Analysis on Distortion Effect of Single Box Double Cell Box Girders Subjected to Lane Load

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Abstract. In view of the particularity that the current analytical theory of box girder distortion is mainly limited to single-cell cross-sections and double-cell cross-sections without cantilever plates, this paper uses the plate element method to establish the distortion control differential equations of single-box and double-cell box girder with cantilever plates, and gives corresponding cross-section geometric parameter calculation formula. The initial parameter method is used to solve the distortion control differential equation, and the analytical solutions of the distortion double moments and distortion angles of a simply supported single-box and double-cell box girder are given. Numerical examples of simply supported box beams show that the analytical solution of distortion warping stress is in good agreement with the ANSYS finite element numerical solution; the concentrated load in the lane load has the greatest impact on the distortion warping stress of the control section, and the mid-span section characteristic points caused by the concentrated load The warping stress at the mid-span cross-section characteristic point of the double-cell box girder is reduced by 11.47% compared with the corresponding single-cell box girder.

Keywords: single box double cell box girder; distortion effect; plate element method; transverse frame stiffness; lane load.

1 Introduction

As an important part of thin-walled box beam theory, the distortion effect has attracted the attention of many scholars at home and abroad in recent years[1-8]. The methods for analyzing the distortion effects of box beams established by different scholars are also different and have not reached unity. Zhang Yuanhai et al[9] applied the energy variation method to study the distortion effect of trapezoidal box beams, and derived a general formula for the distortion of trapezoidal box beams. Xu Xun et al[10] used distortion angle and distortion deflection as parameters to study the distortion effect of box beams under two theories.Wright et al[11]proposed the elastic foundation beam analogy method for analyzing the distortion eddect of box girder, and Hsu et al[12] applied the equivalent elastic foundation beam analogy method to analyze the distortion of box girder. Park et al[13] verified the effectiveness and feasibility of the box girder unit through the analysis of finite element results and experimental results.

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In summary, domestic and foreign scholars have done a lot of complete research on the distortion effects of single-box and single-chamber box beams, but the research on single-box and double-chamber box beams with better anti-distortion capabilities has not been done. There are few, and most of them take box beams under eccentric concentrated load as the research object for analysis, which is quite different from the actual loading situation. Under the action of lane load, the distortion effect produced by long-span thin-walled box girder bridges accounts for a large proportion in the bridge live load design calculation, and is an important issue in the current design of long-span box girder bridges. This paper takes a rectangular single-box double-chamber box girder with cantilever plates as an example to analyze the distortion effect of the box girder under lane load. The plate element method is combined with the ideas of the equivalent frame and the inclination angle displacement method to establish a general form of a single box girder with cantilever plates. The distortion control differential equation of box-double-chamber box girder is given, and its initial parameter solution is given. Combined with numerical examples, the distortion warping stress at the calculation point is compared to study the distortion resistance of single-box and double-chamber box girder, which provides a theoretical reference for the design and calculation of single-box and double-chamber box girder bridges.

2 Analysis object overview

This method is based on the basic assumptions of thin-walled box girder distortion theory, and takes the rectangular cross-section of a single-box and double-chamber box girder as the object for analysis. As shown in Figure 1, The size of the cantilever plate is d,the size of the web is h,and the size of the top and bottom plates is b; The corresponding thicknesses of the top plate, bottom plate, side webs, and middle webs are t_1, t_2, t_3, t_h. A right-handed coordinate system is used, with O as the origin.

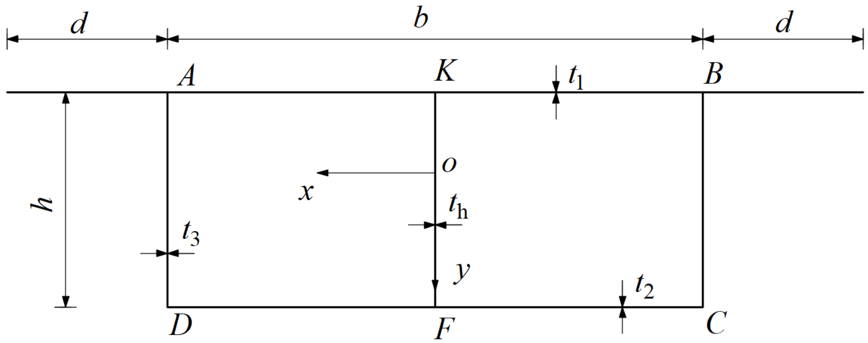

Fig. 1. Section form of single box and double cell box girder

Taking the antisymmetric load P as an example, After decomposition, we get P_{41}, P_{42}, P_{12}, P_{22}, P_{21}, P_{11}, P_{31} and P_{32}, It is antisymmetrically distributed in the box girder section, as shown in Figure 2:

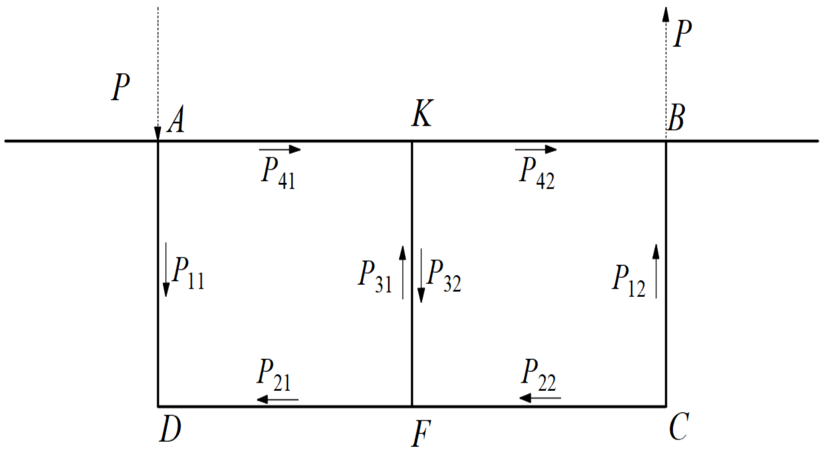

Fig. 2. Decomposition of antisymmetrical distortion load

As shown in Figure 2, the distortion load of a single-box double-chamber box girder is obtained after decomposition of the antisymmetric load P:

$$P_{11} = P_{12} = P_{31} = P_{32} = \frac{P}{2} \tag{1}$$

$$P_{41} = P_{42} = P_{21} = P_{22} = \frac{P}{4} \tag{2}$$

3 Analysis of stress system of single-box and double-chamber box girder

3.1 Analysis of the in-plane force system of each plate element

The box girder section is discretized into multiple plate elements as the research object to analyze the in-plane force system balance between each plate element, as shown in Figure 3. Among them, there are transverse constraint reactions q_{xA}, q_{xB}, q_{xC}, q_{xD}, q_{yA}, q_{yD}, $T_b$dz, $T_u$dz are longitudinal constraint reactions, the in-plane shear and moment increments generated on the top plate micro-segment are dQ_u and dM_u, and the in-plane shear and moment increments generated on the bottom plate micro-segment are dQ_u and dM_u. The resulting in-plane shear force and moment increments are dQ_b,dM_b, and the in-plane shear force and moment increments of the left web are dQ_w,dM_w.

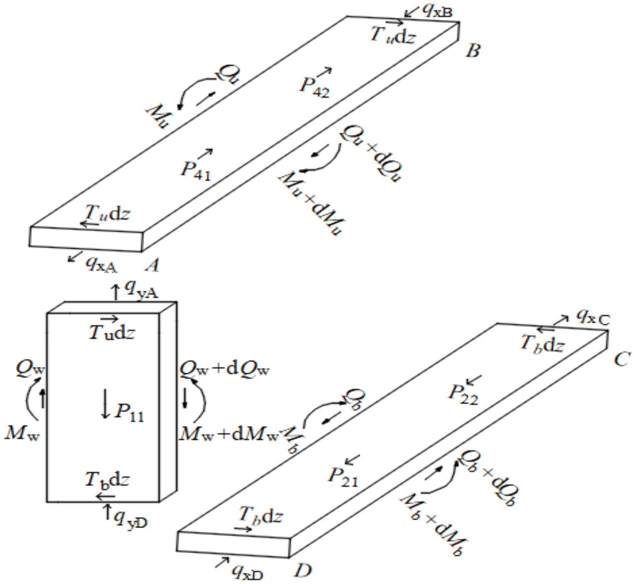

Fig. 3. In-plane force of each plate of box girder

From the balance of the in-plane force system of each plate, after simplification, we can get:

$$\frac{\mathrm{d}^2 M_{\mathrm{w}}}{\mathrm{d}z^2}+\frac{h}{2b}\left(\frac{\mathrm{d}^2 M_{\mathrm{u}}}{\mathrm{d}z^2}+\frac{\mathrm{d}^2 M_{\mathrm{b}}}{\mathrm{d}z^2}\right)+\frac{h(P_{41}+P_{21})}{b}+P_{11}-\left[\frac{h(q_{\mathrm{x1}}+q_{\mathrm{x2}})}{2b}+q_{\mathrm{y}}\right]=0 \quad (3)$$

In the formula: $q_{\mathrm{x1}}=q_{\mathrm{xA}}+q_{\mathrm{xB}}+q_{\mathrm{xK}}$; $q_{\mathrm{x2}}=q_{\mathrm{xC}}+q_{\mathrm{xD}}+q_{\mathrm{xF}}$; $q_{\mathrm{y}}=q_{\mathrm{yA}}+q_{\mathrm{yD}}$.

When the box girder is distorted, each corner point produces displacement and angle change, as shown in Figure 4, where the horizontal displacement of each corner point is Δ_{HA}, Δ_{HB}, Δ_{HC}, Δ_{HD}; The vertical displacement of each corner point is Δ_{VA}, Δ_{VB},Δ_{VC},Δ_{VD}; The angle of each plate after distortion changes to a_1,a_2,a_3.

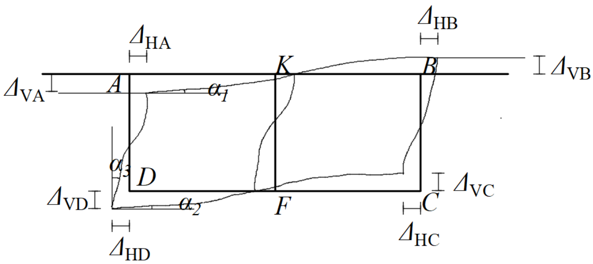

Fig. 4. Distortion of box girder

The distortion angle is represented by the displacement at the corner point and the change in angle of each plate after distortionγ_D, γ_D is

$$\gamma_{\mathrm{D}} = \frac{\alpha_1 + \alpha_2}{2} + \alpha_3 = \frac{\Delta_{\mathrm{VA}} + \Delta_{\mathrm{VD}}}{b} + \frac{\Delta_{\mathrm{HD}} + \Delta_{\mathrm{HA}}}{h} \tag{4}$$

The relationship between the plane bending moment and distortion angle of each plate element is introduced, where J_1, J_2, and J_3 are the in-plane moments of inertia of the top plate, bottom plate, and web respectively. The relationship between the side web bending moment M_w and the distortion angle is:（The expressions of the roof bending moment M_u and the bottom plate bending moment M_b are similar）

$$M_{\mathrm{w}} = -\frac{bEJ_3}{4}\gamma_{\mathrm{D}}'' \tag{5}$$

Calculate the second differential of the distortion angle γ_{D}'' in equation (5), substitute it into equation (3) and simplify it to get

$$-E\frac{J_3 b^2(1+\beta) + h^2(\beta J_1 + J_2)}{4(1+\beta)b}\gamma_{\mathrm{D}}'''' + \frac{h}{b}(P_{41} + P_{21}) + P_{11} - [\frac{h(q_{\mathrm{x1}} + q_{\mathrm{x2}})}{2b} + q_{\mathrm{y}}] = 0 \tag{6}$$

In the formula: β is the ratio of distortion and warpage stress at corner points A and D, which only depends on the cross-sectional size, that is

$$\beta = \frac{b^2 t_2 + ht_3 3b}{\frac{(b+2d)^3 t_1}{b} + ht_3 3b} \tag{7}$$

Analysis of out-of-plane force system of each plate element.

To obtain the distortion control differential equation, equation (6) is simplified through the out-of-plane force system analysis of each plate of the box girder, each plate of the box girder is discrete, and the equivalent frame method is used to analyze the bending moments and rods of each corner point. End shear force, as shown in Figure 5.

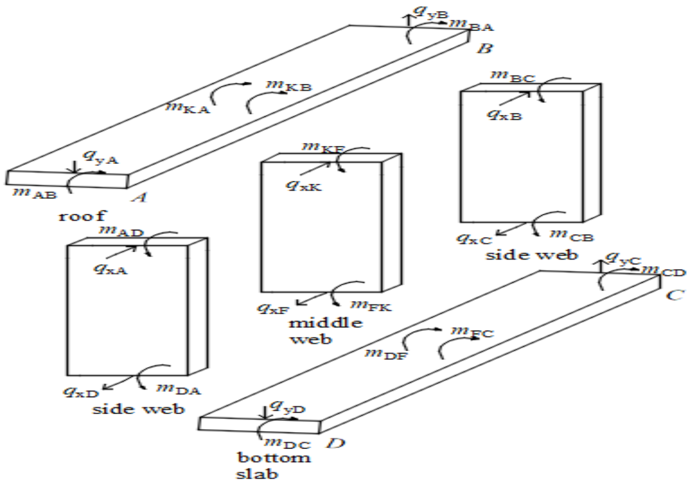

Fig. 5. Out-of-plane force in each plate of box girder

The key point rotation angle expression is obtained by the inclination angle displacement method in structural mechanics, and the back-substitution equal-generation framework represents the corner point bending moment. In the formula, θ_A and θ_D represent the rotation angle displacements at points A and D respectively, I_1, I_2, I_3 are the out-of-plane moments of inertia of the top plate, bottom plate and side web respectively.

$$m_{\mathrm{AB}} = \frac{2EI_1}{b}\left(3\theta_{\mathrm{A}} - 6\frac{\Delta_{\mathrm{v}}}{b}\right) \tag{8}$$

$$m_{\mathrm{DC}} = \frac{2EI_2}{b}\left(3\theta_{\mathrm{A}} - 6\frac{\Delta_{\mathrm{v}}}{b}\right) \tag{9}$$

$$m_{\mathrm{AD}} = \frac{2EI_3}{h}\left(2\theta_{\mathrm{A}} + \theta_{\mathrm{D}} + 6\frac{\Delta_{\mathrm{h}}}{h}\right) \tag{10}$$

$$m_{\mathrm{DA}} = \frac{2EI_3}{h}\left(2\theta_{\mathrm{D}} + \theta_{\mathrm{A}} + 6\frac{\Delta_{\mathrm{h}}}{h}\right) \tag{11}$$

The expressions of θ_A and θ_D can be obtained by simultaneously solving the above formulas based on the balance of corner point bending moments, namely

$$\theta_{\mathrm{A}}=\frac{2\frac{\Delta_{\mathrm{v}}}{b}\left(\frac{3I_1I_2h^2}{I_3{}^2b^2}-\frac{I_2h}{I_3b}+\frac{2I_1h}{I_3b}\right)-2\frac{\Delta_{\mathrm{h}}}{h}\left(1+\frac{3I_2h}{I_3b}\right)}{1+2\frac{\left(I_1+I_2\right)h}{I_3b}+\frac{3I_1I_2h^2}{I_3{}^2b^2}} \tag{12}$$

$$\theta_{\mathrm{D}}=\frac{2\frac{\Delta_{\mathrm{v}}}{b}\left(\frac{3I_1I_2h^2}{I_3{}^2b^2}-\frac{I_1h}{I_3b}+\frac{2I_2h}{I_3b}\right)-2\frac{\Delta_{\mathrm{h}}}{h}\left(1+\frac{3I_1h}{I_3b}\right)}{1+2\frac{\left(I_1+I_2\right)h}{I_3b}+\frac{3I_1I_2h^2}{I_3{}^2b^2}} \tag{13}$$

Substituting the expressions of θ_{A} and θ_{D} into equation (6), the final expression of equation (6) is

$$E\frac{J_3b^2(1+\beta)+h^2(\beta J_1+J_2)}{4(1+\beta)}\gamma_{\mathrm{D}}''''+$$
$$E\frac{24\left(I_3+I_{\mathrm{h}}\right)\left(\frac{6I_1I_2h}{I_3{}^2b^2}+\frac{I_2}{I_3b}+\frac{I_1}{I_3b}\right)}{\left(1+2\frac{I_1+I_2}{I_3}\cdot\frac{h}{b}+3\frac{I_1I_2}{I_3{}^2}\cdot\frac{h^2}{b^2}\right)}\gamma_D=\frac{h}{b}(P_{41}+P_{21})+P_{11} \tag{14}$$

In the formula, I_{h} is the out-of-plane moment of inertia of the mid-web.

4 Distortion differential equation of single box and double chamber box girder

Under the action of distortion load, the distortion control differential equation with the distortion angle γ_{D} defined in this article as the unknown quantity is:

$$EI_{\omega\mathrm{D}}\gamma_{\mathrm{D}}''''+EI_{\mathrm{R}}\gamma_{\mathrm{D}}=m_D \tag{15}$$

In the formula, $I_{\omega\mathrm{D}}$ is the distorted warping moment of inertia of the box girder, I_{R} is the distorted frame inertia moment of the box girder, m_{D} is the distributed distortion moment load.

$\lambda=\sqrt[4]{I_{\mathrm{R}}/4I_{\omega\mathrm{D}}}$, Then equation (15) can be written as:

$$\gamma_{\mathrm{D}}''''+4\lambda^4\gamma=\frac{m_D}{EI_{\omega\mathrm{D}}} \tag{16}$$

According to equation (14), we can get the expression of $I_{\omega\mathrm{D}}$, I_{R}:

$$I_{\omega D} = \frac{J_3 b^2 (1+\beta) + h^2 (\beta J_1 + J_2)}{4(1+\beta)} \tag{17}$$

$$I_R = \frac{24\left(I_3 + I_h\right)\left(\dfrac{6 I_1 I_2 h}{I_3^2 b^2} + \dfrac{I_2}{I_3 b} + \dfrac{I_1}{I_3 b}\right)}{\left(1 + 2\dfrac{I_1 + I_2}{I_3} \cdot \dfrac{h}{b} + 3\dfrac{I_1 I_2}{I_3^2} \cdot \dfrac{h^2}{b^2}\right)} \tag{18}$$

Without considering the influence of external loads, the general solution of the homogeneous differential equation corresponding to Equation (16) is

$$\begin{aligned} \gamma(z) = {} & C_1 \sin(\lambda z)\sinh(\lambda z) + C_2 \cos(\lambda z)\sinh(\lambda z) + \\ & C_3 \sin(\lambda z)\cosh(\lambda z) + C_4 \cos(\lambda z)\cosh(\lambda z) \end{aligned} \tag{19}$$

When a concentrated distortion moment load $\tilde{M}$ is applied to the mid-span of a simply supported box girder with a span of l, the calculation formula for the distortion angle of any section of the half-span $0 \le z \le l/2$ is:

$$\begin{aligned} \gamma(z) = {} & \frac{\eta \tilde{M}}{2\lambda^3 E I_{\omega D}} \sin(\frac{\lambda l}{2}) \sinh(\frac{\lambda l}{2}) \quad [\sin(\lambda z)\cosh(\lambda z) + \cos(\lambda z)\sinh(\lambda z)] + \\ & \frac{\eta \tilde{M}}{2\lambda^3 E I_{\omega D}} \cos(\frac{\lambda l}{2}) \cosh(\frac{\lambda l}{2}) \; [\cos(\lambda z)\sinh(\lambda z) - \sin(\lambda z)\cosh(\lambda z)] \end{aligned} \tag{20}$$

When calculating a simply supported beam subjected to a full-span uniform distortion moment load m_D, the formula for the distortion angle is expressed as

$$\begin{aligned} \gamma(z) = {} & \frac{m_D}{E I_R}\left[\frac{\sinh \lambda l}{\cos \lambda l + \cosh \lambda l} \cos \lambda z \sinh \lambda z - \right. \\ & \left. \frac{\sin \lambda l}{\cos \lambda l + \cosh \lambda l} \sin \lambda z \cosh \lambda z + 1 - \cos \lambda z \cosh \lambda z \right] \end{aligned} \tag{21}$$

In the formula:

$$\eta = \left(\sin^2 \frac{\lambda l}{2} + \sinh^2 \frac{\lambda l}{2}\right) \Big/ \left(\sin^2 \lambda l + \sinh^2 \lambda l\right) \tag{22}$$

5 Numerical examples

The cross-section of a single-box double-chamber box girder with span $l = 50\text{m}$ is shown in Figure 6. The material elastic modulus E=35GPa. According to the specific requirements of the "Highway Bridge Regulations", a single-lane load acts on the top plate of the box girder. The standard value of the uniform load is 10.5kN/m, the standard value of the concentrated load is 360kN, and the load factor is 1.2.

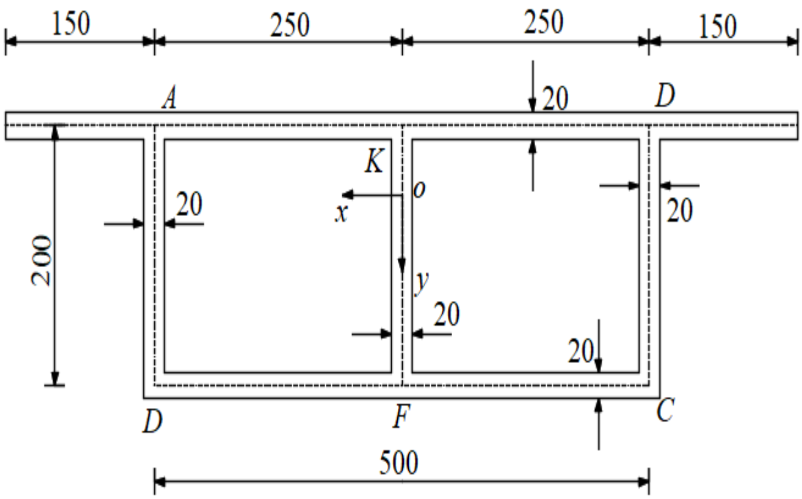

Fig. 6. Section size of box girder (Unit: cm)

Calculate the box girder distortion according to the analytical method in this article, and obtain the geometric parameters of the example box girder. Among them, the warping stress ratio β=0.4154, the distorted warping moment of inertia of the box girder $I_{\omega D}$=4.8097m6, and its distorted frame inertia moment I_R=0.0013m2; the values of the distorted sector coordinates ω_{dA} and ω_{dD} of the corner points A and D are 0.7337m^2 and 1.7663m^2 respectively. Using the calculated values of the above-mentioned cross-sectional geometric characteristics, combined with the distortion angle and distortion double moment calculation formulas in the analytical method in this article, the distortion warpage normal stress of the box girder in the example under uniform load and concentrated load is calculated, and the two loads are The effects under are superimposed to obtain the distortional warping normal stress of the single-box and double-chamber box girder under the lane load, and compared with the finite element results. The distortional normal stress at the corner points of the box girder in the example is listed in Table 1. The finite element model Using the shell-63 unit, from the analysis of the distortion normal stress error at calculation points A and D, the maximum error between the analytical solution and the finite element solution of this method does not exceed 9.7%, and the finite element solution and the solution of this method are in good agreement. The concentrated load in the lane load has the greatest influence on the warping stress of the control section distortion. The warping stress at the characteristic point of the mid-span section caused by the concentrated load reaches 95.3%.

Table 1. Normal distortion stress at calculation point of box girder

Distortion normal stress	concentrated load/kPa	Uniformly distributed load/kPa	Final re-sults/kPa	Ansys re-sults/kPa	tolerance scope%
σ_{dA}	400.335	20.022	421.357	434.700	3.1
σ_{dD}	963.761	48.201	1011.962	1120.491	9.7

To examine the influence of the existence of the mid-web on the warping stress, a single-box single-chamber box girder without a mid-web and a box girder with a mid-web thickness of 0.3m were compared with the examples in this paper. It was found that when the mid-web was added, the mid-span The maximum warpage stress is significantly reduced, about 11.47%, as shown in Figure 7. When the middle web is appropriately thickened to 0.3m, the reduction is approximately 22.88%.

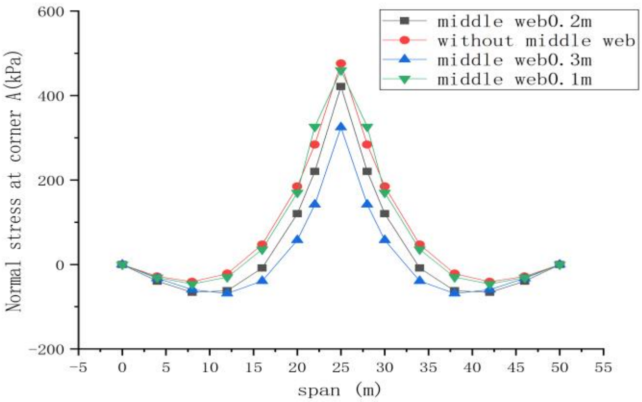

Fig. 7. Changes of distortional normal stress of calculation point

6 Conclusion

1) Based on the plate element method, this paper establishes the distortion-controlled differential equation of a single-box and double-chamber box beam with cantilever plates, and provides its initial parameter solution under the action of lane load. Through comparative analysis, it can be seen that the analytical solution in this paper is consistent with the Ansys finite element The solutions agree well with the maximum error not exceeding 9.7%, which verifies the correctness of the analytical method in this article.

2) The distortion effects caused by concentrated loads and uniform loads in the lane load are analyzed separately. The analysis shows that among the distortion effects of the box girder caused by the lane load, the concentrated load has the greatest impact,

accounting for 95.3% of the total warping stress at the calculation point of the mid-span section.

3) Through the comparative analysis of box girders with and without mid-webs, it can be seen that when the box girder has mid-webs, compared with the box girder section without mid-webs, the calculated warping stress at the corner points has a significant decreasing trend, where the web thickness is At 0.2m, the normal stress at the calculation point is reduced by approximately 11.47% compared to the single-box single-chamber box girder.

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