



A three-dimensional crack under complex loading conditions

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Abstract. In this paper, finite element (FE) analysis is performed for the asymmetric four-point bend specimen under mixed mode fracture loading conditions. This test configuration can be used in studies of mixed mode I/II and mixed mode I/II/III crack problems. In order to study mixed-mode I/II loading, two alternatives of changing the crack position and changing the support position are considered in FE analysis and the obtained results are compared. For mixed mode I/II/III loading, an important parameter, the proportion of stress intensity factors is studied for two cases, i.e. changing the inclined crack location and altering the crack inclination plane in two different directions. It is shown that for mixed-mode I/II, the results are more sensitive to changing the position of support than crack plane. For mixed-mode I/II/III changing the position of the inclined crack can generate a better variation of stress intensity factors than having the specimen with a crack of two inclination angles which is also difficult to fabricate.

Keywords: Asymmetric four-point bend, mixed mode I/II/III, finite element analysis, stress intensity factors.

1 INTRODUCTION

Cracked components in machines and structures can cause catastrophic consequences such as collapse of civil structures as well as sudden failures in various engineering applications like fuel tankers, pipelines, and railway tracks. All of these catastrophic events which cost lives and money underline the need for in-depth investigations and comprehensive analysis in the field of fracture mechanics. In real applications, fracture seldom occurs in the simplest way of pure mode I. This is because of complex stress distribution around the crack tips which makes the crack deformation as a superposition of different modes, so the study of mixed mode fracture has received much attention in the literature. Meanwhile, most of the research studies performed in the past are focused on mixed mode I/II, mixed mode I/III, or mixed mode II/III because these conditions are more convenient to be experimentally carried out and less complex in the analytical and numerical investigation than mixed mode I/II/III which needs complicated crack geometry or loading devices.

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There are some test configurations for the study of mixed mode fracture such as compact tension-shear (CTS), 3Point or 4point bending tests with symmetric and antisymmetric loading configurations [1,2]. The geometry of the test specimen can also be varied to achieve mixed-mode fracture such as using an angled crack specimen [3,4]. Each of these test configurations has its pros and cons and they have been widely used in numerical analysis and experimental studies by many researchers. The aim of this study is to investigate the Asymmetric four-point bend (ASFPB) loading test configuration for both mixed mode I/II and mixed mode I/II/III loading to provide a more detailed understanding of crack behavior under complex loading conditions.

2 METHODS

2.1 Mixed mode I/II

Asymmetric four-point bend loading has been extensively used in the investigation of mixed mode I/II fracture. For this aim, one of the typical ways is shifting the position of the crack plane or changing the position of one of the supports [5,6]. By shifting the crack location from the middle of the ASFPB specimen (S_0) which can be seen in Fig. 1a, in-plane bending moment (mode I) is also present as well as in-plane shear (mode II) at the crack plane. On the other hand, changing the location of roller support (d in Fig. 1b) changes the distribution of shear and moment in the entire length of the specimen, so there are both moment and shear (mode I and II) at the crack plane (at the middle of the specimen), which makes this configuration an appropriate case for mixed-mode I/II fracture studies.

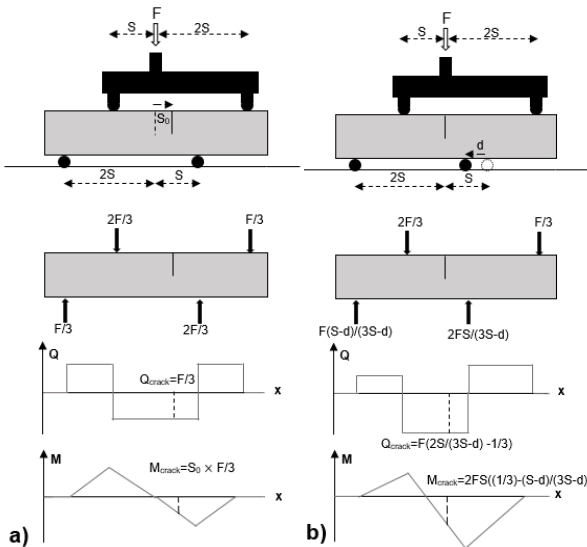


Fig. 1. Schematic of ASFPB specimen a) shifting the crack location from the middle plane b) changing the location of support

2.2 FE modeling of specimen under mixed mode I/II loading

In order to investigate the ASFPB specimen and calculate the stress intensity factors (K_I , K_{II}), the commercial FE code ABAQUS is used. For analysis of the mixed mode I/II, the 2D meshed FE model is used which can be seen in Fig. 2. An eight-node quadrilateral plane stress element is used for meshing the entire specimen except for the area around the crack tip where the collapsed six-node triangle plane stress elements are employed in order to consider the singularity in the vicinity of the crack tip.

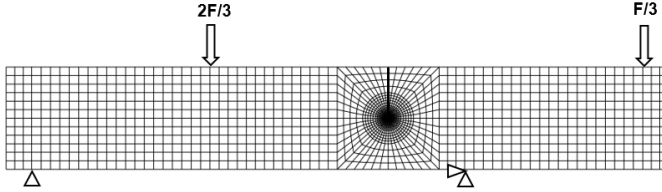


Fig. 2. FE model of ASFPB specimen under mixed mode I/II loading

The dimensions of the specimen are based on the SENB specimen as given in the ASTM [8], with a length of 90 mm, a width of 20 mm, a thickness of 10 mm, and an initial crack length of 10 mm. The value of S (in Fig. 1) which is related to the distance between the supports is considered 20 mm.

There are empirical equations for calculating stress intensity factors of mixed mode I/II (K_I , K_{II}) when the position of the crack is shifted from the middle of the specimen [5,7]. These two equations can be seen in eq. (1) and eq. (2) where Y_I and Y_{II} can be obtained from [5], η is S_0/W and the Q is the shear force at the crack plane ($F/3$). Having the stress intensity factors from eq. (1) and eq. (2) makes the verification of the FE results possible.

$$K_I = \frac{\eta Q \sqrt{\pi a}}{Wt} Y_I \quad (1)$$

$$K_{II} = \frac{Q \sqrt{\pi a}}{Wt} Y_{II} \quad (2)$$

2.3 Mixed mode I/II/III

ASFPB specimen can also be used to investigate mixed mode I/II/III. Kui [9] proposed an ASFPB specimen with a crack inclined in both thickness and width directions (α , β) to study mixed mode I/II/III fracture, as schematically represented in Fig. 3a. Also, previous studies proved that under bending moment at the crack plane, if the initial crack is rotated through the thickness, both mode I and mode III are present [10]. As a consequence, by shifting the crack from the middle of the sample or changing the support position and also creating an angle for the initial crack in the thickness direction mixed mode I/II/III loading can be obtained (see Fig. 3b).

It is worth noting that in the ASFPB test configuration, eliminating mode II (shear force) and creating mixed mode I/III is impossible due to the nature of the test configuration. Additionally, since mode III is dependent on the in-plane bending moment

(mode I) at the inclined crack position, creating mixed mode II/III loading conditions is not possible.

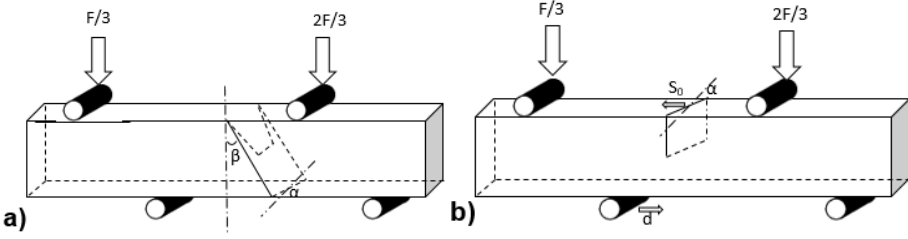


Fig. 3. Schematic of ASFPB specimen for mixed mode I/II/III with a) two crack inclination angles [9] b) changing the position of inclined crack or roller support

2.4 FE modelling of specimen under mixed mode I/II/III loading

For FE modeling of ASFPB specimen which has the crack with inclination angle in thickness direction, the 3D model was used in ABAQUS. As it can be seen in Fig. 4, in order to mesh the 3D model, 20-node quadratic brick elements are used and very fine elements in the vicinity of the crack front are employed because of high stress gradient. To produce the square root singularity of the stress/strain field, the singular elements with the middle nodes at quarter-point positions are considered in the first ring of elements surrounding the crack tip.

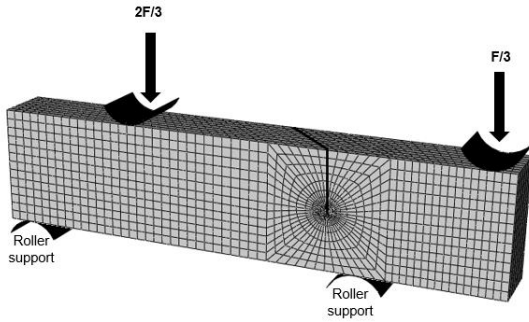


Fig. 4. FE model of ASFPB specimen under mixed mode I/II/III loading

After modeling, three stress intensity factors (K_I , K_{II} , K_{III}) are obtained from ABAQUS. These stress intensity factors can be used to calculate normalized stress intensity factors (eq. (3)). These parameters which indicate the proportion of modes in mixed mode loading are used to compare the FE results with the results of previous work.

$$K_I^n = \frac{|K_I|}{|K_I|+|K_{II}|+|K_{III}|} \cdot K_{II}^n = \frac{|K_{II}|}{|K_I|+|K_{II}|+|K_{III}|} \cdot K_{III}^n = \frac{|K_{III}|}{|K_I|+|K_{II}|+|K_{III}|} \quad (3)$$

3 RESULTS AND DISCUSSION

In this section, first the FE results obtained from the case of mixed mode I/II loading are validated. Then, the stress intensity factors for mixed mode I/II/III cases are presented.

From Fig. 5, it can be seen that the stress intensity factors from FE analysis of mixed mode I/II loading in cases of shifting the crack position (S_0) with 0,2,4,6,8, and 10 mm (points on Fig. 5) agree well with the results of eqs. (1) and (2), which proves the validity of FE results.

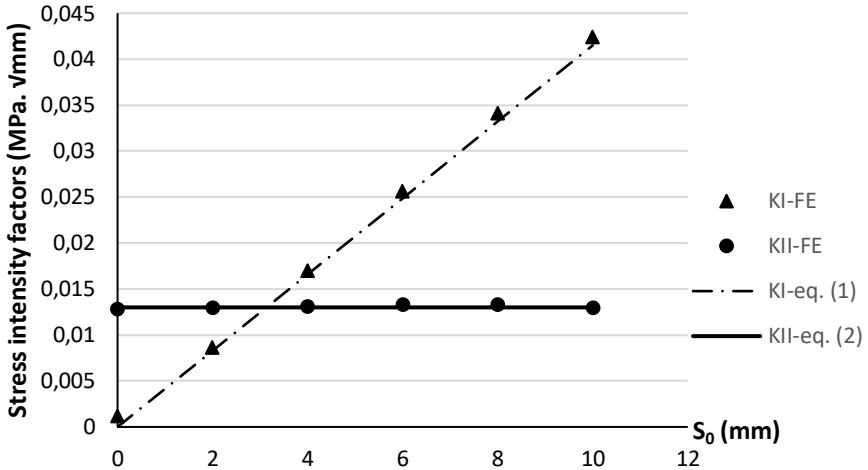


Fig. 5. Stress intensity factors (K_I , K_{II}) obtained from eqs. (1) and (2) and FE modeling

Fig. 6 indicates the stress intensity factors of ASFPB specimens under mixed mode I/II loading in two cases of shifting crack plane (S_0) and changing the position of roller support (d). It can be seen that K_I increases more sharply by changing the position of support than shifting the crack plane. This can be due to the fact that the moment (M in Fig. 1) is linearly correlated to S_0 while it's nonlinearly related to the position of support (d). The value of mode II stress intensity factor (K_{II}) is constant with changing S_0 because the shear is not changing but when the roller support moves (changing d) K_{II} varies because the value of in-plane shear alters.

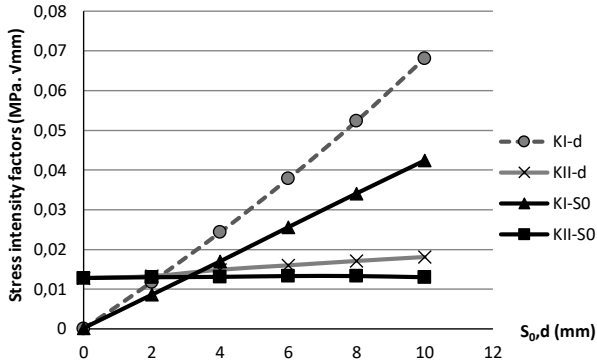


Fig. 6. The variations of stress intensity factors with respect to the positions of crack plane (S_0) and roller support (d)

To analyze the proportion of modes in mixed mode I/II/III for ASFP specimen, the normalized stress intensity factors (from eq. (3)) obtained from two methods, i.e. the specimen with two crack inclination angles (α , β) presented in [9] and the specimen with an angled crack shifted from the middle of the specimen are compared in Table 1. From Table 1 it can be observed that in the second method (i.e. shifting the location of inclined crack plane) the proportion of mode III is greater than in the first method. In this regard, it can be conceived that having two crack inclination angles doesn't generate mode III considerably implying that it is not an ideal option to study mixed mode I/II/III fracture. In contrast, changing the position of inclined crack effectively altered the mode III which makes it a suitable method for mixed mode I/II/III investigations with considerable contribution from mode III deformation.

Table 1. Normalized stress intensity factors of ASFP specimen obtained from two methods.

Method	Crack Angles (α , β)	S_0 (mm)	K_I^n	K_{II}^n	K_{III}^n
ASFP specimen with two crack inclination angles [9]	$\alpha=45$, $\beta=26.5$	0	0.355	0.516	0.129
ASFP specimen with two crack inclination angles [9]	$\alpha=45$, $\beta=45$	0	0.474	0.342	0.184
ASFP specimen with a shifted angled crack ($S_0 \neq 0$)	$\alpha=45$, $\beta=0$	5mm	0.346	0.379	0.276
ASFP specimen with a shifted angled crack ($S_0 \neq 0$)	$\alpha=45$, $\beta=0$	10mm	0.321	0.275	0.404

It is therefore recommended to use the ASFPB specimen with an inclined crack plane shifted from the center of the specimen to study various mixed mode I/II/III conditions rather than using an ASFPB specimen with two crack inclination angles which not only

is incapable of generating a large mode III proportion but also is difficult to be prepared for experimental purposes.

4 CONCLUSIONS

It is shown that the results for stress intensity factors obtained from modeling of ASFPB specimen under mixed mode I/II loading are different for two alternatives of changing the locations of the crack plane and the roller support. This is because the in-plane shear force and bending moment at the crack plane are different in these two cases. When the support position is shifted the results vary more rapidly, so more accurate measurement is needed to determine the support positions during the test.

To investigate the ASFPB specimen under mixed mode I/II/III loading, the comparison between the results obtained from these two methods shows that shifting the location of the inclined crack can generate mixed mode I/II/III with a larger proportion of mode III than having a specimen with a crack inclined in two directions. It is worth mentioning that the other drawback of the latter method is the difficulty of sample preparation because the crack needs to be generated in a complex twisted plane, when preparing the test specimen.

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