Mathematical analysis of propeller flight simulation.

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Abstract. The thesis focuses on the analysis and calculation of the aerodynamic lift force of the propeller that is applied to the drone. The unmanned aerial vehicle, a drone, can be remotely controlled or capable of autonomous flight using pre-programmed flight plans or more complex dynamic autonomous systems. The main input is a model of the propeller, in this particular case a model of the three-bladed propeller that is used on the drone. The thesis summarizes the theoretical knowledge of the physics of flight. Subsequent analysis and mathematical calculation establishes the dependence of the lift force on the rotation of the propeller, which is shown in the table. The result of the work is the maximum load that the propeller can sustain in the process of flight due to the aerodynamic lift force.

Keywords: drone, propeller, mathematical analysis, aerodynamic lift force

1 Basic principle, physics of flight

A quadcopter or quadrotor is an unmanned aerial vehicle with six degrees of freedom. Indeed it can move in the direction of each axis and also rotate around all of them. The movement of the drone is based on electric motors that set the blades into rotational motion. Likewise, the blades generate lift on this basis. [2,7]
The lift force can be determined using the equation to calculate the lift:

$$F_y = \frac{1}{2} . c_y . \rho . S . v^2 \tag{1}$$

- F_y [N] –aerodynamic lift force
- ρ [kg·m^{-3}] – air density
- v [m·s^{-1}] – speed
- S [m^2] –blade leaf surface
- Cy [-] – lift coefficient

It is clear from the formula that it is only possible to influence the magnitude of the lift force altering the design of the blade blades or by the speed of their rotation. Obviously

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it is not possible to increase the surface area of these blades or their angle of attack in flight. Control of the motion is accomplished by increasing and decreasing the speed of the individual motors depending on the action required. [1]

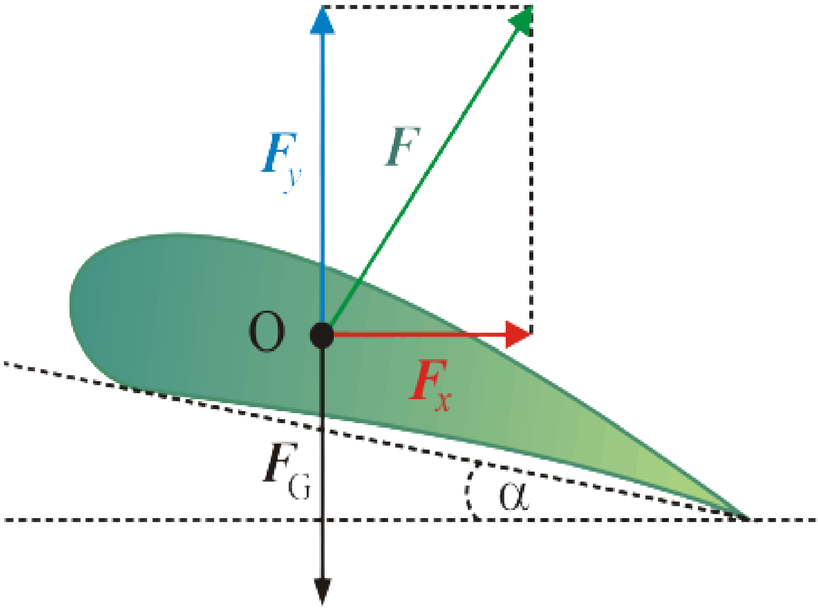

Fig. 1. Forces acting on the propeller wing [1]

In addition to the lift force, there is also a drag aerodynamic force acting on the wing. This force counteracts the movement of the propeller blades. Propeller motors must be sized so that the drag aerodynamic force does not affect their desired speed. The drag aerodynamic force can be determined using Eq:

$$F_x = \frac{1}{2}.c_x.\rho.S.v^2 \tag{2}$$

- F_x [N] – aerodynamic drag force
- ρ [$kg \cdot m^{-3}$] – air density
- v [$m \cdot s^{-1}$] – speed
- S [m^2] –blade leaf surface
- C_x [-] – coefficient of drag

The lift coefficient (denoted as Cy in the formula) is a dimensionless coefficient. Its magnitude is affected by the angle of the body (propeller) with respect to the axis of flow of its Reynolds number and Mach number.
The dependence of the lift coefficient on the angle of attack can be seen in the following table (we will also use the table as a basis for our calculations). [12]

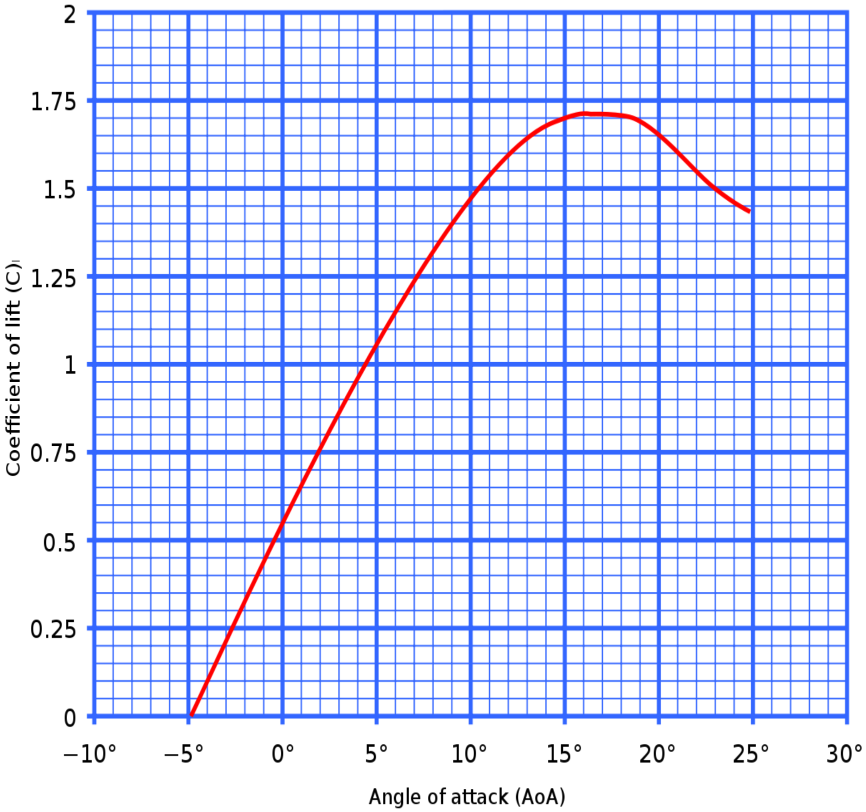

Fig. 2. Dependence of lift coefficient on angle of attack [2]

2 Propeller 3D model

The simulation and calculations will be performed on a 90mm diameter three-bladed propeller, which is commonly used in drones and quadcopters. The propeller together with the connection to the electric motor can be seen in the following figures. [5,13]

Fig. 3. Propeller with electric motor

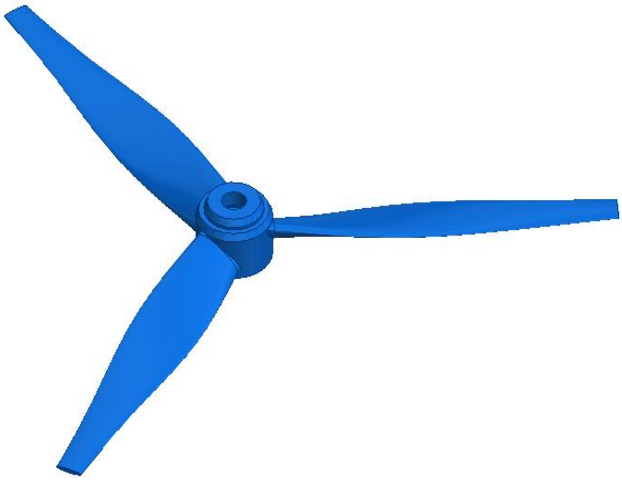

Fig. 4. Propeller

3 Effect of speed on propeller lift

Since most drones do not have the ability to tilt the propeller flaps, the biggest parameter that affects the lift aerodynamic force of the propeller is the propeller electric motor speed. The effect of increasing the speed is to increase the lift aerodynamic force, which allows the drone to "fly" off the ground. [6,9,11]

The simulation is carried out by calculations and substitutions into the formulas that were presented in the first chapter.
Since a crucial parameter that affects the aerodynamic lift force is the angle of attack of the propeller, which varies in the propeller profile, the propeller blade is divided into several parts for the sake of the calculations. For each part the lift force is calculated depending on its circumferential speed. Finally, the lift forces acting on each part of the propeller are added and averaged. [4,8,10]

For the sake of clarity, one part is calculated in detail at specific speeds. The subsequent results of the calculations of the other parts at specific speeds are given in the table.

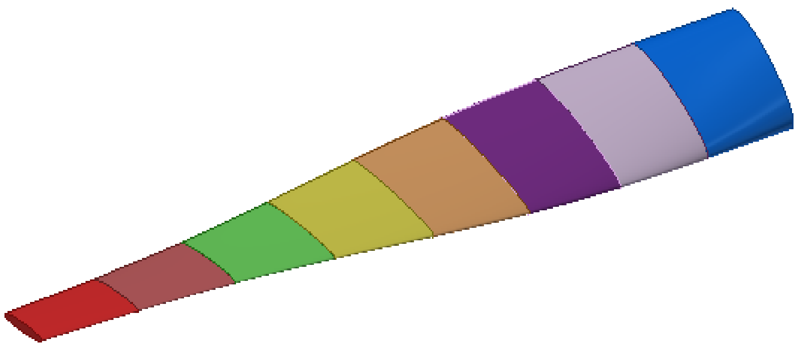

Fig. 5. Propeller blade divided into 8 parts

3.1 Illustrative calculation of one propeller part

The basis for the calculation will be the formula mentioned above:

$$F_y = \frac{1}{2} . c_y . \rho . S . v^2 \quad (3)$$

The calculation described above is valid for the blue part of the propeller (the part closest to the rotor). It is a 5mm long part whose edge is located at a distance of 5mm from the centre of the rotor.
The aerodynamic lift force is calculated at 2000 propeller revolutions per minute. The angle of attack alpha is 21 degrees. The angle is measured using Autodesk Inventor 3D software.

$$F_y = \frac{1}{2} . c_y . \rho . S . v^2 \quad (4)$$

- C_y = 1,65 (coefficient for an angle of attack of 21 degrees, read from the table for the dependence of the lift coefficient on the angle of attack)
- ρ = 1,29 $kg.m^{-3}$ table value
- S = 0,00847 m^2 (area measured using Autodesk Inventor 3D)

The speed depends on the circumference described by a given part of the propeller at a given speed. The speed must be in basic units, i.e. m.s-1.

- Speed (v) for 1 rotation per second = 2 . π . R = 2 . π . 10 = 62,83 mm . s-1
- Speed (v) for rotation per minute = 62,83 . $\frac{2000}{60}$ = 2,094 m . s-1

The aerodynamic lift force for the first part is then:

$$F_y = \frac{1}{2} . 1{,}65 . 1{,}29 . 0{,}00847 . 2{,}07^2 \quad (5)$$

$$F_y = 0{,}03946 \text{ N}$$

3.2 Calculated aerodynamic lift force values

Using the system of formulas that have been entered, the lift forces of 1 propeller blade have been calculated (the propeller has 3 blades). The forces are shown in the table below.

Table 1. Illustrative calculation of one propeller part

Rotation of propeller (RPM)	Aerodynamic lift force of one propeller blade (N)	Aerodynamic lift force of propeller (N)
1000	0,0396	0,1188
2000	0,1587	0,4761
3000	0,3571	1,0713
4000	0,6349	1,9047
5000	0,992	2,976
6000	1,4286	4,2858
7000	1,9444	5,8332
8000	2,5397	7,6191
9000	3,2143	9,6429
10000	3,9683	11,9049

4 Results and discussion

As can be seen, the aerodynamic lift force increases exponentially with increasing speed. At the lowest speed (1000 RPM) the force is very small, almost negligible. On the contrary, at the highest speeds studied (10000 RPM), the aerodynamic force of the three-bladed propeller approaches the 12N limit. After substituting in the formula for calculating the gravitational force Fg, we find that at 10000 r.p.m.-1 the propeller is able to keep in the air a weight of 1.2177kg (including the weight of the propeller and the actual structure).

$$F_g = \mathrm{m} \,.\, \mathrm{g} \tag{6}$$

- F_g [N] – gravitational force
- m [kg] – weight of the body
- g [m . s^{-2}] – gravitational acceleration

$$\mathrm{m} = \frac{F_g}{g} = \frac{11{,}9094}{9{,}780} = 1{,}2177 \text{ kg} \tag{7}$$

In the attached graph you can see in more detail the increasing aerodynamic lift force as a function of speed.

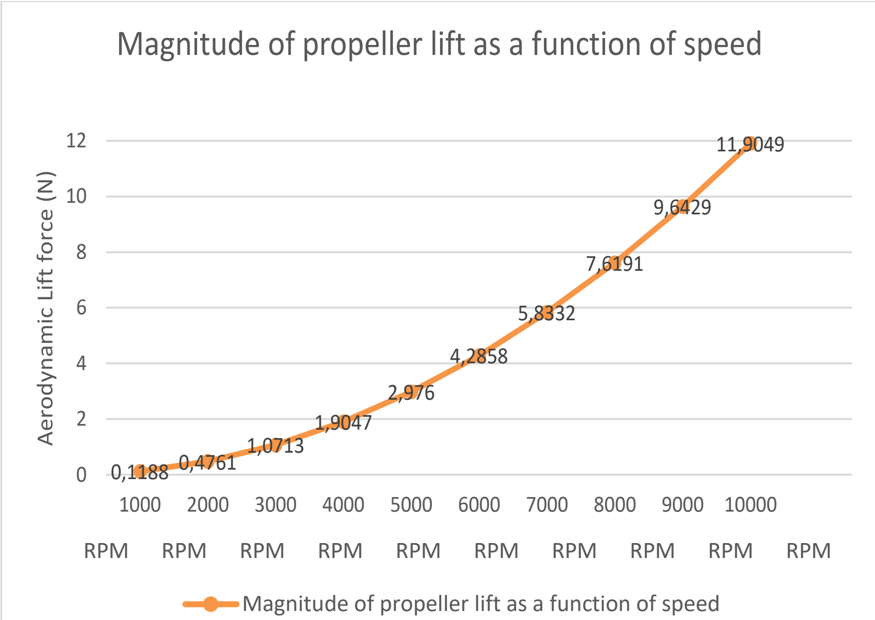

Fig. 6. Magnitude of propeller lift as a function of speed

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