNM on increasing BAM.

Furthermore, total effect analysis was carried out to assess the total effect of a variable on other variables either directly or indirectly. The total effect is presented in the Table 5

Table 5. Total effect

	BAM	**BNM**	**BTM**
BAM			
BNM	0.581		0.691
BTM	0.709		

With the resulting total effect value, it can be stated that the total effect of BNM on BAM, both directly and indirectly affects is 0.581 or 58.1%. This value comes from the sum of 9.1% of the direct effect (path coefficient) and 4.90% of the indirect effect. By considering the value of the total effect, it can be seen that increasing BNM has an effect of more than 50% on increasing BAM and BTM.

The next test on the inner model is to test the structural model. Structural model testing using R^2 is used to measure the effect of BNM on BTm and BAM. The value of R^2 is presented in Table 6.

Table 6. R^2 results

	R Square	**R Square Adjusted**
BAM	0.600	0.595
BTM	0.477	0.474

Based on Table 6 it can be stated that BNM can have a positive influence on BAM by 60% and 47.7% on BTM. With an R Square Adjusted value that is close to the R^2 value, it can be said that BNM as an exogenous variable has a positive influence on increasing BAM and BTM as an endogenous variable in the moderate category To see whether the effect is substantive, it is continued by looking at the value of f^2 (effect size), considering that there is a mediating variable between BNM and BAM. Table 7 below shows the value of f^2.

Table 7. Effect size (f^2)

	BAM	BNM	BTM
BAM			
BNM	0.011		0.913
BTM	0.657		

Based on the values in Table 7, it shows that BNM has a large direct effect on increasing BTM (0.913) but has a weak effect on directly increasing BAM (0.011). The existence of BTM as a moderating variable between BNM and BAM has a significant effect on increasing BAM so that the effect is larger (0.657) and falls into the large effect category. So it can be said that BTM as a moderating variable has a strategic role in increasing BAM.

Analysis of effect values in the outer model and inner model needs to be compared with the test results using bootstrapping. Bootstrapping analysis includes assessing the level of significance or probability of direct effects, indirect effects, total effects, and R^2. A summary of the results of bootstrapping significance is shown in Table 8

Table 8. Bootstrapping result

Category	p value	level of significance
Total effects	0.000	significance
R^2	0.000	significance
f^2	0.643 for effect BNM to BAM	not significance
	0.01 for effect BNM to BTM	significance

Based on Table 8, it shows that the test value is significant for the total effect and R^2. There is one value that is not significant, namely the direct effect (f^2) of BNM on BAM. This also confirms the low effect size results in Table 7 which shows that the direct effect of BNM on increasing BAM is very low.

The analysis results obtained from the outer model and inner model showed that the effect of BNM (beliefs about the nature of mathematics) on BTM (beliefs about teaching mathematics) is positive and is in the moderate category but the direct effect of is very low on BNM (beliefs about the nature of mathematics) on BAM (beliefs about assessment in learning mathematics) which means that increasing beliefs about the nature of mathematics does not have a significant effect on increasing beliefs about assessment in learning mathematics. Different results were shown when beliefs about teaching mathematics became a moderating variable between beliefs about the nature of mathematics and beliefs about assessment in learning mathematics. Beliefs about the nature of mathematics which have a positive effect on increasing beliefs about teaching mathematics can indirectly have a significant effect on increasing beliefs about assessment in mathematics learning.

2 Conclusion

4.1 The effect of beliefs nature of mathematics on beliefs about teaching of mathematic

The results of data analysis show that beliefs about the nature of mathematics have a positive effect on beliefs about teaching mathematics. This is in accordance with the assumption that mathematics teacher beliefs, one aspect of which is beliefs about the nature of mathematics, are important things that can influence teachers in managing their learning [5]. This is because beliefs about the nature of mathematics are the basis for teachers' mental models in understanding and teaching mathematics [6] and can encourage learning practices that respect mathematics as a habit of mind and understand and appreciate the role of mathematics in human affairs [7].

For elementary school teachers with different educational backgrounds, it is very important to consider what they believe about mathematics when teaching, because what teachers believe about mathematics has a significant correlation with what they teach [8], [9] and to feel satisfied with the learning carried out and motivate the achievement of learning outcomes and better interactions in the classroom [10]. A major concern for teacher education programs is the fact that primary school teachers are thought to often lack confidence in their understanding of the nature of the mathematical concepts being taught, which has an impact on the quality of the assessments carried out. [11]. Therefore increasing beliefs about nature of mathematics for teachers can encourage a metacognitive ability to control cognitive aspects in solving mathematical problems, designing mathematics learning for teachers, and being able to change situations that are not pleasant in learning into something fun [12]. It must be acknowledged that mathematics practiced in the classroom is different from the practice of mathematicians in finding a theory or application, so it is recommended that beliefs about the nature of mathematics can be considered to accommodate possible differences in views about this matter and school mathematics can be more focused [13].

4.2 The beliefs nature of mathematics and its effects on beliefs about assessment

Assessment in mathematics learning has an important role to provide information to teachers, parents and students regarding the mathematical abilities that have been achieved and to help make improvements and improve them in the future [14], [15]. Until now there is no information regarding the direct relationship between beliefs about the nature of mathematics and beliefs about assessment in mathematics learning among teachers. Some existing research is always related to the teaching and learning process. It can be understood that the assessment carried out by teachers regarding knowledge of concepts, rules or mathematical skills is very dependent on learning activities. This strengthens the results obtained in this research that the direct relationship between beliefs about the nature of mathematics and beliefs about assessment in learning mathematics is very weak.

4.3 The effect of beliefs teaching mathematics as moderating variable on beliefs nature of mathematics and beliefs about assessment in mathematics learning

The research results show that beliefs about the nature of mathematics has a weak direct effect on beliefs about assessment in mathematics learning, but this effect becomes strong when beliefs about mathematics teaching become a moderating variable for these two variables. This illustrates that increasing beliefs about the nature of mathematics can have an indirect effect on increasing teachers' confidence in providing assessments in mathematics learning. This strengthens the assumption that there is a relationship between beliefs about mathematics, especially beliefs about the nature of mathematics, learning, and assessments carried out by teachers in mathematics learning. [16]. Beliefs about assessments carried out in learning play a role in forming an example of how to apply mathematical facts or rules in solving everyday problems. So mathematics can be considered to provide benefits to make life better in the future.

For elementary school teachers, assessment is not just part of the learning process as a student assessment [17], but it also includes a sense of responsibility towards the students' parents [18]. This encourages teachers' belief in the importance of assessment to be greater as part of putting into practice the knowledge acquired by students in their lives. Very important for teachers to consider the nature of mathematics in learning. Because this can construct beliefs regarding the mathematics assessments carried out in learning [19].

The results of this research support the idea that we must at the same time pay more attention to how the education program for prospective mathematics teachers must be modified [20] by encouraging beliefs about mathematics that can be used as one of the materials in educational and training activities for mathematics teachers or teachers. who teaches mathematics in elementary schools [13]. This is because confidence in what teachers teach can be influenced by their education, so this is important to pay attention to in better mathematics teacher education programs in the future. [21], [22]

This research has implications for understanding that increasing teachers' mathematics beliefs, especially beliefs about the nature of mathematics, can have an impact on teachers' teaching abilities and classroom management, including the assessments they carry out. So this can be a concern in the education of prospective teachers or training related to teaching mathematics. It is hoped that increasing knowledge about mathematics and its learning can be accompanied by an open awareness of beliefs about the nature of mathematics, the importance of mathematics, and the use of mathematics in everyday life.

References

[1] V. Hatisaru, “Views and Beliefs in Mathematics Education,” *Views Beliefs Math. Educ.*, no. November, 2018, doi: 10.1007/978-3-030-01273-1.

[2] B. Rott, “Teachers’ behaviors, epistemological beliefs, and their interplay in lessons on the topic of problem solvings,” *Int. J. Sci. Math. Educ.*, vol. 18, no. 5, pp. 903–924, 2020, doi: 10.1007/s10763-019-09993-0.

[3] Y. W. Purnomo, “A scale for measuring teachers’ mathematics-related beliefs: A

validity and reliability study," *Int. J. Instr.*, vol. 10, no. 2, pp. 2–38, 2017, doi: 10.12973/iji.2017.10120a.

[4] J. F. Hair, T. M. Hult, C. Ringle, and M. Sarstednt, *A Primer on Partial Last Squares Structural Equation Modelling (SEM-PLS)*. America: SAGE Publications, 2017.

[5] C. Y. Charalambous, "Working at the intersection of teacher knowledge, teacher beliefs, and teaching practice: a multiple-case study," *J. Math. Teach. Educ.*, vol. 18, no. 5, pp. 427–445, 2015, doi: 10.1007/s10857-015-9318-7.

[6] P. Ernest, "The knowledge, beliefs and attitudes of the mathematics teacher: a model," *J. Educ. Teach.*, vol. 15, no. 1, pp. 13–33, 1989, doi: 10.1080/0260747890150102.

[7] A. G. Sawyer, "Factors influencing elementary mathematics teachers' beliefs in reform-based teaching," vol. 26, no. 2, pp. 26–53, 2018.

[8] N. M. Aljaberi and E. Gheith, "In-service mathematics teachers ' beliefs about teaching , learning and nature of mathematics and their mathematics teaching practices," vol. 7, no. 5, pp. 156–173, 2018, doi: 10.5539/jel.v7n5p156.

[9] Q. Shi, S. Zhang, and E. Lin, "Relationships of new teachers' beliefs and instructional practices: comparisons across four countries," *Action Teach. Educ.*, vol. 36, no. 4, pp. 322–341, Jul. 2014, doi: 10.1080/01626620.2014.948228.

[10] H. N. Perera and J. E. John, "Teachers' self-efficacy beliefs for teaching math: Relations with teacher and student outcomes," *Contemp. Educ. Psychol.*, vol. 61, p. 101842, 2020, doi: https://doi.org/10.1016/j.cedpsych.2020.101842.

[11] G. Philippou and C. Christou, "A study of the mathematics teaching efficacy beliefs of primary teachers BT - Beliefs: A hidden variable in mathematics education?," G. C. Leder, E. Pehkonen, and G. Törner, Eds. Dordrecht: Springer Netherlands, 2002, pp. 211–231. doi: 10.1007/0-306-47958-3_13.

[12] J. Maasz and W. Schlöglmann, *Beliefs and attitudes in mathematics education: New research results*. Nedherlan: Sense Publisher, 2009.

[13] K. Beswick, "Teachers' beliefs about school mathematics and mathematicians' mathematics and their relationship to practice," *Educ. Stud. Math.*, vol. 79, no. 1, pp. 127–147, 2012, doi: 10.1007/s10649-011-9333-2.

[14] D. Carless, G. Johin, and N. F. Liu, *How assessment supports learning: learning-oriented assessment in action*. Hongkong: Hongkong University Press, 2006. [Online]. Available: https://books.google.co.id/books?hl=id&lr=&id=Ud5ahKq7At8C&oi=fnd&pg=PR9&dq=assessment+without+learning&ots=eph6iASI2L&sig=4XfHl9CnNoQ1T2BzQyDblDVWTjo&redir_esc=y#v=onepage&q=assessment without learning&f=false

[15] National Research Council, *Measuring What Counts : a conceptual guide for Mathematics assessment*. Washington, 1993. doi: 10.17226/2235.

[16] G. Martínez-Sierra, J. García-García, M. Valle-Zequeida, and C. Dolores-Flores, "High school mathematics teachers' beliefs about assessment in mathematics and the connections to their mathematical beliefs," *Int. J. Sci. Math. Educ.*, vol. 18, no. 3, pp. 485–507, 2020, doi: 10.1007/s10763-019-09967-2.

[17] G. T. L. Brown, "Teachers' conceptions of assessment: implications for policy and professional development," *Assess. Educ. Princ. Policy Pract.*, vol. 11, no. 3, pp. 301–318, Sep. 2004, doi: 10.1080/0969594042000304609.

[18] S. Nisbet and E. Warren, "Primary school teachers' beliefs relating to mathematics, teaching and assessing mathematics and factors that influence these beliefs," *Math. Teach. Educ. Dev.*, vol. 2, pp. 34–47, 2000.

[19] K. P. Tamba and U. P. Harapan, "The Relationship between pre-service elementary school mathematics teachers ' beliefs about epistemology of mathematics , teaching and learning , and mathematics assessment," vol. 11, no. June, pp. 30–41, 2021, doi: 10.25273/pe.v11i1.8311.

[20] F. K. Lester, "Preservice teachers ' beliefs about the nature of mathematics," 2004.

[21] A. Viholainen, M. Asikainen, and P. E. Hirvonen, “Mathematics student teachers ’ epistemological beliefs about the nature of mathematics and the goals of mathematics teaching and learning in the beginning of their studies,” vol. 10, no. 2, pp. 159–171, 2014, doi: 10.12973/eurasia.2014.1028a.

[22] N. R. 1 Prada, G. 2 Marmolejo, and S. 3 Mulema, “Belief system towards mathematics present in future teachers . A comparative study,” vol. 41, no. 35, pp. 197–212, 2020.