Vibration Analysis of Multifunctional Laminated Glass Composite Panel

Reza Moezzi¹*, Jindrich Cyrus¹, Jan Koci¹

¹ Institute for Nanomaterials, Advanced Technologies and Innovation, Technical University of Liberec, Czech Republic
Reza.Moezzi@tul.cz

Abstract. This paper aims to investigate on laminated glass composite panel (LGCP) including at least one viscoelastic interlayer which can resolve several design problems. LGCP can be offered to increase strength stiffness characteristics of the panel or sound damping proficiencies. In this study, first the laminated glass’ young’s modulus is calculated by an explicit experiment, then the natural frequencies are estimated by analytical model and have been captured by modal analysis with impact hammer as an experiment. Finally, a novel method performed by sound excitation under certain frequencies to determine natural modes of the beam according to sand’s shape on the panel. The results, comparisons and conclusion are shown and discussed.

Keywords: Laminated Glass Composite, Natural Frequency, Eigenmode, Impact Hammer Test, Acoustics, LABVIEW.

1 Introduction

Laminated glass composite panel (LGCP) is used in structures where glass is the foremost load bearing element with at least one viscoelastic or flexible plastic interlayer. It can solve multiple design problems to offer increasing strength/stiffness characteristics of the panel [1] or sound damping competences of the structure [2]. Additionally, the viscoelastic can be used as a polymeric encapsulant for integrating solar cell layers [3]. Recent technological advances in the design of the interlayer such as EVA (ethylene vinyl acetate), PET (polyethylene terephthalate), PVB (polyvinyl butyral), Transparent thermoplastic materials (TPU), Acoustic film, microperforated materials, etc., are well studied [4][5]. Also interlayer contained within laminated glass composite panel, have revealed an enhancement in acoustic performance. These novel solutions can be implemented for noise control in a wide range of practical applications. In the case of PV modules, the main factor determining the operational life is most commonly the encapsulation material [6]. Therefore, strategy of interlayers of the LGCP is one of the key concepts to deal with different applications. The laminated glass composite panels are widely used as safety glass in building and housing products etc. [7] since it offers a strong barrier against forced entry. LGCP is preferable in cases where it is important to ensure integrity of the whole sheet after breakage. Current study deals with evaluation of vibration behavior, also investigate
of multifunctional LGCP natural frequencies. This interlayer has a considerable effect on natural frequencies of the panel. According to application purposes, some frequencies which match up the resonance frequencies of the panel should be avoided to optimize the design factors and minimize the radiation of the panel. There are several conventional methods to determine the panel’s natural frequencies, such as hammer based experiment with accelerometers, FEM analysis etc.

In this paper, first an outline of the LGCP design problems is overviewed. Then an experimental procedure to find the young’s modulus of the panel is described. Furthermore, analytical approach to investigate on natural frequencies is described. Hammer impact test experiment shows the deviation of the results with analytical approach and a novel experiment based on acoustics excitation reveals a reliable estimation of resonance frequencies. Finally, a brief comparison on various methods, their advantages, and disadvantages, are given in the conclusion section.

2 Problem Overview

In general, the LGCP can be modeled as a sandwich structure with glass top and bottom layers and viscoelastic/plastic core. Contrary to traditional sandwich structures here the top and bottom layers consist of one tempered glass layer, but the interlayer is a multilayer laminate with more complex mechanical behavior. The LGCP structures, like windows, etc. may contain several interlayers. The theory of the traditional sandwich structures is well established [8], but its direct application for LGCP structures may lead to inaccurate results due to special features of these structures. In the case of classical sandwich applications, the ratio of shear moduli of the core and skin is in the range of 10\(^{-2}\) and 10\(^{-1}\). In the case of LGCP structures like PV modules the corresponding ratio fits to the range amongst 10\(^{-5}\) and 10\(^{-2}\) [1]. Also, classical sandwich structure contains thick core and thin multilayer skins. The LGCP structure includes thick one-layer skins and thin multilayer core. Thus, the basic concepts of the sandwich theory can be employed, but modeling of the interlayer(s) needs special attention.

Recently, the layer-wise theory-based approach is proposed in [9] for analysis of the LGCP structures. In [10] the system of partial differential equations providing extension to the classical plate theories has been derived. An analysis of the plate strip was included as a case study. It was concluded that the applicability range of the classical plate theories can be related to the importance of the shear rigidity parameter. Applicability of the first-order shear deformation theory is analyzed in [11] based on findings of higher order layer-wise theory. In [12] the FE (finite element) formulation based on user-defined element and layer-wise theory has been introduced. The obtained results are validated against closed form solutions. In these discussed above papers, the main attention is paid to bending analysis and vibration analysis is not properly studies, therefore current study is focused on vibration analysis of the LGCP.
as case study. The experimental research using impact hammer with accelerometers via LABVIEW, acoustics sound-based and theoretical analysis are performed.

3 Calculation of Young’s Modulus

The microstructure of composite materials are designed in terms of their macroscopic elements, for example, fibers in a homogeneous matrix material. By adjusting the selection of fibers, the volume fraction, and their alignment; the mechanical properties are changed to meet certain design conditions.

![Diagram of uniaxial fiber-reinforced composite material](image1.png)

**Fig. 1.** uniaxial fiber-reinforced composite material

![Diagram of stress and strain](image2.png)

**Fig. 2.** stress carried out by the fibers and the matrix

The fig.1 shows a uniaxial fiber-reinforced composite, and fig.2 demonstrates how the stress on the composite is carried out by the fibers and the matrix. In typical settings, the fiber has a greater Young's modulus than the matrix, and for the continuous fibers (where the strain is equal in the fiber and the matrix) the matrix stress is lower
than the fiber stress. According to 'rule of mixtures' as equation (1), The Young's modulus of the composite is:

\[ E_C = E_F V_F + E_M V_M, \]

\[ (V_M + V_F) = 1 \rightarrow V_M = (1 - V_F) \quad (1) \]

Where \( E_C, E_F, E_M \) are the moduli of composite, reinforcement and matrix respectively, and \( V_F \) and \( V_M \) are the volume fraction of particles and matrix, correspondingly.

The elastic modulus alongside the fiber direction can be under control by choosing the volume fraction of the fibers.

Considering \( E_M = 70 \text{Gpa}; E_F = 5.8 \text{Mpa}; V_F = 2 \% \) according to manufacturing datasheet of the glass, the Young's modulus of our LGCP calculated based on equation (1) would be 68 GP.

To verify young’s modulus of the LGCP, a simple experiment also is done as shown in Fig.3

![Fig. 3. Experiment to calculate Young's Modulus](image)

Considering \( L=1 \text{ m}; h=0.006; b=0.1; I=1/12bh^3; W=mg; g=9.81 \), Three different weights on the plate (as beam) are applied and respective young modulus’ have been calculated. (Table 1).

<table>
<thead>
<tr>
<th>M (gr)</th>
<th>Y (mm)</th>
<th>E (Gp)</th>
</tr>
</thead>
<tbody>
<tr>
<td>163.6</td>
<td>4.42</td>
<td>67.2</td>
</tr>
<tr>
<td>200.5</td>
<td>5.32</td>
<td>68.4</td>
</tr>
<tr>
<td>163.6+200.5</td>
<td>9.75</td>
<td>67.8</td>
</tr>
<tr>
<td>623.5</td>
<td>16.38</td>
<td>69.1</td>
</tr>
<tr>
<td><strong>Average Young Modulus (Gp)</strong></td>
<td><strong>68.12</strong></td>
<td></td>
</tr>
</tbody>
</table>
4 Analytical Approach

In this part, the classical lamination plate theory method (CLPT) which is an extension of the Kirchhoff-Love classical plate theory, is considered to laminated glass composite beam. The displacement in the three directions supposed have the following equation (2):

\[
\begin{align*}
\text{u}(x,y,z,t) &= u_0(x,y,t) - z \frac{\partial w_0}{\partial x} \\
\text{v}(x,y,z,t) &= v_0(x,y,t) - z \frac{\partial w_0}{\partial y} \\
\text{w}(x,y,z,t) &= w_0(x,y,t)
\end{align*}
\]

where the assumed variables \((u_0, v_0, w_0)\) are displacements in the \(x, y, z\) directions, respectively, of a point on the midplane of the plate. This theory ignores transverse normal effect and transverse shear, and it considers in-plane stretching deformations and bending. The equations of motion can be written as equation (3)

\[
\begin{align*}
\frac{\partial N_{xx}}{\partial x} + \frac{\partial N_{xy}}{\partial y} &= I_0 \frac{\partial^2 u_0}{\partial t^2} - I_1 \frac{\partial^2}{\partial t^2} \left( \frac{\partial w_0}{\partial x} \right) \\
\frac{\partial N_{xy}}{\partial x} + \frac{\partial N_{yy}}{\partial y} &= I_0 \frac{\partial^2 v_0}{\partial t^2} - I_1 \frac{\partial^2}{\partial t^2} \left( \frac{\partial w_0}{\partial y} \right)
\end{align*}
\]

where \(N_{xx}, N_{xy}, \text{and } N_{yy}\) are the force resultants per unit length in three dimensions and the mass moments of inertia \(I_0, I_1, \text{and } I_2\) are calculated as equation (4)

\[
\begin{pmatrix}
I_0 \\
I_1 \\
I_2
\end{pmatrix} = \int_{-h/2}^{+h/2} \begin{pmatrix}
1 \\
z \\
z^2
\end{pmatrix} \cdot \rho_0 \cdot dz
\]

where \(h\) is the total thickness of the plate and \(\rho_0\) is the relevant density.

Therefore, the force resultants per unit length in three dimensions matrix can be as equation (5)

\[
\begin{pmatrix}
N_{xx} \\
N_{yy} \\
N_{xy}
\end{pmatrix} = \int_{-h/2}^{+h/2} \begin{pmatrix}
\sigma_{xx} \\
\sigma_{yy} \\
\tau_{xy}
\end{pmatrix} dz
\]

But in the simplest condition, the laminated glass composite beam can be considered as a uniform beam in the boundary condition free-free. So, the characteristics equation can be followed as:

\[
\cos(\alpha_2 L) \cos h(\alpha_2 L) = 1
\]
Which the solution to above PDE is as equation (7) as follows:

$$(\alpha_2 L)_n = 4.73004, 7.85321, 10.9956, \ldots \frac{(2n+1)\pi}{2} \quad (7)$$

With eigen values of

$$\omega_1 = \left(\frac{4.73004}{L}\right)^2 \sqrt{\frac{D_{11}}{I_0}} \quad (8)$$

It’s important to notice that formulation above is given for the case neglecting the rotary inertia and not including axial compression load.

## 5 Experiments

The stiffness/strength characteristics and the sound attenuation performance of a single glass panel can be tested in the lab with different approaches. In the study, modal analysis was made. The excitation with a modal hammer as electromechanical vibrator and loudspeaker were applied. The eigenmodes were visualized using an optical approach.

### 5.1 Impact Hammer

The most common excitation instrument for modal analysis for such plate, is an impact hammer. While it is fairly a straightforward technique to use, it’s hard to acquire reliable findings. The ease of this method is appealing because it needs simple hardware and it offers quicker measurement time. Applying the impact pulse is illustrated in Fig.4 which includes a hammer and accelerometer connected to the plate.

![Impact Hammer Test](image.png)

**Fig. 4: Impact Hammer Test**
Because the force is an impulsive force, the energy amplitude level applied to the composition is a function of the velocity and the mass of the hammer. According to the linear momentum concept, which is described as mass, times velocity the linear impulse is equivalent to the incremental difference in the linear momentum. Since, it is difficult to control the velocity of the hammer, therefore, the force amount is typically controlled by adjusting the mass. On the other hand, the frequency subject of the energy which is applied to the composition is a function of the contacting surfaces stiffness and, to a reduced extent, the hammer’s mass. The contacting surfaces stiffness influences the force pulse’s shape which defines the frequency subject and based on the frequency responses, the dominant peaks can be estimated as natural frequencies of the plate.

LABVIEW as shown in Fig.5 is programmed to receive signals from accelerometers which has linked to the glass beam and the output is captured as Fig.6.

![Fig. 5. LABVIEW program for hammer test](image-url)
5.2 Acoustics Sound Excitation

This sound excitation technique suggests a cosine acoustics wave that affect with the LGCP surface without any connection. The advantage of this method rather comparison with using hammer impact test, is its contactless process. (Figure 7) The frequency and magnitude of excitation are tuned to avoid non-linearity or excitation of nonlinear normal modes [13].
The vibration signal excitation is produced by a signal generator via computer on a specific frequency and some sands are distrusted on the panel randomly. By changing the frequency, the sands start to move. When excitation frequency meets close to the natural frequencies of the plate, sands start to be cumulative in the modal point of the beam. (The experiment video is available here: https://www.youtube.com/watch?v=N6uEA11Nq4w)

6 Results And Discussion

The stiffness/strength characteristics and the sound attenuation performance of a single glass panel can be improved mainly by increasing the thickness, since the variation of the material properties is limited. Laminated glass composite panel (LGCP) with at least one flexible plastic/viscoelastic interlayer resolves several design problems offering increased stiffness/strength characteristics and sound damping capabilities. One important characteristic of LGCP is the fact that it provides better post failure safety due to ability to hold broken-off pieces of glass together. In combination with thermal and chemical treatment of glass sheets interlayer provides more possibilities to tailor the LGCP to specific needs of the application. Wide range of possible interlayer materials exist, so the design containing layers with very different properties can be combined. However, if the properties are very different then it also causes some problems and limitations that must be addressed in the research. In the current study the vibration analysis of the laminated glass plate strip is performed by use of analytical study, hammer impact test and sound acoustics based as experiments. The obtained results are found to be in good agreement with experimental study performed and with analytical solution derived for the particular case study considered.
plate strip with free-free boundary conditions and the obtained results can be summarized as Table 2.

<table>
<thead>
<tr>
<th>Method</th>
<th>Analytical Solution</th>
<th>Hammer Test</th>
<th>Sound Excitation - Sand</th>
</tr>
</thead>
<tbody>
<tr>
<td>1st NF</td>
<td>32</td>
<td>32</td>
<td>33</td>
</tr>
<tr>
<td>2nd NF</td>
<td>88</td>
<td>89</td>
<td>90</td>
</tr>
<tr>
<td>3rd NF</td>
<td>173</td>
<td>175</td>
<td>181</td>
</tr>
</tbody>
</table>

In the case of analytical solution, the eigenvalue problem was converted into solution of transcendental algebraic equation. Based on experimental results with hammer test, the damping coefficient on time domain according to Fig 8 and equation (9) has been computed.

\[
\zeta = \frac{1}{\sqrt{1 + \left(\frac{2\pi}{\delta}\right)^2}} = 0.001
\]

Where

\[
\delta = \frac{1}{n} \ln \left(\frac{x(t)}{x(t + nT)}\right)
\]  

(9)

Fig. 8. Damping coefficient calculation
7 CONCLUSIONS

The vibration of laminated glass composite panel (LGCP) is investigated experimentally in this article and the analytical solution is considered to compare the experimental results. Advanced equipment and software are employed for measuring natural frequencies of LGCP. Impact hammer and acoustics-based methods were applied to characterize the properties of the panel. The future study is related with development of advanced plate model for LGCP with interlayers. One modern approach for describing the viscoelastic behavior of the material is use of fractional viscoelastic model which can be employed for modeling interlayer(s).

8 acknowledgment

The result was obtained through the financial support of the Ministry of Education, Youth and Sports of the Czech Republic and the European Union (European Structural and Investment Funds - Operational Programme Research, Development and Education) in the frames of the project “Modular platform for autonomous chassis of specialized electric vehicles for freight and equipment transportation”. Reg. No. CZ.02.1.01/0.0/0.0/16 025/0007293

9 References


Open Access This chapter is licensed under the terms of the Creative Commons Attribution-NonCommercial 4.0 International License (http://creativecommons.org/licenses/by-nc/4.0/), which permits any noncommercial use, sharing, adaptation, distribution and reproduction in any medium or format, as long as you give appropriate credit to the original author(s) and the source, provide a link to the Creative Commons license and indicate if changes were made.

The images or other third party material in this chapter are included in the chapter's Creative Commons license, unless indicated otherwise in a credit line to the material. If material is not included in the chapter's Creative Commons license and your intended use is not permitted by statutory regulation or exceeds the permitted use, you will need to obtain permission directly from the copyright holder.