Applying Particle Swarm Optimization Algorithm to Solve Securities Portfolio Based on Utility Maximization

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Abstract. In this paper, Particle Swarm Optimization (PSO) is introduced into the decision-making problem of securities portfolio. Particle swarm optimization algorithm is simple and easy to implement, and it is not easy to fall into local optimization due to random search. Based on the introduction of particle swarm optimization algorithm, the portfolio objective function is constructed. In the empirical process, the income and risk factors are taken into account at the same time, and a new utility maximization portfolio objective function is constructed by referring to the investor utility function, which makes the results more realistic. The application example shows that the particle swarm optimization algorithm can solve the problem of portfolio optimization accurately and quickly.

Keywords: particle swarm optimization algorithm； securities portfolio； utility maximization

1 Introduction

After more than 60 years of continuous revision and development, the portfolio theory of securities has formed a relatively mature system. Since Markowitz (1952) first proposed the return rate as a random analysis of portfolio returns, the mean-variance theory has become the cornerstone of the development of modern portfolio theory. Based on this theory, the problem of optimal portfolio can be summed up as solving an extreme value problem under multiple constraints by applying the mathematical programming method of operational research. Most of the relevant studies of domestic

R. Chitrakar et al. (eds.), *Proceedings of the 4th International Conference on New Computational Social Science (ICNCSS 2024)*, Advances in Social Science, Education and Humanities Research 844,
https://doi.org/10.2991/978-2-38476-230-9_34

and foreign scholars are carried out according to this idea [1-3]. The actual solving process of such problems is often very complicated.

With the rapid development and continuous improvement of social economy and computing technology, some new methods with nonlinear and distributed computing capabilities based on artificial intelligence technology have also been applied to more and more fields to achieve fast and efficient solution of complex problems. In the field of finance, some scholars have begun to use artificial intelligence technology to solve securities portfolio. Oh et al. (2006) [4], Wang et al. (2022) [5] have used genetic algorithms, Solis et al. (2022) [6] 、Chen et al. (2018) [7] have used simulated annealing algorithm, Adosoglou et al. (2021) [8] 、Deng et al. (2021) [9] have used neural network algorithm and bee colony algorithm respectively to solve the portfolio problem. After introducing these artificial intelligence algorithms into the portfolio problem, the simplicity of solution is improved. However, the implementation steps of these algorithms are quite tedious. Even the genetic algorithm, which is highly respected in the field of optimization technology, also requires complex crossover, mutation and selection operations, making it difficult for these optimization technologies to meet the actual optimization needs, and most optimization technologies are prone to fall into local optimization, which greatly reduces the accuracy.

In this paper, Particle Swarm Optimization (PSO) algorithm will be used to study the portfolio problem. This bionic algorithm is relatively simple and easy to implement, and it is not easy to fall into local optimization due to random search. In addition, on the components of the portfolio objective function, we use the variable of investor utility to measure the effect of the portfolio, including both the return and the size of risk, and comprehensively analyze the effect of the constructed portfolio. The application of PSO algorithm will further enrich the methods and theories of portfolio optimization. We have verified the effectiveness of this algorithm through an example.

2 Particle swarm optimization algorithm

Particle Swarm Optimization (PSO) is based on the principle of complex adaptive system (CAS), which was developed by Kennedy and Eberhart in 1995 through the research and analysis of bird swarm system.

2.1 Mathematical expression of particle swarm optimization

In the PSO algorithm, the solution of the research problem is regarded as a point in the D dimension space, also known as a particle, and the parameter waiting for optimization adjustment is the particle position. Each particle gets a fitness value according to the objective function and moves at a certain speed and direction. The optimal solution found by each particle itself is called the individual extreme value, and the optimal solution found by particle swarm is called the global extreme value. These particles fly at a certain speed and direction in the whole space. Through the experience of the particles themselves and the experience of the particle swarm, they constantly adjust the flight angle and speed, and update the individual extreme value and the global extreme value at the same time, until the end condition is reached, the cycle is completed, and the optimal solution is obtained.

The mathematical expression of the PSO algorithm is: let the search space be D dimension, the number of particles in the particle swarm be m, and the position of the ith particle is $X_i = (x_{i1}, x_{i2}, x_{i3}, \ldots, x_{iD})$; The velocity of the ith particle is $V_i = (v_{i1}, v_{i2}, v_{i3}, \ldots, v_{iD})$; The position of the optimal solution experienced by the ith particle in space is $P_i = (p_{i1}, p_{i2}, p_{i3}, \ldots, p_{iD})$;The position of the optimal solution experienced by the particle swarm in space is $P_g = (p_{g1}, p_{g2}, p_{g3}, \ldots, p_{gD})$. The PSO algorithm updates the particle position as follows:

$$v_{id}(t+1) = \omega \times v_{id}(t) + c_1 \times r() \times [p_{id}(t) - x_{id}(t)] + c_2 \times r() \times [p_{gd}(t) - x_{id}(t)]$$

$$x_{id}(t+1) = x_{id}(t) + v_{id}(t+1) \quad 1 \le i \le m, \quad 1 \le \mathrm{d} \le \mathrm{D} \qquad (1)$$

Where: r () is a random number between [0, 1], ω It is a non-negative number, called inertia factor, with a value range of 0.1~0.9, which represents the individual cognition of the particle to itself, and is the weight of the tracking particle. Its value is usually set to 2, which represents the social cognition of the particle to the particle swarm, and is the weight of the tracking particle swarm. Its value is usually set to 2.

2.2 Implementation process of particle swarm optimization

Step 1: initialize the position and velocity of m particles randomly;

Step 2: evaluate the fitness of each particle, store the current position and fitness of each particle in the individual extreme value of each particle, and store the position and fitness of the individual with the best fitness of all particles in the global extreme value;

Step 3: update the formula according to the position and velocity to update the position and velocity of particles;

Step 4: For each particle, compare its fitness with the best position it has experienced. If it is better, take it as the current best position;

Step 5: For each particle, compare its fitness with the best position that all particles have experienced, and if it is better, take it as the best position of all particles at present;

Step 6: If the termination condition (usually the preset operation precision or iteration number) is met, the search will stop and the result will be output; Otherwise, return to step 3 to continue searching.

3 Establishment of portfolio objective function based on particle swarm optimization algorithm

Markowitz's portfolio model needs to be based on the assumption that short selling of securities is allowed in the securities market. At present, because there is no short selling mechanism in China's securities market, investors cannot directly use the portfolio model to calculate. In addition, due to the random variables that investors face when investing, in addition to the situation of income, there are also corresponding risks. If we only consider the single factor of income or risk, we cannot reach the optimal state. Therefore, we have used the utility of investors as a variable to measure the effect of the investment portfolio, including both the income and the size of the risk, and comprehensively analyzes the effect of the constructed investment portfolio. The utility function of investors is as follows:

$$U = (1-\lambda)E(r) - \lambda\sigma \quad (2)$$

U is the utility value; λ is the risk aversion degree of investors. The greater the λ, the greater the risk aversion degree of investors, and the smaller the corresponding utility. The larger U is, the greater the utility of the constructed portfolio is, the better the effect is. Therefore, it is necessary to find the group with the largest utility value U under different portfolio conditions. However, the objective function of PSO particle swarm optimization algorithm generally seeks the minimum value. Therefore, the objective function of the constructed portfolio is deformed, let $\min f(x) = -U$, the result is as follows:

$$\min f(x) = \lambda \cdot \left(\sum_{i=1}^{n}\sum_{j=1}^{n} x_i x_j \sigma_{ij}\right)^{1/2} - (1-\lambda)\cdot\sum_{i=1}^{n} x_i r_i \quad (3)$$

$$0 \le \lambda \le 1$$

$$\forall i, x_i \ge 0$$

$$\sum_{i=1}^{n} x_i = 1$$

Where, σ_{ij} represents covariance, and when i=j, σ_{ij} represents variance. r_i represents the expected yield of the stock.

4 Empirical analysis

4.1 Data selection and processing

From five different industries, mining, wholesale and retail, construction, real estate and manufacturing, we have selected one Shanghai A-share stock as the object of empirical research, namely Sinopec (600028), Liaoning Chengda (600739), Seiko Steel (600496), Wolong Real Estate (600173), and Harbin Pharmaceutical (600664). Select the closing price from October 8, 2021 to September 20, 2022 as the original data, and the datas is from Juyuan database.

We use the above closing price sequence to calculate the yield of the stock, and divide the closing price of the day by the closing price of the previous day and subtract 1 to get the yield of the stock on that day.

$$r = \frac{P_i - P_{i-1}}{P_{i-1}} = \frac{P_i}{P_{i-1}} - 1 \quad (4)$$

According to the yield of five stocks, the average daily yield of each stock can be calculated:

TABLE 1 DAILY YIELD OF EACH STOCK

Stock code	600028	600739	600496	600173	600664
Yield	0.00017	0.00043	0.0011 2	-0.00012	0.00076

Calculate the variance and covariance of each stock through the yield series of each stock, and the results are as follows:

TABLE 2 VARIANCE-COVARIANCE MATRIX OF STOCKS

	600028	600739	600496	600173	600664
600028	0.00021	0.00015	0.00011	0.00015	0.00010
600739	0.00015	0.00054	0.00024	0.00024	0.00023
600496	0.00011	0.00024	0.00060	0.00023	0.00018
600173	0.00015	0.00024	0.00023	0.00048	0.01339

We have used matlab 7.0 software to program the portfolio problem and calculate the PSO particle swarm optimization algorithm.

4.2 Determine the parameters of the model

First, we compare different particle numbers, inertia weights and maximum iteration times to determine a relatively reasonable parameter ratio. Then, in the case of obtaining the optimal parameters, the corresponding optimal investment strategies of investors under different risk preferences are discussed respectively.

1) The particle number N is different

TABLE 3 OBJECT FUNCTION VALUES WITH DIFFERENT PARTICLE NUMBERS

Number of particles	30	40	50	60
Objective function	0.0084	0.0082	0.0078	0.0082

According to the test, when other parameters remain unchanged, when the number of particles is 50, the target function can take a smaller value, so the number of particles N is set to 50.

2) Inertia weight ω Different

TABLE 4 OBJECTIVE FUNCTION VALUES WITH DIFFERENT INERTIA WEIGHTS

Inertia weight	0.45	0.5	0.55	0.6
Objective function	0.0086	0.0078	0.0079	0.0081

According to the test, when other parameters remain unchanged, when the inertia weight is 0.5, the target function can take a smaller value, so the inertia weight ω Set to 0.5.

3) The maximum number of iterations n is different

TABLE 5 OBJECTIVE FUNCTION VALUES WITH DIFFERENT ITERATION TIMES

Maximum number of iterations	80000	100000	120000	150000
Objective function	0.0091	0.0078	0.0078	0.0077

According to the test, when the maximum number of iterations exceeds 100000, the objective function will not change almost if other parameters remain unchanged. In order to improve the operation speed, the maximum number of iterations n is set to 100000.

According to repeated tests, the parameter setting of particle swarm optimization algorithm is finally determined as particle number N=50, inertia weight ω= 0.5, when the maximum number of iterations n=100000, the obtained objective function value can be the minimum, and the parameter configuration is the best.

4.3 Determination of optimal portfolio

Particle swarm optimization algorithm can be used to select the optimal portfolio to meet the needs of investors to maximize utility. However, different investors will have different optimal strategies in the face of even the same securities. Therefore, according to investors' attitudes towards risk and return, we have discussed different λ In this case, what is the allocation of asset weight and determine the optimal portfolio.

1)If investors are more concerned about risk than income, suppose λ= 0.6, the result is as follows.

TABLE 6 OPTIMAL PORTFOLIO UNDER RISK PREFERENCE

Indicators	Risk appetite	Income preference	Risk function value	Income function value	Objective function value
Result	0.6	0.4	0.00810	0.00016	0.0079
Stock code	600028	600739	600496	600173	600664
Investment weight	0.6027	0.0953	0.1445	0	0.1575

When λ= 0.6, according to the weight shown in the table, the ratio of asset portfolio can reach the best, and the objective function value is 0.0079.

2)If investors have the same attitude towards risk and income, assume λ= 0.5, the result is as follows.

TABLE 7 OPTIMAL PORTFOLIO UNDER THE SAME RISK AND RETURN PREFERENCES

Indicators	Risk appetite	Income preference	Risk function value	Income function value	Objective function value
Result	0.5	0.5	0.00700	0.00059	0.0064
Stock code	600028	600739	600496	600173	600664
Investment weight	0.6223	0.0118	0.0983	0	0.2676

When λ= 0.5, according to the weight shown in the table, the ratio of asset portfolio can reach the best, and the objective function value is 0.0064.

3) If investors are more concerned about income than risk, suppose λ= 0.4, the result is as follows.

TABLE 8 OPTIMAL INVESTMENT PORTFOLIO UNDER PREFERENCE INCOME

Indicators	Risk appetite	Income preference	Risk function value	Income function value	Objective function value
Result	0.4	0.6	0.0054	0.00037	0.0050
Stock code	600028	600739	600496	600173	600664
Investment weight	0.5422	0.1461	0.1982	0	0.1136

When λ= 0.4, according to the weight shown in the table, the ratio of asset portfolio can reach the best, and the objective function value is 0.0050.

To sum up, according to λ= 0.4 to achieve the best investment method. At this time, the objective function is minimized under the consideration of both income and risk.

By studying the sample data of China's securities market, we have constructed a securities investment model that conforms to China's actual situation. The model not only considers that short selling is not allowed in China's securities market, but also discusses different investors' different preferences for risk and return, making the model applicable to different types of investors. Finally, an example is solved by particle swarm optimization, which proves the practicability of the model, and also shows that particle swarm optimization can effectively solve the portfolio model, find the optimal asset allocation strategy, and complete the investment decision.

5 Conclusion

Using portfolio can reduce investment risk and improve the effectiveness of investment. However, there are many ways to determine the weight of each stock in the portfolio to meet the utility maximization. On the basis of fully considering the respective returns of five stocks from different industries and their associated risks, we have applied particle swarm optimization algorithm to solve the optimal weight ratio by generating particles of particle swarm optimization to continuously update the speed and position, so as to guide investors to build their portfolios. Compared with other random algorithms, Particle Swarm Optimization (PSO) solves the problems that gradient descent method is easy to fall into local extremum and slow convergence speed. Moreover, this algorithm has no centralized control constraints, and its distribution form is parallel, so the system is more robust. In the empirical process, the income and risk factors are taken into account at the same time, and a new objective function is constructed by referring to the investor utility function to make the results more realistic.

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