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Efficient Geotechnical Reliability-based Design Updating Using Direct Monte Carlo Simulation And Sample Reweighting

Xing Peng, Qirui Ma*, Congpeng Zhang

Changjiang institute of survey, planning, design and research, Wuhan, Hubei Province, 430010, China

*Corresponding author's e-mail:maqirui1101@foxmail.com

Abstract. This paper proposes an efficient reliability-based design (RBD) updating procedure for geotechnical structures, which integrates the Expanded RBD method based on Monte Carlo Simulation (MCS) with a sample reweighting approach. Within the proposed procedure, the Expanded RBD is utilized to perform the preliminary design, which can obtain the failure probability (P_f) for all designs using a single MCS run rather than trial-and-error procedure. During the updating process, P_f under different design scenarios can be expressed as a weighted sum of failure sample values using sample reweighting approach. Equations are derived for integrating Expanded RBD with sample reweighting approach to evaluate P_f of geotechnical structures. The flow chart of the proposed procedure is presented and illustrated through rock slope design example. The results are validated against those from direct MCS runs.It is shown that sample reweighting can evaluate the P_f accurately and the computational efficiency is improved significantly compared with direct MCS. With the aid of sample reweighting approach, design updating can be realized effectively and efficiently using the proposed procedure.

Keywords: Reliability-based design; Monte Carlo simulation; Sample reweighting.

1 Introduction

Recently, several full probabilistic design approaches based on Monte Carlo simulation (MCS) have been developed for geotechnical structures[1]. These approaches can provide satisfactory designs for a given design situation(e.g., statistics and probability distributions of loads and geotechnical parameters) .As the design situation changes, the design of the geotechnical structure should be updated accordingly. Such design updating is cumbersome as MCS-based full probabilistic design approaches are used in geotechnical RBD, because repeated MCS-based probabilistic analyses are needed to redesign the geotechnical structure corresponding to different design situations. When

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MCS is directly applied in full probabilistic design, this problem becomes more profound because it lacks efficiency and resolution at low probability levels (e.g., those close to P_T) that are of great interest in design practice.

This article develops an effective method for updating geotechnical RBD, which uses direct MCS to obtain design results in various design scenarios. The proposed method consists of two main steps: (1) executing a direct MCS run to obtain a preliminary design for a given design situation; (2) update the designs corresponding to various design scenarios based on the direct MCS samples generated in the first step, to avoid repeated simulations of different design scenarios. The proposed method only requires one direct MCS operation to obtain the final design in various design situations, achieving cost-effective updates of geotechnical RBD. This article first describes the two main steps of the proposed method. Then, a rock slope design example is used to illustrate the proposed method.

2 Preliminary Reliability-based Design Using Monte Carlo Simulation

The goal of RBD is determining the design (D) to meet the reliability requirement and economic requirement for varying design scenarios implying probabilistic characterizations. D may be a single design parameter (i.e., width of foundation) or a combination of several design parameters (i.e., height and angle of slope). Consider that there is a total of N_D designs in design space,for a given design D_i(i= 1, 2, …,N_D) and probabilistic characterizations θ_0, the failure probability P_f of geotechnical structure can be represented in the form of conditional probability $P(F|D_i,\theta_0)$. The design process is then revised as a process of evaluating conditional probabilities $P(F|D_i,\theta_0)$ corresponding to designs and comparing them with the target failure probability P_T. Feasible designs are those with $P(F|D_i,\theta_0)\leq P_T$. An Expanded RBD method[2] is adopted herein to evaluate $P(F|D_i,\theta_0)$ for all possible designs in design space. In the Expanded RBD method, the design parameters are assumed to be independent and distribute uniformly within the design space. The probability density function (PDF) of D_i is expressed as:

$$P(D_i) = \frac{1}{N_D} \tag{1}$$

In this paper, random variables are used to represent design parameters in order to calculate $P(F|D_i,\theta_0)$ without trail-and-error procedure. Let $\boldsymbol{X}$ be a set of uncertain geotechnical parameters involved in RBD, the conditional failure probability $P(F|D_i,\theta_0)$ can be expressed by:

$$P(F \mid D_i, \theta_0) = \int P(F \mid \boldsymbol{X}, D_i, \theta_0) f(\boldsymbol{X} \mid D_i, \theta_0) d\boldsymbol{X} \tag{2}$$

where $P(F|\boldsymbol{X},D_i,\theta_0)$ is the conditional failure probability given uncertain geotechnical parameters $\boldsymbol{X}$, specified design parameters D_i and probabilistic characterization θ_0; $f(\boldsymbol{X}|D_i,\theta_0)$ is the joint PDF of $\boldsymbol{X}$ conditional on D_i and θ_0. Note that $\boldsymbol{X}$ and design parameters D are mutual independent and $\boldsymbol{X}$ is only determined by θ_0, joint PDF $f(\boldsymbol{X}|D_i,\theta_0)$ can be simplified to be $f(\boldsymbol{X}|\theta_0)$. A single MCS run with N_T random samples of $\boldsymbol{X}$ and D, is used to calculate $P(F|D_i,\theta_0)$. Eq. (2) is further written as:

$$P(F \mid D_i,\theta_0) = \int P(F \mid \boldsymbol{X}, D_i, \theta_0) f(\boldsymbol{X} \mid \theta_0) d\boldsymbol{X} \approx \frac{1}{N_{Di}} \sum_{j=1}^{N_{Di}} I(D_i, \boldsymbol{X}_j) \tag{3}$$

where N_{Di} is the number of samples with a given design D_i; X_j ($j = 1, 2, \ldots, N_{Di}$) are random sample of $\boldsymbol{X}$; $I(\cdot)$ is an indicator function of the occurrence of failure: $I(D_i, \boldsymbol{X})$ is taken as the value of 1 if failure occurs (i.e., $G(D_i, \boldsymbol{X}) < 0$) with a given design D_i; otherwise, it is equal to 0. Thus the value of $\sum_{j=1}^{N_{Di}} I(D_i, \boldsymbol{X}_j)$ equals to the number of failure samples for a given design D_i.

Once the failure probability $P(F|D_i,\theta_0)$ is properly evaluated and compared with P_T, a pool of feasible designs, with corresponding $P(F|D_i,\theta_0) \le P_T$, can be obtained. The economic requirement is then considered to finalize the design among feasible ones, and the final designD_{F0} is the one with minimum construction cost.

3 Efficient Design Updating Using Sample Reweighting

Consider, for example, a new probabilistic characterizations θ and joint PDF $f(\boldsymbol{X}|\theta)$. Similar to Eq. (2), the conditional probability $P(F|D_i,\theta)$ can be expressed as:

$$P(F \mid D_i, \theta) = \int P(F \mid \boldsymbol{X}, D_i, \theta) f(\boldsymbol{X} \mid \theta) d\boldsymbol{X} \tag{4}$$

For estimation of $P(F|D_i,\theta)$, it seems inevitable that a new set of N_T samples should be generated from $f(\boldsymbol{X}|\theta)$ and the performance function have to be evaluated N_T times in Expanded RBD. This resampling and recalculation can be very time consuming. A sample reweighting approach has been proposed herein to update failure probability for each design efficiently using sample reweighting.

The generated samples and corresponding outputs (e.g., indicator function values) of MCS in preliminary design can be reused in the following updating. Instead of generating another set of samples from $f(\boldsymbol{X}|\theta)$, $P(F|D_i,\theta)$can be estimated using samples generated from $f(\boldsymbol{X}|\ \theta_0)$ via a change of probability distribution[3, 4]

$$P(F|D_i,\theta)=\int P(F|X,D_i,\theta_0)\frac{f(X|\theta)}{f(X|\theta_0)}f(X|\theta_0)dX\approx\frac{1}{N_{Di}}\sum_{j=1}^{N_{Di}}I(D_i,X_j)w(X_j;\theta,\theta_0) \quad (5)$$

Where:

$$w(\boldsymbol{X}_j;\theta,\theta_0)=\frac{f(\boldsymbol{X}_j|\theta)}{f(\boldsymbol{X}_j|\theta_0)} \quad (6)$$

is a weighting factor, being the ratio of two probability density functions $f(\boldsymbol{X}_j|\theta)$ and $f(\boldsymbol{X}_j|\theta_0)$.

Note that the estimations of $P(F|D_i,\theta)$ and $P(F|D_i,\theta_0)$, using Eqs. (5) and (3), respectively, share the same set of samples $\boldsymbol{X}_j$ and indicator function $I(\cdot)$ values. These quantities need not be evaluated repeatedly and only the weights $w(\boldsymbol{X}_j;\theta,\theta_0)$ need to be evaluated using Eq. (6) with known $f(\boldsymbol{X}_j|\theta)$ and $f(\boldsymbol{X}_j|\theta_0)$. It is evident that the computational effort is substantially reduced. Note that, for a sample with specific design D_i which is not failure, the corresponding value of $I(D_i,\boldsymbol{X}_j)w(\boldsymbol{X}_j;\theta,\theta_0)$ is equal to 0 according to the definition of indicator function. Therefore only the weighting factors of failure samples contributes to the evaluation of $P(F|D_i,\theta)$. That is to say, the geotechnical practitioners only need to store the failure samples and calculate the weighting factors of them,the computational cost is minimal.

Once the updated $P(F|D_i,\theta)$ for possible designs are evaluated, a pool of feasible designs, whose corresponding $P(F|D_i,\theta)$ are not larger than P_T according to the reliability requirement, can be obtained. Then the most economical design D_F is selected as the final design.

4 Illustrative Rock Slope Design Example

The rock slope design example adopted in this paper is the Sau Mau Ping rock slope in Hong Kong. The geometrical design of this rock slope, with slope height and slope angle as design parameters, is focused on to demonstrate the proposed RBD updating method. The geometry of the Sau Mau Ping slope is illustrated in Fig. 1. The slope before remediation has a height H of 60 m and an overall slope angle ψ_f of 50°[5]. The potential failure plane is inclined at 35° (ψ_p=35°). The specific weights of rock and water are γ=26kN/m^2 and γ_w = 10kN/m^2, respectively.

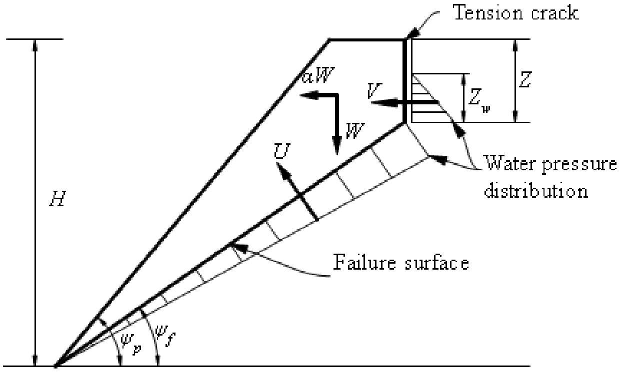


Fig. 1. Illustration of the Sau Mau Ping slope

4.1 Deterministic model

The two-dimensional limit equilibrium model[6] is adopted herein as the deterministic model for rock slope performance analysis with single failure mode. The deterministic formulation for the factor of safety (*FS*) is calculated by:

$$FS = \frac{cA + [W(\cos\psi_p - \alpha\sin\psi_p) - U - V\sin\psi_p]\tan\varphi}{W(\sin\psi_p + \alpha\cos\psi_p) + V\cos\psi_p} \tag{7}$$

where c is the cohesive strength along the sliding surface; A is the base area of wedge; W is the weight of rock wedge located on the failure surface; ψ_p is the angle of failure surface; α is the gravitational acceleration coefficient defined by the ratio of horizontal to gravitational acceleration; U is the uplift pressure generated by the water pressure on failure surface; V is the horizontal force caused by water in tension crack; φ is the friction angle of sliding surface.

The intermediate terms for computing *FS* are obtained as following:

$$A = (H - z)/\sin\psi_p \tag{8}$$

$$W = 0.5\gamma H^2\left\{[1-(z/H)^2]\cot\psi_p - \cot\psi_f\right\} \tag{9}$$

$$U = 0.5\gamma_w z_w A \tag{10}$$

$$V = 0.5\gamma_w z_w{}^2 \tag{11}$$

$$i_w = z_w/z \tag{12}$$

where z is the depth of tension crack, z_w is the depth of water in the tension crack, H is the height of the entire slope, ψ_f is the entire horizontal angle of slope, and i_w is percentage of the depth of tension crack filled with water.

Based on Eq. (7), the corresponding performance function can be expressed as:

$$G(H,\psi_f,\boldsymbol{X}) = FS - 1 \tag{13}$$

Failure occurs when $G(H, \psi_f, \boldsymbol{X}) < 0$.

4.2 Uncertainty modeling

Variables $\{c, \varphi, z, i_w, \alpha\}$ should be considered as random in the RBD of the slope. Table 1 summarizes the probabilistic characterizations of these random variables. The variables c, φ and z are assumed to follow normal distribution; i_w and α are assumed to follow truncated exponential distribution. Furthermore, c and φ are assumed negatively correlatedwith a correlation coefficient $\rho_{c,\varphi}$; z and i_w are also likely to have negative correlation with a correlation coefficient $\rho_{z,i_w} = -0.5$[7] In this design, the means $\{\mu_c, \mu_\varphi\}$, COVs $\{\delta_c, \delta_\varphi\}$ and correlation coefficient $\rho_{c,\varphi}$ of c and φ need to be evaluated by site investigation.

Table 1. Statistics of random variables for rock slope example

Variable	Distribution	Mean	COV	ρ
c	Normal	μ_c(kPa)	δ_c	$\rho_{c,\varphi}$
φ	Normal	μ_φ(°)	δ_φ	
z	Normal	14 (m)	0.21	−0.5
i_w	Exponential with mean 0.5, truncated to [0,1]			
α	Exponential with mean 0.08, truncated to [0,0.16]			

For the two design parameters of the rock slope, a discrete design space is considered.The slope height H will be selected from the range of 50 m to 60 m with an increment of 0.2 m, and the slope angle ψ_f will be selected from the range of 44° to 50° with an increment of 0.2°[8]. Thus, design parameters $H$and ψ_f can be conveniently modeled in the discrete domain with finite number of designs (i.e.,N_D = 1581 in this example).

4.3 Preliminary designs using MCS

In the preliminary design, here consider, for example, an extreme design scenario under which geotechnical practitioners know nothing (i.e., distribution type, correlation) but the value ranges of c and φ. For simplification, c and φ are considered to be uniform distribution within these ranges.

The ranges of c and φ are [0kPa, 250kPa] and [15°, 75°], respectively. Thus θ_0 represents that c and φ are two independent variables following uniform distribution within ranges[0kPa, 250kPa] and [15°, 75°], respectively; meanwhile the other variables' statistics are taken as given in Table 1.

Expanded RBD is executed to accomplish the preliminary design. In this study, β_T = 2.5 (i.e., $P_T = 6.2 \times 10^{-3}$) is adopted as the target reliability index and a sample size of 10^8 (i.e., $N_T = 10^8$) is used to improve further the resolution.

Since there are as many as 1581 designs, Fig. 2 shows the failure probability P_f for selected designs with slope height H=50m, 54m, …, 58m obtained from Expanded RBD and direct MCS. The curves for direct MCS is obtained by repeatedly running a MCS with 10^6 random samples for each design. These curves are used to benchmark the accuracy of the curves obtained from Expanded RBD. It can be seen that the P_f estimated from Expanded RBD agree well with those obtained from direct MCS, and P_f increases with the increase of both the H and ψ_f as expected. Note that the minimum P_f is about 0.0632 and no design achieves the reliability requirement (i.e., $P_f < P_T$), which can be attributed to the assumption that c and φ follow uniform distribution. In addition, as many as 9,820,000 failure samples are recognized and stored in this stage to implement the followed design updating with new probabilistic characterizations.

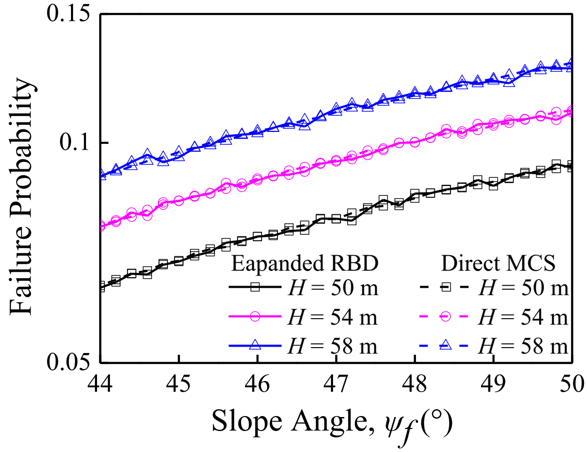


Fig. 2. Failure probability of rock slope at various slope height H from Expanded RBD and direct MCS in preliminary design

4.4 Updated designs using sample reweighting

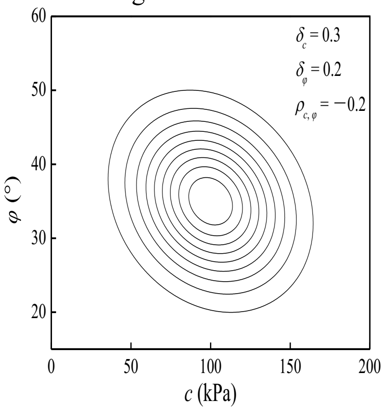
In this design, the mean of c and φ are ascertained to be 100 kPa and 35°, respectively. For illustrative purposes, four design situations with different values of δ_c, δ_φ and $\rho_{c,\varphi}$ as summarized in Table 2 are considered for rock slope RBD updating. Fig. 3 presents the contours of joint PDF of c and φ corresponding to different design scenarios. Comparing Figs. 3(a) and (c), Figs. 3(b) and (d), it can be noted that the ranges of c and φ with large COVs are wider than those with small COVs. Comparing Figs. 3(a) and (b), Figs. 3(c) and (d), it can be seen that more negatively correlation between c and φ implies that unfavorable strength combinations are less likely to occur than if c and φ are less negatively correlated. Such differences may lead to significant differences in RBD updating results, which will be discussed later.

Table 2. Values of δ_c, δ_φ, $\rho_{c,\varphi}$ under various design scenarios

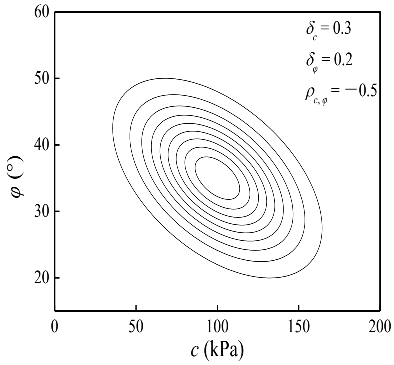
Scenario ID	δ_c	δ_φ	$\rho_{c,\varphi}$
Scenario I	0.3	0.2	−0.2
Scenario II	0.3	0.2	−0.5
Scenario III	0.2	0.14	−0.2
Scenario IV	0.2	0.14	−0.5

Then design scenarios listed in Table 2 are selected to update the design. For example, if Scenario I is selected, the new probabilistic characterizations θ represents that δ_c = 0.3, δ_φ = 0.2 and $\rho_{c,\varphi}$ =−0.2, the other random variables' statistics stay the same as listed in Table 1. In the updating process, the 9,820,000 failure samples stored in preliminary design are used to evaluate the P_f of all possible designs under different design scenarios by sample reweighting. For comparison, direct MCS with 10^6 random samples for each design is executed to estimate the P_f. Fig. 4 plots the estimated P_f under different design scenarios obtained from both sample reweighting and direct MCS. It is evident that the results from sample reweighting are consistent with those obtained directly from MCS. It also can be seen that the P_f for a given rock slope increases with the COVs of c and φ, and decreases with the negative correlation between c and φ. Note that instead of computational cost repeated reliability analysis, sample reweighting allows the P_f to be obtained through a single MCS run, which has already been performed in preliminary design.

Fig. 4 also shows $P_T = 6.2\times 10^{-3}$, feasible designs are those that fall below the P_T shown in the figures. The cost can be approximated as the volume of rock mass that must be excavated. By checking all the feasible designs, the least cost one can be chosen as the final design.



(a) Scenario I



(b)Scenario II

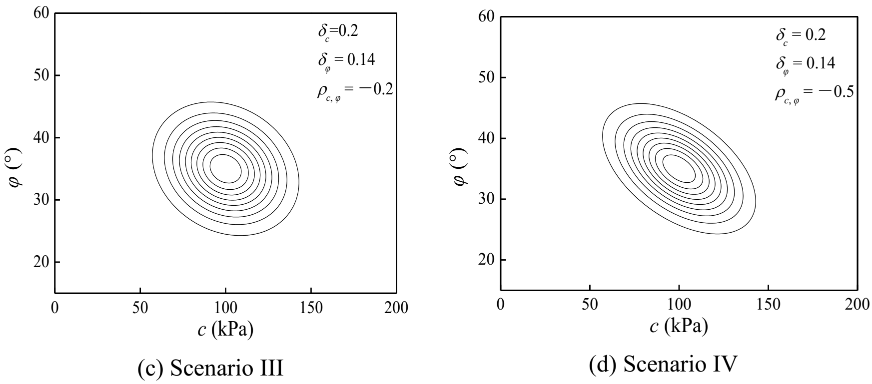


Fig. 3. Contour of joint PDF of c and φ for different design scenarios

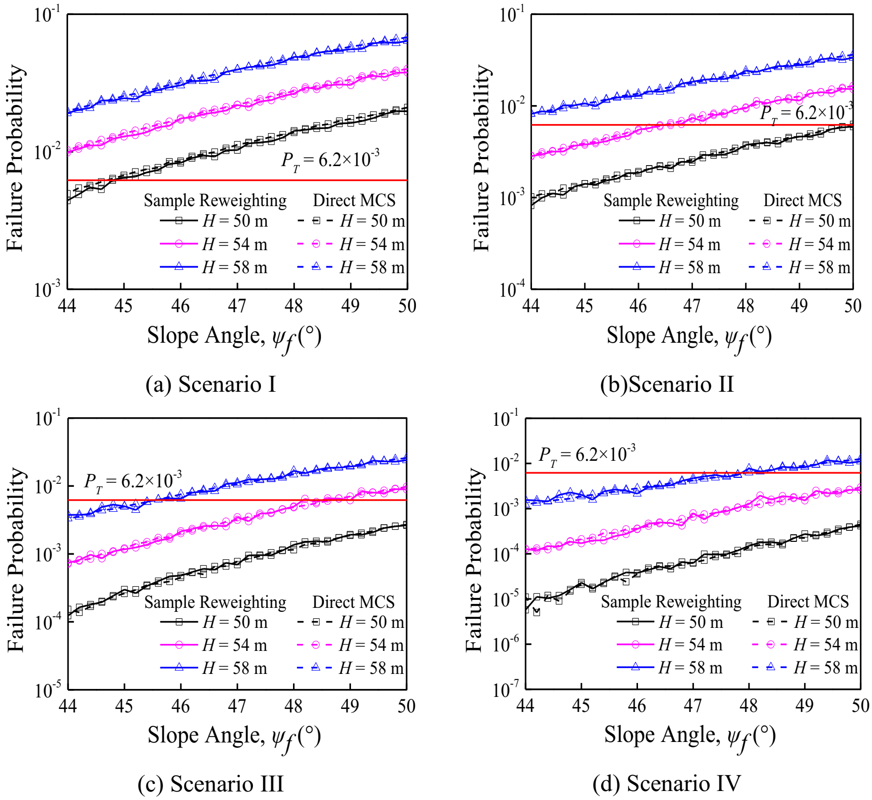


Fig. 4. Failure probability of rock slope for selected designs from sample reweighting and direct MCS under various design scenarios

5 Conclusions

This paper proposes an efficient reliability-based design (RBD) updating procedure for geotechnical structures, which integrates the Expanded RBD method based on Monte Carlo Simulation (MCS) sample reweighting. Equations are derived for evaluate failure probability (P_f) of geotechnical structures by integrating Expanded RBD with sample reweighting approach. The proposed procedure is presented and illustrated through a rock slope design example. Several conclusion can be drawn from this study:

1. The proposed procedure deliberately decouples the traditional deterministic analysis and reliability analysis. By this means, the reliability analysis can proceed as an extension of deterministic analysis in a non-instructive manner. In addition, by using the proposed procedure only one single MCS run, instead of trial-and-error procedure, is needed to accomplish the preliminary design and following design updating. It is easy-to-follow for the geotechnical practitioners with limited training in probability theory and statistics.
2. The P_f of geotechnical structures can be evaluate the accurately and efficiently using sample reweighting approach. The Pf obtained from sample reweighting are fairly consistent with those from direct MCS runs, which indicates that the sample reweighting approach is validate. Moreover,since sample reweighting approach properly reuses the failure samples generated from preliminary design stage, rather than running MCS repeatedly, the computational efficiency is improved significantly.
3. The proposed procedure can realize the design updating under different design scenarios. A design calculation using the proposed procedure is equivalent to a sensitive study on P_f versus the design parameters. It allows the geotechnical practitioners to adjust target failure probability, without additional calculation efforts, to accommodate the specific needs of a particular project. However, such adjustment is not possible for current RBD codes without re-calibrations.
4. The P_f of a given rock slope decreases with the negative correlation between the cohesion and friction angle. If such correlation is not taken into consideration, the reliability of rock slope will be underestimated, which results in a conservative final design with higher cost.
5. It is worth using the method proposed in this article to establish the link the between final design savings and site investigation. The costs associated with final design obtained from the proposed approach may decrease with increasing site investigation efforts.

Acknowledgments

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