Comparison of American, European, and Exotic Options and Pricing Methods

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Abstract. As a matter of fact, the valuation of options is a fundamental element of financial markets and investment in general especially in recent years. With this in mind, this study provides an overview and analysis of three commonly used options as well as the methods used to price them. To be specific, these options include the Binomial tree model, the Black-Scholes Model, and Monte Carlo simulations. Additionally, recent research on option pricing is presented, highlighting various aspects such as flexibility, pricing premiums, market predictability strategy, and investor suitability. At the same time, this study examines the constraints of existing research and identifies prospective domains for further study. According to the analysis, the main aim of this study is to introduce and contrast three different choices along with their respective pricing systems. In the meantime, it also aims to extend current research concerns and provide guidance to future researchers, which shed light on guiding further exploration of option pricing.

Keywords: American Options, European Options, Exotic Options, Binomial Tree Model, Monte Carlo Simulations.

1 Introduction

Options as significant financial instruments provide critical means for managing risk and leveraging opportunities in modern financial markets. The practice of option-like trading dates all the way back to ancient Greece and Rome when they were used in agricultural commerce. However, not until the 17th century did an option resembling today's features emerge during the Tulip Mania in the Netherlands. The provision of insights into the early speculative market and option trading practices can be considered invaluable [1]. The establishment of formal options trading gradually materialized during the 18th and 19th centuries. Although option contracts were traded on the London Stock Exchange, they lacked the standardized language and regulatory oversight that contemporary options markets now possess. The formation of the Chicago Board Options Exchange (CBOE) in the 1970s played an influential role in the advancement of contemporary options. The CBOE, as the first organized options exchange, standardized and regulated the market, thereby enhancing accessibility and significantly increasing liquidity [2]. Concurrently, the introduction of the Black-

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Scholes model in the same year, originally employed for evaluating European options, signified a noteworthy progression in the evaluation of option value. This groundbreaking financial model, initially devised by Fischer Black and Myron Scholes, and subsequently enhanced by Robert Merton, provided a robust theoretical framework for estimating option prices in a risk-neutral setting [3]. Accordingly, the options market experienced substantial growth during the latter half of the 20th century. This growth was accompanied by the introduction and standardization of a diverse array of options, including European and Exotic options, reflecting the increasing complexity and diversification of financial instruments. This expansion also led to a corresponding increase in research on option-related subjects. The study of options is crucial for achieving better risk management, understanding market dynamics, and developing investment strategies. To date, numerous esteemed scholars have conducted insightful research on option evaluation. This paper will focus on three commonly used options: American, European, and Exotic.

The evaluation of American options has always presented a complex challenge due to their ability to be exercised at any time prior to expiration. Initially, approximation techniques were employed to determine their value. However, in 1979, the industry experienced a revolution with the introduction of the binomial options pricing model by Cox, Ross, and Rubinstein [4]. This model uses a continuous-time framework to calculate the value of the option at different time intervals leading up to its expiration. Over time, researchers have developed more accurate pricing methods, such as lattice and finite difference methods, particularly for options with dividend payments. Presently, experts continue to explore numerical and analytical methods to enhance efficiency and improve pricing accuracy.

The Black-Scholes-Merton model, which came about in 1973, serves as the groundwork for evaluating the value of European options. This model provides an analytical solution for pricing options under the assumption of continuous volatility and no dividends. Further research has enlarged the model to involve varying market conditions, such as stochastic volatility, jump-diffusion processes, and the inclusion of dividends. Noteworthy contributions in this context incorporate Heston's effort on stochastic volatility models [5] and Merton's effort on jump-diffusion models [6]. Recently, there has been a particular emphasis on refining volatility modeling, which involves utilizing machine learning methodologies to make predictions on volatility and explore the implications of market microstructure on option pricing.

Exotic options have recently garnered increased attention on account of their intricate characteristics and distinct structures, which encompass path dependency and diverse payoffs. Considerable investigation has been carried out on specific classifications of unconventional financial instruments, including Asian, Barrier, and Lookback options. Monte Carlo simulations are extensively employed in the evaluation of intricate options due to their adaptability and capacity to accommodate numerous varieties of payoffs and dependencies on the underlying asset's trajectory. The objective of this research is to examine the concept of options and their pricing methodologies, encompassing American, European, and exotic options. The investigation will encompass the attributes of each type of option and their present valuation techniques. Furthermore, recent research on pricing various options will be deliberated. Ultimately, the study will juxtapose different types of options and yield novel insights into comprehending them.

2 Option Descriptions and Pricing Method

Options have a pivotal function as monetary instruments within current markets, furnishing an expansive selection of tactics for both investors and traders. Within the extensive gamut of accessible alternatives, there are three noteworthy types: American, European, and Esoteric alternatives. Each classification possesses unique characteristics and valuation methodologies.

American options grant the holder the privilege, without compulsion, to purchase a call option or sell a put option with an underlying asset on any given period before the expiration date at a fixed price. The ability to exercise the option at any point before to expiration is what sets American options apart from their European counterparts. The pricing of American options becomes more complex due to this feature. The Binomial Tree Model, which can effectively simulate the varying exercise times of the option, is a widely adopted approach for pricing such options. The construction of the binomial tree involves the discretization of time into a series of discreet increments. This process enables the modeling of potential price movements for the underlying asset at each step. At each node within the tree structure, two possible price movements are considered: an upward motion and a downward motion. Initiating from the terminal nodes of the tree (when it ends), the profits associated with the options are established by computing the disparity between the strike price of the option and the asset price at each node. In the case of American options, the potential for premature execution must be considered at every step. Consequently, the prices of the options are determined in a reverse recursive manner within the tree. At every node, the option's value is calculated by discounting the expected value of the option's payoffs in the next period. This calculation considers both the risk-neutral probabilities and the decision regarding early exercise for American options [7]. This method is particularly favored for its adaptability in adjusting for dividends and other factors that influence option pricing.

European options, in contrast, possess the characteristic of being exercisable solely at the specific point of expiration. This exercise point serves to streamline the process of determining their pricing methodology. The Black-Scholes Model (BS Model), an innovative equation formulated by Fischer Black and Myron Scholes in 1973, is used as the conventional approach for pricing European options. The tool offers a theoretical valuation of European-style options, taking into account several parameters such as the option's strike price, the present value of the underlying asset, the remaining time before expiration, the volatility of the asset, and the risk-free interest rate [8]. The Black-Scholes Model assumes a lognormal distribution of prices, continuous trading, and the absence of dividends throughout the option's lifespan. Its extensive acceptance and utilization can be ascribed to its ability to furnish a closed-form resolution for option pricing, a feature that entails less computational intensity when compared to the approaches implemented for American options.

Exotic options, which deviate from the conventional American and European options in terms of their payoff structure, underlying assets, or expiration dates, form a distinct category within the realm of options. This diverse category encompasses a wide array of financial products, including Barrier options, Asian options, and Lookback options, each tailored to suit specific investment strategies or risk preferences. The unique attributes and intricacies of exotic options often necessitate the use of specialized pricing models, given their bespoke nature. In pursuit of an accurate valuation, Monte Carlo simulations (MC) frequently serve as the go-to approach. This method involves the repetition of simulating the underlying asset's price trajectory numerous times, thereby enabling the determination of the option's payoff under various scenarios, and ultimately establishing its fair value. This is commonly achieved by employing a stochastic process such as Geometric Brownian Motion. This process encompasses critical parameters such as drift (mean return), volatility, and the risk-free rate. Subsequently, following the initial event, a substantial quantity of arbitrary trajectories of valuations for the fundamental resource are generated, meticulously adhering to the pre-established probabilistic pattern throughout the entire duration of the lifespan of the option. Each path represents a distinct potential future scenario for the asset's price. The exotic option's payoff is then calculated for each simulated path. In the end, the mean return is calculated for the exotic option's payoff, considering its specific terms, such as barrier levels, lookback features, Asian averaging, and others. This discounted average value serves as the MC estimate for the exotic option price. The inherent versatility of Monte Carlo simulations renders them an optimal choice for pricing options that exhibit path-dependent features or involve multiple underlying assets.

3 New Developments in Option Pricing

Apart from the more conventional approach to pricing options discussed above, there have been recent advancements in evaluating American, European, and Exotic options. These novel researches are designed to illuminate diverse aspects of option valuation, thereby contributing to a more extensive comprehension of this intricate monetary concept. These advancements have cleared the path for a more thorough investigation into the complexities of option pricing, empowering analysts and experts alike to explore unexplored territories within this domain.

3.1 American Option

The most recent research for pricing American Options using the binomial tree model has seen significant advancements. A recent study presented a nonparametric predictive inference for pricing American options using the binomial tree model, offering a helpful approach for determining their prices [9]. The current exposition elucidates the American option valuation technique rooted in the binomial tree, delving into the realm of imprecise statistical analysis. In this context, the Nonparametric Predictive Inference (NPI) framework is employed to infer uncertain probabilities pertaining to the underlying asset's movements. By incorporating this imprecision, the process of learning from data is enhanced, surpassing the limitations imposed by the utilization of constant risk-neutral probabilities. Utilizing the concept of the backward pricing strategy. Let $h_t = (x)$denote the instant value of the American option at time t, $0 \leq t \leq T$, given $S_t = x$. Then $h_t(x) = x - K_c$ for a call option and $h_t(x) = K_p - x$ for a put option. $V_t(x)$ is the option value at time $t \in \{0,1,\ldots,T-1\}$ given $S_t = x$. The American option value at time t is

$$V_t(x) = \max\{h_t(x), B(t,t+1)V_{t+1}(S_{t+1}|S_t = x)\} \tag{1}$$

$$V_T(x) = \max\{h_T(x), 0\} \tag{2}$$

Here $B(t,t+1)$ is the discount factor between times t and $t+1$. By employing the concept of recursion, it is easy to derive the valuation of the American option during its initial time, denoted as $V_0(x)$. This particular value represents the anticipated price of this American option.

An additional research article proposed a novel numerical technique for valuing American-style options using the HODIE (high order via differential identity expansion) finite difference scheme. The study delves into the intricate difficulty of determining the appropriate valuation for American options, a task that becomes even more challenging due to the deteriorating nature of the Black-Scholes operator when the asset price reaches zero, as well as the presence of non-smooth payoff functions. In order to circumvent the computational inaccuracies that are commonly encountered when employing conventional techniques, a novel HODIE finite difference scheme is introduced, which effectively addresses the linear complementarity problem associated with American put options. Also, the study employs a singularity-separating technique to overcome the non-smoothness intrinsic in the payoff function, thereby substantially boosting the precision and reliability of the pricing methodology used. This method provides a robust approach for valuing American options, ensuring accuracy and efficiency in the valuation process [10]. Another interesting research utilizes the Logistic Regression in the MC method to value American options. The different situations that have been considered include cases where the option is in, out, and at the money. This exploration aims to give valuable viewpoints on the practical utilization of machine learning techniques in the pricing of options. Consider the scenario of whether early exercise is applicable or not for each stock price trajectory is $f_i = (1{:}\ early\ exercise; -1{:}\ not\ early\ exercise)$, and let P be the probability of each path and S is the simulate stock price:

$$P = E(f_i = -1|S_i) \tag{3}$$

$$\text{Odds ratio} = \frac{P}{1-P} = \frac{\text{probability of early exercise}}{\text{probability of continuation}} \tag{4}$$

$$L_n \frac{P}{1-P} = f(S_i{:}\ simulated\ stock\ price) = \beta_0 + \beta_1 S \ldots + \beta_k S^k + \varepsilon \tag{5}$$

$$P = \frac{1}{(1+e^{\beta_0+\beta_1 S\ldots+\beta_k S^k})} \tag{6}$$

By showcasing the potential of advanced computational methods in valuation models, this research underscores the significance of leveraging such approaches in the realm of option pricing [11].

3.2 European Option

Recent research studies conducted within recent years have made considerable advancements in the field of European option pricing valuation. Prathumwan and Trachoo explored the intricacies involved in option pricing within the market by thoroughly examining the multifaceted nature of the two-dimensional time fractional-order BS equation when applied to a European put option [12]. The explicit solution of the two-dimensional time-fractional BS model is provided as:

$$\begin{aligned}\omega(x,y,t) &= \max(E-(\beta_1 e^x+\beta_2 e^y),0)+e^{x+y}t^\alpha \\ &+ \max(\beta_1 e^x,0)\frac{t^\alpha\sigma_1^2}{2}E_{\alpha,\alpha+1}\left(\frac{t^\alpha\sigma_1^2}{2}\right)+\max(\beta_2 e^y,0)\frac{t^\alpha\sigma_2^2}{2}E_{\alpha,\alpha+1}\left(\frac{t^\alpha\sigma_2^2}{2}\right) \\ &+\left[e^{x+y}\left(\frac{\sigma_1^2}{2}+\frac{\sigma_2^2}{2}+\rho\sigma_1\sigma_2\right)\Gamma(\alpha+1)t^{2\alpha}E_{\alpha,2\alpha+1}\left(t^\alpha\left(\frac{\sigma_1^2}{2}+\frac{\sigma_2^2}{2}+\rho\sigma_1\sigma_2\right)\right)\right] \\ &-e^{x+y}\Gamma(\alpha+1)t^\alpha E_{\alpha,\alpha+1}\left(t^\alpha\left(\frac{\sigma_1^2}{2}+\frac{\sigma_2^2}{2}+\rho\sigma_1\sigma_2\right)\right)\end{aligned} \tag{7}$$

Where:

$$E_{\gamma,\eta}(z)=\sum_{k=0}^{\infty}\frac{z^k}{\Gamma(\gamma k+\eta)} \tag{8}$$

is the generalized Mittag-Leffler function, in which γ and η are constants. Their research efforts involved a meticulous and comprehensive analysis of the dynamic factors that influence the pricing of options, providing a holistic understanding of the underlying dynamics that drive this phenomenon in the financial markets. Furthermore, a framework was proposed by Fadugba, Adeniji, and Kekana, which is based on the Mellin transform. This framework is designed to offer a straightforward solution for the Black-Scholes-Merton European Put Option Model on Dividend Yield, utilizing a Modified-Log Payoff Function [13]. The Mellin transform (MT) offers a direct solution for the BSM European Put Option Model on Dividend Yield with Modified-Log Payoff Function, providing a straightforward approach to solving this specific financial model. Utilizing MT proves to be highly effective when dealing with intricate functions, making it a comparable option to other well-known transforms such as Laplace and Fourier. The consequences of the research indicate that the Modified-Log Payment Function, which is used in this specific scenario, surpasses the more commonly employed log payment function and closely aligns with the linear payment function. Interestingly, an intriguing relationship emerges between the Dividend Yield and the prices of the European Put Option with the Modified-Log Payoff Function, as an increase in Dividend Yield leads to a corresponding increase in the prices of these options. Consequently, the Mellin transform proves to be a suitable method for valuing European Put Options on the Modified-Log Payoff Function with consideration for Dividend Yield, as it provides the ability to directly solve the pricing model in terms of market price, thereby offering a comprehensive approach to pricing these options. This approach offers a novel and innovative method by incorporating dividend yield and modified-log payoff function into the option pricing model. It significantly enhances the flexibility and applicability of European option valuation, enabling it to be effectively used in various market conditions.

3.3 Exotic Option

As for Exotic options, Lyons, Nejad, and Arribas conducted an analysis of exotic options, explicitly focusing on their prices in a discrete-time model-free setting [14]. This research approach allowed them to make estimations using rough path signatures. The research aims to expand the concepts of model-free finance to discrete-time frameworks for the purpose of pricing exotic derivatives. In this case, it introduces the notion of signature payoffs as a fundamental principle for replicating and determining the pricing of exotic options. Using this approach, they anticipated signatures from market prices, thereby effectively allowing them to characterize the market's underlying dynamics. It is important to note that this method builds upon ideas presented in contemporary research and holds significant potential for applications in the field of risk management. To validate the effectiveness of the approach, they conduct a series of numerical experiments, which present concrete evidence demonstrating the viability and effectiveness of the proposed approach. It not only contributes valuable insights to the study of model-free finance and exotic derivatives but also offers a promising avenue for exploration and implementation in the field of risk management.

Qiao et al. conducted a study that developed a pricing formula for exotic options using a comprehensive logarithmic technique, comparing its price advantage to classical European options and exotic options with a linear position [15]. The research presents a novel approach to hedging European options, which is characterized by a proactive strategy incorporating a general logarithmic position strategy. By allowing for continuous trading of underlying assets, this new exotic option enables traders to mitigate risks effectively. The study findings suggest that exotic options implementing a logarithmic method in general can result in more substantial reductions in option premiums compared to those applying a linear position strategy. Furthermore, the cost benefit of the suggested unconventional choice is especially noticeable for traders with little initial money. Hence, this research brings forth valuable perspectives on options trading and provides a feasible option for market participants looking to optimize their risk management strategies.

4 Comparative Analysis

Regarding options, it is essential to note that American options hold a position of prominence due to their unparalleled flexibility, which allows for the execution of the option at any point in time. This characteristic renders American options particularly advantageous in scenarios marked by market volatility or instances where the anticipation of dividends plays a significant role. However, the flexibility of American options comes at a cost. It is essential to acknowledge that American options generally come with a higher premium in comparison to their European counterparts. This disparity can be attributed to the elevated level of risk assumed by the seller of the American option, which arises from the potentiality of early exercise. It is worth emphasizing that this distinctive characteristic of American options proves to be particularly advantageous in the context of stocks that distribute dividends. It is notable that investors who possess American call options possess the prerogative to exercise their options immediately prior to the stock's dividend disbursement, thereby securing the dividend payment for themselves. Besides, American options are widely utilized and can be observed across a multitude of underlying assets including stocks, indexes, and ETFs. These options are highly regarded and widely used in the US stock market because of their extraordinary versatility and huge trading volume. Moreover, individual investors are particularly inclined towards American options, as they offer a range of advantages and opportunities for strategic decision-making.

Conversely, European options possess a distinct feature that sets them apart from their American counterparts by virtue of having a predetermined date on which the option can be exercised, consequently lacking the same degree of adaptability found in American options. European options usually come with a lower premium, reflecting the fact that the holder cannot take advantage of market changes before the expiration date. Although European options have certain limitations, they continue to be greatly coveted in specific markets, particularly in the realm of forex and commodities trading. These options possess a sense of simplicity and predictability that renders them more appropriate for strategies centered around precise forecasts pertaining to market conditions on the expiration date. It is worth noting that European options hold the status of being the standard option type of index options, which are financial derivatives based on stock market indexes. European options cannot be physically delivered or exercised in the traditional sense, as they solely rely on cash settlement. European options present a more direct and uncomplicated approach to settling contracts, wherein the underlying security is exchanged for cash upon expiration.

Exotic options emerge as a highly appealing option due to their ability to offer customized structures that yield exceptional returns. However, it is crucial to acknowledge that exotic options represent the most intricate form of option, necessitating a comprehensive understanding of their intricacies and the utilization of more complex calculation methods for both pricing and comprehension purposes. Exotic options offer a significant level of versatility, making them extremely suitable for investors with specific needs or as a tool to reduce specific risks. These options can be meticulously designed to tackle market conditions that cannot be adequately addressed by conventional options. Exotic options possess intrinsic flexibility and customization. However, this advantage is accompanied by a higher level of complexity and generally less liquidity when compared to ordinary options. Experienced investors and institutions often utilize exotic options due to their advanced features, which allow them to meet specific risk management requirements or take advantage of market fluctuations that cannot be effectively addressed with regular options. Proficiency in the deployment of these options necessitates a deep understanding of the fundamental dynamics of the market and the detailed specifics of the option contract.

In fact, American options offer a significant level of versatility, granting the holder the ability to execute options at any time before its expiration. European options only allow exercise on the expiration date, but exotic options have exercise conditions that are determined by their distinctive structure. Besides, it often commands higher premiums due to their flexibility, while European options are generally cheaper. Exotic options vary widely in pricing, depending on their complexity and specific features. European options are more predictable for strategies focused on the expiration date. American options offer more strategies due to their flexibility. Exotic options provide customized strategies for specific market conditions or needs. American options are suitable for investors who value flexibility and are willing to pay a higher premium. European options are preferred by those who have a specific market view at expiration and seek lower premiums. Exotic options are for sophisticated investors or institutions with unique market predictions or hedging needs.

5 Limitation and Future Prospects

Many financial models, including the Black-Scholes model, have several flaws due to their reliance on naive or unrealistic assumptions. These models make the assumptions that market volatility is constant, asset prices follow a log-normal distribution, and there are no transaction costs or market frictions. However, these assumptions can lead to inaccuracies in the pricing of financial instruments, mainly in markets that exhibit significant volatility or possess unique characteristics that differ from the assumptions made by the models. Advanced models, which attempt to tackle the unrealistic assumptions inherent in simpler models, such as stochastic volatility models or models with jump diffusion, tend to exhibit higher complexity. This complexity arises from the inclusion of additional factors that capture the inherent randomness or sudden changes in asset prices. Consequently, accurately calibrating these advanced models to market data becomes a challenging task, as it requires the estimation of a larger number of parameters. The difficulty in accurately calibrating these models might lead to mispricing or model risk, where the model fails to precisely capture the true behavior of the financial markets. Financial markets are often analyzed using conventional models. However, these models tend to overlook the influence of market sentiment, investor behavior, and psychological factors on market dynamics. Behavioral biases and herd behavior can significantly affect the functioning of financial markets, but these factors are typically disregarded by conventional models. These models may not be able to accurately capture the complex dynamics that underlie the pricing of options. With the advent and rise of novel asset categories, exemplified by the proliferation of cryptocurrencies, coupled with the introduction of innovative financial tools, the effectiveness and relevance of prevailing models may be called, as their direct applicability to these emerging markets might be compromised or necessitate substantial recalibrations to ensure the provision of precise valuations.

Future investigation in the realm of option valuation is expected to concentrate on tackling the existing constraints and accommodating the progressive environment of financial markets. Incorporating the principles of machine learning and artificial intelligence has the potential to produce more flexible and advanced techniques. It can facilitate a thorough comprehension and examination of market trends, identification of volatility patterns, and recognition of complex interrelationships among multiple variables. With the exponential growth and widespread adoption of blockchain technology, coupled with the surging popularity of cryptocurrencies, there has been a growing interest in creating advanced models that can accurately and efficiently calculate the values of options in these new and highly volatile markets. Exploration of various models that integrate the fundamental principles of behavioral finance has the potential to furnish a significantly more precise comprehension of the intricate dynamics that govern the market. By taking into consideration the complex nuances of investor psychology, encompassing emotions such as fear, greed, and herd mentality, it is plausible to imbue option pricing models with a heightened degree of realism and, consequently, augment their predictive capabilities. Eliminating these psychological factors may widen the gap between theoretical models and market participants' actual behavior and decision-making processes, thereby hindering a more comprehensive and nuanced understanding of market dynamics.

6 Conclusion

In the financial market, it is of utmost importance for individuals to possess a profound comprehension of the intricacies associated with American, European, and Exotic options. Each of these classifications presents exclusive benefits and hurdles, ranging from the versatility of American options to the simplicity inherent in European options, as well as the distinctive attributes exhibited by Exotic options. The selection between these options is contingent upon the investor's strategic methodology, tolerance for risk, and the prevailing conditions of the market. Thus, acquiring a comprehensive understanding of option trading assumes paramount significance. There has been an increasing emphasis placed on the undertaking of investigations concerning option pricing, specifically directed towards the enhancement of precision and customization. The emergence of financial derivatives has also brought forth a plethora of novel challenges. These recent investigations are gradually aligning with the trajectory of market development. Furthermore, these novel inquiries address existing gaps in various domains and offer fresh remedies to intricate predicaments. The primary objective of this study is to present and contrast three distinct options and their corresponding pricing methodologies. These results expand upon current research concerns and avenues, providing an appraisal and directions for future researchers.

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