Comparison of Option Pricing Based on Black-Scholes and Binomial Tree: Evidence from Moutai's Share Price

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Abstract. As a matter of fact, option pricing is a vital area in mathematical finance, and various academic pricing methods has shed light on specific option types' pricing, among which Black-Scholes model and Binomial Tree model are the most widely-used measures. However, recent debates have been concentrated on the utility and efficiency of those two measures. Moreover, the newly developed Chinese capital market have been developing its option transaction mechanisms to a more mature level. Based on those two models' theoretical framework, this study acquired real time data of Moutai from Shanghai Stock Market Exchange (SSE) and use Python code to simulate the option price in the next period based on two different models respectively. According to the analysis, though share price from SSE might not reflect the overall Chinese capital market's condition, these results would contribute to mend the research gap of empirical application of option pricing models in the Chinese market.

Keywords: Black-Scholes Model, Binomial Tree Model, Chinese Capital Market, Kweichow Moutai Company.

1 Introduction

Option has become one of the most important derivatives in financial market. The first options called "Halters" were created in ancient Greece market where the merchants could lock a price for goods that were not yet to be sold, minimizing the risk of price fluctuations. Over the years, the role of options has developed from a tool of merchant to an important risk management tool as well as one of the most important derivative securities, whose value is contingent on the price of a stock [1], and could allow game players in the stock market to hedge against potential high financial risks. Based on the difference in exercise time, options could be divided into American option, European option and Bermudan options, with the former two types of options are relatively mature and widely used. In order to satisfy the diversified demand of investors, derivatives are no longer limited to standard options, exotic options such as Asian Options, Compound Options and Super share Options have been introduced and further studied [2]. In Chinese emerging capital market, the option transactions

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have maturely formed since 2015, and its developing pricing models used by various dealers have shown potential for growth [3].

More than a century after French financial expert Louis Bachelier published the first dissertation about option pricing Theoria de la Speculation in 1900, which is now recognized as a landmark in the history of mathematical finance [4] and the precedent of applying Brownian Motion to the financial literature, option pricing method have experienced major improvements, including Markowitz proposing mean-variance portfolio theory in 1952 [5], Boness' assumption, in which the stock yield follows a lognormal distribution [6] incorporated with risk factors, and finally Black-Scholes Model [7], indicating a breakthrough in the research area of option valuation. B-S model bases on the assumption that the underlying asset price follows Brownian Motion, and proposes that the returns of the portfolio, which is constructed by both option contract and underlying asset, is equivalent to that of the risk-free bonds. Thus, this model gives an analytical formula for option pricing for the first time. After that, experts have been trying to improve the deficiency in the assumption of the B-S model as the underlying assets' price follows an unrealistic normal distribution, and the following Merton Model, Heston's and Bates' model allowed option pricing to embrace a modern era. Besides, in 1979 Cox, Ross, Robinstein and Sharpe proposed a more visualized and simpler Binomial Tree Model in calculating American Option, whose random walk calculates the value of options at each time step based on the probability of the different option prices. By iterating forward, the final value of the option at the starting time is obtained. This relatively simple method could be used in estimating the value of European and American options. Nowadays, combined with Monte-Carlo Simulation [8] and machine learning techniques [9], option's prices could be evaluated more accurately.

Normally, the buyers and sellers of options might use various finance mathematical models to calculate option prices. The motivation of this article lays in the lack of implementation of the Black-Scholes Model and Binomial Tree model to the Chinese emerging capital market. Combining the characteristics of the national capital market might indicate different explanatory effects, enriching academic research in the field of stock pricing. Due to the differences in the calculation of the option prices in those models, the article first gives a brief introduction to these two option pricing models, then in the data selection section, the relevant share price of Kweichow Moutai company as well as the risk-free interest rate and vitality rate are obtained and analyzed by Python using B-S model and Binomial Tree Model, with the former combined with the Monte-Carlo simulation method. Eventually, the call option price is calculated and compared between those models base on the captured data.

2 Option Descriptions and Pricing Method

2.1 European Option and B-S Model

The timing of maturity date could divide the types of options into European Option and American Option. As is known to all, European Option is a path-independent option, as it could be only exercise at maturity date, thus its payoff depends only on

the strike price (k) as well as its option price(S_T) at maturity date, thus its payoff amount, or the presumable deposits paid to guarantee the future transaction at time t could be defined by below formula:

$$\text{Payoff}\,(S_T) = \max\{0, S_T - k\} \tag{1}$$

Considering the interest risk free, compounded interest rate r and the Probability Measure, one could derive the price of European Option at time 0 based on the assumption that the current price of the option is the expectation of the payoff:

$$S_0 = e^{-rt}\,E[S_T] \tag{2}$$

In this formula, $E[S_T]$ is the expectation payoff of the option, and e^{-rt} is the discounted factor that transfers the price of the option in time t to time 0. To determine the price more accurately, B-S Model is used, which could only be used in estimating derivates that have definite exercise date, such as European Options and share options [10]. B-S Model includes those parameters: the maturity time t, underlying asset's price S_0 at time 0, determined strike price k, risk-free interest rate r, and votality rate σ, which is calculated based on the assumption that the price of the underlying assets follows a Brownian Motion with constant drifts. The price of the call option (C_0), for instance, could be expressed in below formula:

$$C_0 = S_0\,N(d_1) - k \cdot e^{-rt}\,N(d_2) \tag{3}$$

In which d_1 and d_2 could be determined by those equation:

$$d_1 = \frac{ln(S/k)+(r+\sigma^2/2)t}{\sigma\cdot\sqrt{t}} \tag{4}$$

$$d_2 = d_1 - \sigma \cdot \sqrt{t} \tag{5}$$

The conditions suitable for B-S model are: 1) Stock prices follow a lognormal distribution; 2) The risk-free interest rate is constant; 3) There are no arbitrage opportunities in the financial market; 4) Option should not be exercised before maturity date (options are European Option); 5) No dividends paid; 6) There are financial assets existed for continuous transaction. After the B-S Model was proposed, China's relevant research area has been developed vigorously. In 2002, Liu and others had classified and comparing different pricing methods including B-S model [11]. Moreover, mathematical ideas of Fluidics Science were introduced in B-S Model calculation [12], which was transformed into a standard parabolic partial differential equation, and then a difference scheme was obtained and its stability was proved. In 2014, Jiang and others proposed a finite difference method based on adaptive Laplace transform to solve Black Scholes equation [13]. Apart from research on B-S Model, Binomial Tree Model, Monte-Carlo Simulation, and neutral network are also applied in the forthcoming study of option pricing problems.

2.2 American Option and Binomial Tree Model

On the flip side, American Option's privilege is that it could be exercised at any date before or at the expiration date, thus it is path-dependent. Besides, due to its pre-exercise nature, American Option could be given the right to exercise at a higher price. Therefore, research on the pricing of American options has attracted much attention [14]. One way to price American Option is using the Binomial Tree model, which could be used widely in almost all kinds of option pricing [15]. Binomial Tree Model could be demonstrated in Fig. 1.

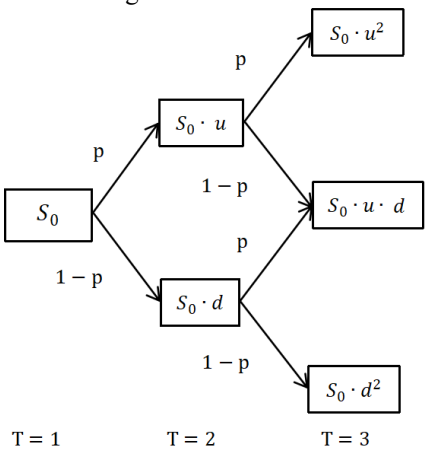

Fig. 1. Binomial Tree Model (Photo/Picture credit: Original).

In this model, the initial price S_0 at time 0 would experience downwards and upwards based on the risk-neutral possibility p, therefore, from the initial time to the maturity date, binary tree could be extended to countless possiblities eventually. For each step of extension, the price of the underlying asset (option price) would have two possible conditions, price upward $S_0 \cdot u$ and price downward $S_0 \cdot d$, in which u and d could be determined by votality rate $\sigma \cdot \sqrt{t}$. Thus, for risk-neutral probabilities (p), one has:

$$S_0 = e^{-rt}[S_0 u \times p + S_0 d \times (1-p)] \tag{6}$$

Consequently, p would be derived by this formula:

$$p = \frac{e^{rt} - d}{u - d} \tag{7}$$

$$1 - p = \frac{u - e^{rt}}{u - d} \tag{8}$$

In an effort to estimate the original price S_0, every option price $(\max\{(S_T - k), 0\})$ at the end of the 'branches' at maturity date should be calculated. Based on the（2）formula, determined price at maturity time (for instance, $T = 3$) could deduce the prices at two points at $T = 2$, and those two prices could finally derive price

at time 0. Through this repeated backward derivation, each option could be determined by numerous probabilities of the option price in the maturity date. Therefore, in each time step the estimated option price could be calculated, which is suitable for prediction in regard to American Option's uncertainty in exercise time.

2.3 Relationship Based on Assumption

In fact, there are similarities between those models. From a perspective of probability distribution, B-S Model's equation is deduced by calculating the area of the probability distribution composed by the numerous random walk of nonlinear trend. In terms of Binomial Tree Model, if the intervals between each time step could be minimized to zero, then the branches and different outcomes of the prices at time t would be innumberable. As binomial distribution could be considered as normal distribution when there are endless breakdowns at each step, B-S and Binomial Tree Model could be regarded as based on the same basic probability assumption. Nevertheless, due to the differences in assumptions and backgrounds, they exist as two independent and crucial models in finance market.

3 Empirical Analysis

3.1 Sampling

The criteria for the data selection is 1) large scale of share issuance with a listing time of more than 2 years, 2) sufficient trading volume and good liquidity, so the price could fully reflects the market environment, 3) the company has a good credit record, and it is in an upward stage, which has good profitability and ability to the short-term debt repayment. The data selected in this article is from Kweichow Moutai Company's September 2nd 2015 to November 17th 2023's share price (opening and closing price). Along with China's capital market's further development, Moutai Company, founded in 2017, which is one of the largest international baijiu (Chinese liquor) companies in Chinese capital market, has also been listed in SSE 50, which is 50 top stocks in Shanghai Stock Exchange with best daily average market cap and liquidity, in September, 2023. Firstly, to increase sampling consistency, over 13,000 pieces of share price data including date, daily opening price and closing price of Moutai was obtained from Tdx software, which is one of the largest security information platforms. Using Python, Moutai's Share close price is plotted as Fig. 2. Daily yield is presented in Fig, 3) and by calculating yield's standard deviation, the volatility (1.672%) from the daily yield is acquired. However, due to the large number of sample and length of years, share price before 2021.1.4 is abandoned, as its yearly rise reflects little about the current share price situation. Thus, volatility after year 2021 is calculated as 0.0174. To verify Moutai's share return's reliability, Moutai's daily yield return is standardized and fitted (seen from Fig. 4). The bar chart above revealed that although even standardized, Moutai's share price has experienced a surge in almost ten years, its daily yield follows an approximate normal distribution, which ful-

fills the B-S assumptions in previous. Thus, the applicability and reliability of share data are verified.

Fig. 2. Moutai's close price (Photo/Picture credit: Original).

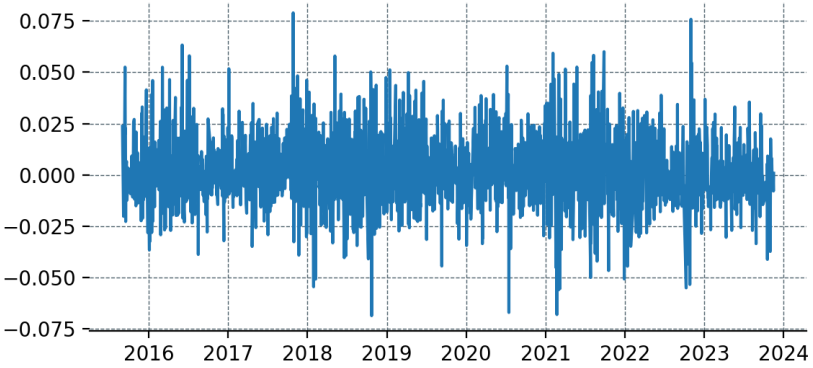

Fig. 3. Moutai's Daily Yield Curve (Photo/Picture credit: Original).

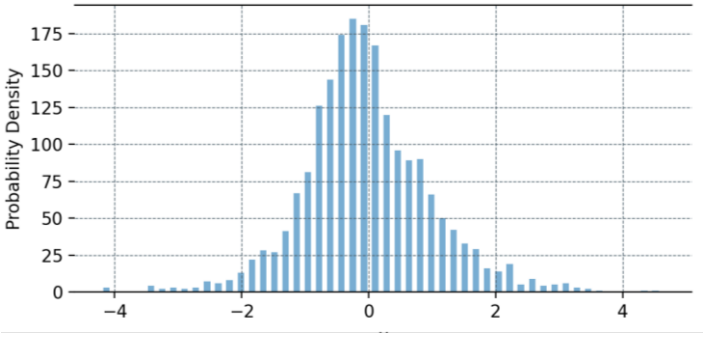

Fig. 4. The Daily Yield of Moutai's Share follows a normal distribution (Photo/Picture credit: Original).

3.2 Share Price Simulation Based on Binomial Tree Model

This section takes Moutai stock as the subject matter and calculates the share price in November 2023 based on the Binomial Tree Model. Firstly, an iterative Binomial Tree Model is constructed, where the mean closing price of Moutai is 1128.63 RMB, and the highest and lowest daily closing share price are 2601.0 RMB and 188.71 RMB respectively, and the standard deviation is 651.199. Due to the large price flotation depicted in Fig. 3, this study first uses the logarithm of daily closing price to narrowing the difference (given in Fig. 5), and the standard deviation of logarithm of daily closing price is 0.74. The second option to calculate the σ is using daily yield rate, in Fig. 1, the sheer rising trend of JD's daily closing price is observed to stopped nearby 2021. Thus, the data used to process Binomial Tree Model dates from 2021.1.4 to 2023.11.17 (seen from Fig. 6), which mean is 1851.314, deviation is calculated as 170.324. With regard to the upward trend of Moutai's share price, the article simulate the price based on the call option assumption. As the share closing price at November 17th 2023 is 1753.42, the Strike price K is set at 1700, risk-free rate is set at 2.3118%, rounded at 0.02, which is China's treasury bond three-month yield rate at November 17th 2023, acquired at macroview.club. Maturity time T is set at 1, and the iteration steps number is set at 1000.

Fig. 5. Logarithm Form of Daily Share Price (Photo/Picture credit: Original).

Fig. 6. Closing share price after 2021.1.4 (Photo/Picture credit: Original).

First, the volatility of share price is calculated as the standard deviation of $\log closing\ price - \log opening\ price$, and the standard deviation of the volatility after 2011.1.4 is 0.0174, using the formula $S_0 = e^{-rt}[S_0 u \times p + S_0 d \times (1-p)]$, firstly a 4-step Binomial Tree random walk structure is built (seen from Fig. 7), and then the branches and steps are increased for better accuracy. Moreover, the standard deviation decreases as the iteration time N increase, but the complexity and time length of calculation will be increase as well. After experimenting a forward and reverse iteration of the Binomial Tree Model, the simulated Moutai option price at time T is 87.01, and the standard deviation of call option price is 41.211. In the second option, daily Yield rate is calculated after 2021.1.4, and its standard deviation is also 0.0174. Thus, those two methods calculating volatility have the same results in simulating option price.

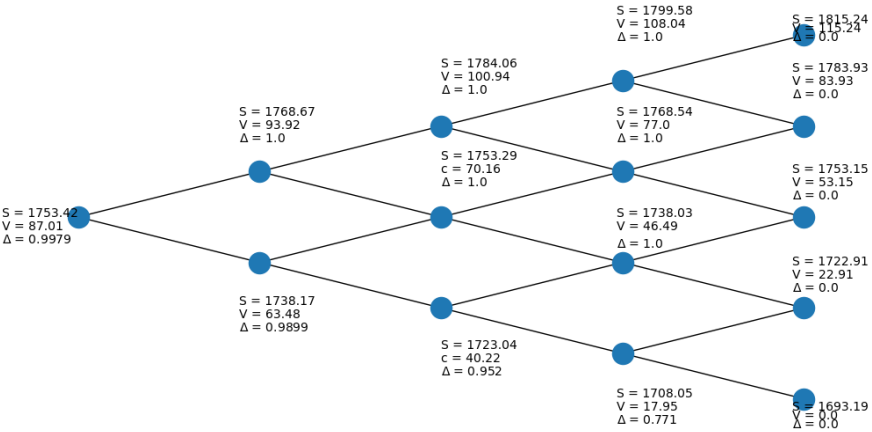

Fig. 7. Simulated Binomial random walk (4 steps) (Photo/Picture credit: Original).

3.3 Share Price Simulation Based on B-S Model

The data based on the B-S model simulation is also closing share price after January 4th, 2021 to remain consistency. Using B-S formula, S_0, K, r, T and σ are as the same as those used in the Binomial Tree Model. To start with, the result that using single Black-Scholes formula without Monte-Carlo random generation is 87.097. Using random number seeded, a share price random-walking path is built. Using the iteration of every random walk, a mean price of the share price at time T is simulated as 88.831, and the standard deviation of simulated closing price is calculated as 33.182.

4 Conclusion

The results reveal that combined with Monte-Carlo simulation, B-S model has lower standard deviation of random walk results than Binomial Tree Model. Moreover, the call option price simulated by B-S model is more precise than the Binomial Tree Model as the iteration times is more than the Binomial Tree Model. As regard to the

accuracy aspect, since Binomial Tree Model iterates in discrete time gaps, thus the effectiveness and accuracy of Binomial Tree Model is less than B-S Model. Moreover, the complexity of simulation of Binomial Tree Model is higher than B-S Model. Nevertheless, B-S Model has less range for application as it could only be used in European Option pricing. Thus, in reality, the optimal pricing strategy should base on the advantages of both option pricing models. The empirical research based on Moutai Company's data have given insights into the comparison between B-S model and Binomial Tree model. However, SSE's share information might not cover the overall condition of Chinese newly-developed capital market. Nevertheless, this article would amend the research gap for the lacking of empirical analysis for B-S and Binomial Tree model in Chinese market.

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