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Calculation of Coordinates of an Internal Gearing Fillet Manufactured by a Pinion-type Shaper Cutter with Rounded Tip

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Abstract. The solution is made for the pinion-type shaper cutter with several parameters (AMC, tip rounding, helix angle, etc.). Normally, for the production of internal teeth, a shaper wheel with a so-called "sharp tips" is used. This means that the tool tip curves are $\rho_{a0} = 0$. When using rounded tool tips for manufacturing ($\rho_{a0} > 0$), the solution is somewhat more difficult, especially in the case of helical gears. The coordinates of the ring gear fillet are usually solved in the simplest way – by the generating process. The calculation step is the constant angle of rotation $\Delta\varphi_0$ of the tool or $\Delta\varphi_2$ of the ring gear. However, the obtained points of the fillet do not have equidistant distances, which is not so suitable for the FEM calculations that usually follow. Therefore, a method will be introduced to allow the conversion of these coordinates into coordinates meeting the condition of equidistance.

Keywords: Internal Gearing Fillet, Shaper Cutter, Rounded Tip.

1 Introduction

Solution is made for a Shaper cutter with any parameters (Addendum Modification Coefficient, tip rounding, helix angle etc.). This type of tool is not used as much as a tool with a sharp crest That means the tip rounding of a Shaper cutter $\rho_{a0} = 0$ (fig. 1). Solution for this sharp crest is in [1].
When a tool with rounded tip ($\rho_{a0} > 0$) is used for manufacturing, solving is rather complicated, especially for helical gears. Tooth shape of a tool is in the fig. 2. Design of a tool tip for manufacturing can be made by different ways – see [2]. An actual regrinding of a tool face has a big influence – the size of its addendum modification coefficient x_0 is changing. The calculation of parameters d_{L0}, d_{S0} and c_{a0} is performed in [2]. Rounding ρ_{a0} will be considered in the normal section (for helical gears) for next calculations. Circular rounding in the transverse section is not usual one and solving would be more complicated.

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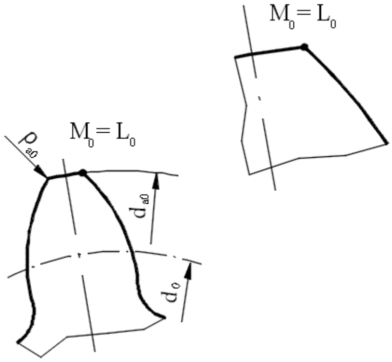

Fig. 1. Sharp crest of the tool

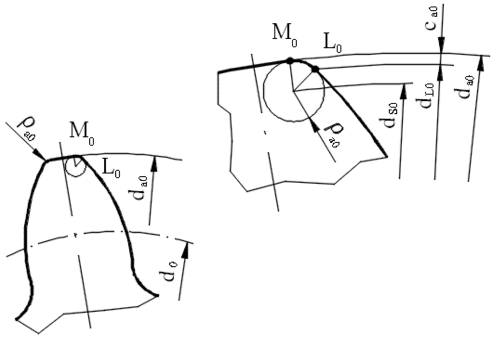

Fig. 2. Rounded tip of the tool

A mesh of a tool (z_0) and a manufactured gear with internal gearing (z_2) is solving for very calculation of coordinates of z_2 fillet. Both are in the figure 5 when they achieved a final (manufacturing) central distance a_{w0} [3]. Two rotating system of coordinates are established here. Tool's system of coordinate origin is in its center O_0 and the axis y_0 is identical to the axis of its tooth space. Workpiece's system of coordinate origin is in the center of a gear with internal gearing (ring gear) and the axis y_2 is identical to the axis of the machined tooth. Starting position for a calculation is considered for machining of the first point of the fillet – M_2. This point lies on the root diameter of the ring gear and it is the start of its fillet at the same time - fig. 3.

Corresponding tool's point M_0 lies on its tip circle and it is the start of its tip rounding - fig. 2. Position of the tool and the workpiece (after joining of these two points - fig. 5) is given by the vertical line $O_2 – O_0 – C_0 – M$. During a subsequent manufacturing of the fillet rounding by the Fellow cutter rotation clockwise this fillet is created from the point M_2 (root circle) to the point L_2 (end of the involute of the ring gear). For next calculations it is important to connect the just manufactured point by a line going through the point C_0. In the case of the point L_2 it is a meshing line. In the case of the point M_2 it is a connecting line $M_2 – C_0 – O_2$. For next points of the ring gear fillet this line has a general position limited by angles α_{w0} a $\pi/2$. Points M_2 and L_2 (start and the end of the fillet) see fig. 3.

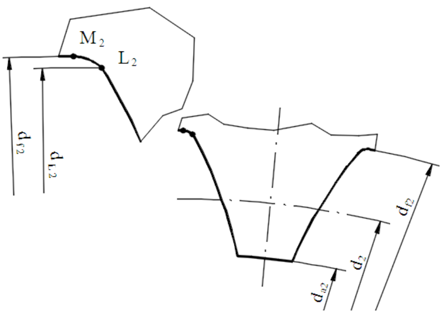

Fig. 3. Ring gear fillet

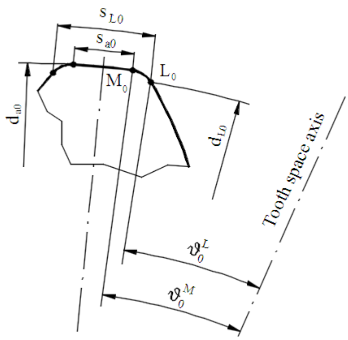

Fig. 4. Tool tooth thickness angles

2 Analysis for spur gears

Before the very solution it is necessary to specify thickness angles of the tool tooth (measured from the axis of its tooth space) relevant to points M_0 and L_0 – see fig. 4. These angles are applying by practice [4] –

$$\upsilon_0^L = \frac{\pi}{z_0} - \frac{s_{tL0}}{d_{L0}} \tag{1}$$

$$\upsilon_0^M = \frac{\pi}{z_0} - \frac{s_{ta0}}{d_{a0}} \tag{2}$$

First it is necessary to choose number of points of the fillet (n_p) for a determination of their coordinates. Total turning angle of the tool when the whole fillet is manufactured – φ_0^Σ, or a total turning angle of the workpiece – φ_2^Σ is then divided to n_p-1 spacing. Both of these angles can be used. Relation between them can be applied –

$$\varphi_0 = -\varphi_2 \cdot u_0 \tag{3}$$

Starting position $M_0 \equiv M_2$ for manufacturing of the fillet is in the fig. 5. The start angle for the ring gear then –

$$\varphi_2^M = -\frac{\varphi_0^M}{u_0} = -\frac{\upsilon_0^M}{u_0} \tag{4}$$

End position $L_0 \equiv L_2$ for manufacturing of the fillet is in the fig. 5. The end angle for the ring gear then –

$$\varphi_2^L = -\frac{\alpha_{tL0} - \alpha_{tw0} + \upsilon_0^L}{u_0} \tag{5}$$

Total turning angle of the ring gear when the whole fillet is manufactured –

$$\varphi_2^\Sigma = -\frac{\alpha_{tL0} - \alpha_{tw0} + \upsilon_0^L - \upsilon_0^M}{u_0} \tag{6}$$

And an appropriate division (a calculation step) –

$$\Delta\varphi_2 = \frac{\varphi_2}{n_p - 1} \tag{7}$$

$$\Delta\varphi_0 = -\Delta\varphi_2 \cdot u_0 \tag{8}$$

The next step is a calculation of a position of a point Y_0 of a tool tip (which one is just in the contact to the manufactured ring gear fillet – see fig. 7) between points M_0 and L_0 (see fig. 4). It is necessary to determine a position of the center of the tool tip rounding radius S_0 for precise assessment of the position of this point. This will be solved in a normal section. When helical gears are used – solving is made for virtual spur gears $z_{0n} - z_{2n}$. It means that next explanation will be shown for spur gears. When a tool is rotating for an angle $\Delta\varphi_0$ the center of a tool tip radius S_0 is rotating with it. A position of this common point Y_0 is in fig. 7.

For assessment of the position of the common point $Y_0[i]$ it is necessary to calculate an appropriate diameter $d_{Y0}[i]$ and the angle from the vertical axis $\xi[i]$. Diameter d_{S0}, the distance $O_0 - C_0 = 0{,}5 \cdot d_{w0}$, and the angle $\varphi_0[i]$ are known. For it is applied $\varphi_0[i] = i \cdot \Delta\varphi_0$.

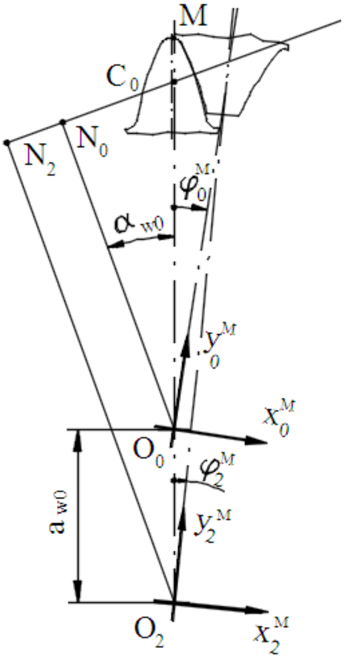

Fig. 5. Start point

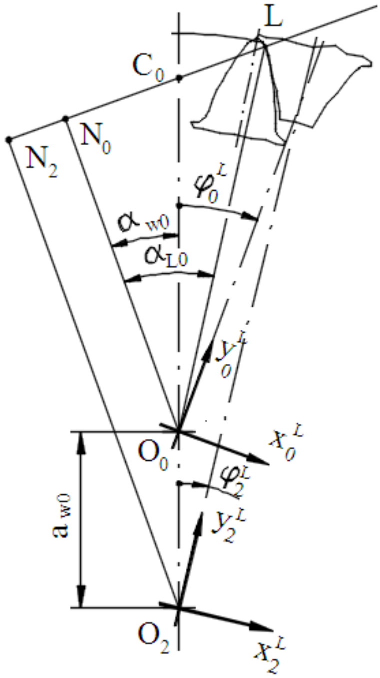

Fig. 6. End point

Distance $C_0S_0[i]$ is calculated as the first one –

$$C_0S_0[i] = \sqrt{\left(\frac{d_{w0}}{2}\right)^2 + \left(\frac{d_{S0}}{2}\right)^2 - d_{w0} \cdot \frac{d_{S0}}{2} \cdot cos\big((i-1) \cdot \Delta\varphi_0\big)} \quad (9)$$

Then the angle $\omega[i]$ is determined –

$$\omega[i] = arcsin\left(\frac{0{,}5 \cdot d_{S0}}{C_0S_0[i]} \cdot sin\big((i-1) \cdot \Delta\varphi_0\big)\right) \quad (10)$$

Now it is possible to enumerate distance of the point $Y_0[i]$ from the center of the tool -

$$d_{Y0}[i] = 2 \cdot \sqrt{\left(C_0S_0[i] + \rho_{a0}\right)^2 + \left(\frac{d_{w0}}{2}\right)^2 - \left(C_0S_0[i] + \rho_{a0}\right) \cdot d_{w0} \cdot cos(\pi - \omega[i])} \quad (11)$$

And the relevant rotation of the point $Y_0[i]$ from the vertical axis –

$$\xi[i] = arcsin\left(\frac{C_0S_0[i] + \rho_{a0}}{0{,}5 \cdot d_{Y0}[i]} * sin(\pi - \omega[i])\right) \quad (12)$$

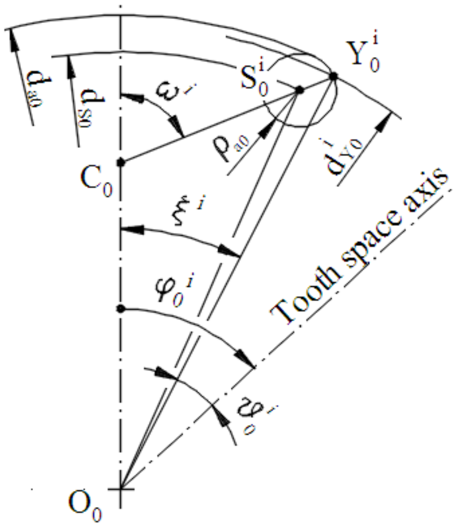

Fig. 7. Tool tooth contact point Y_0

For next calculations it is good to know an actual thickness angle and relevant rotation angles –

$$\upsilon_0[i] = \upsilon_0^M + (i-1)\cdot\Delta\varphi_0 - \xi[i] \quad (13)$$

$$\varphi_0[i] = \upsilon_0^M + (i-1)\cdot\Delta\varphi_0 \quad (14)$$

$$\varphi_2[i] = -\frac{\varphi_0[i]}{u_0} \quad (15)$$

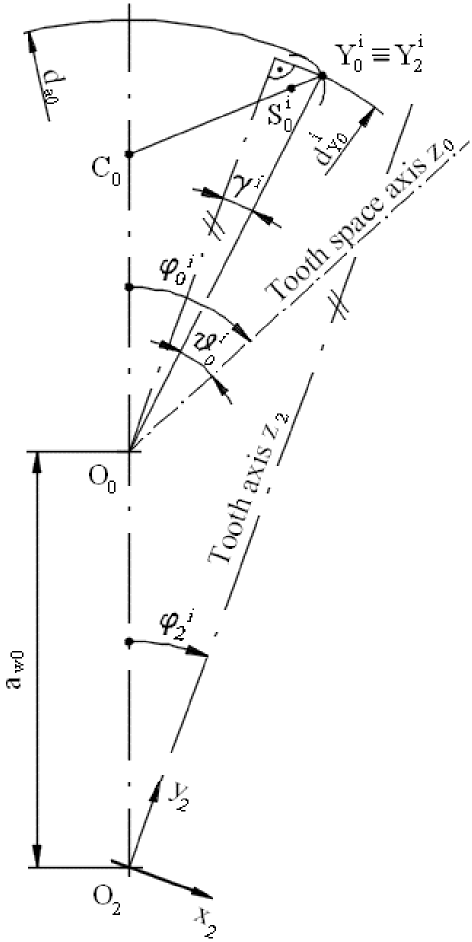

Fig. 8. Relation $z_0 - z_2$

For calculation of coordinates of a ring gear fillet just one angle is missing (fig. 8) –

$$\gamma[i] = \varphi_0[i] - \varphi_2[i] - \upsilon_0[i] \quad (16)$$

And searched coordinates then –

$$x_2[i] = -0{,}5\cdot d_{Y0}[i]\cdot sin(\gamma[i]) - a_{w0}\cdot sin(\varphi_2[i]) \quad (17)$$

$$y_2[i] = 0{,}5\cdot d_{Y0}[i]\cdot cos(\gamma[i]) - a_{w0}\cdot cos(\varphi_2[i]) \quad (18)$$

3 Transformation of coordinates to coordinates with equidistant spacing

Ring gear fillet coordinates gained in the foregoing part are completed by the simplest way – by generating system. The constant angle of slewing of a ring gear $\Delta\varphi_2$ or of a tool $\Delta\varphi_0$ is a step of a calculation. Common method is used for solving. Points of the profile gained by this method do not have an equidistant spacing which is not good for next FEM calculations - for some workpiece and tool parameters, the distances between individual points of the ring gear fillet are extremely large. The method which allows a transformation of these coordinates to those ones which fulfill the condition of equidistant spacing will be brought in. Since the fillet is a general curve, it is not easy to find analytically corresponding (various) slewing $\Delta\varphi_0$, such as distances of adjoining points of the involute (ΔL_p) were always the same. It is preferential to enter constant (equidistant) spacing between points describing a tooth contour into stress or deformation FEM tasks.

Direct analytical calculation of such coordinates is very tough. Therefore, it is better to use already computed coordinates with non-equidistant spacing between adjoining points and to transform them to the new coordinates with constant spacing between adjoining points. Finding of corresponding angles $\Delta\varphi_i$ is possible to do by using so called computing "brute" force. A data file of profile coordinates with non-equidistant spacing is hence available. When transforming, a rule must be kept that the fillet must be divided into the same number of points for an equidistant spacing.

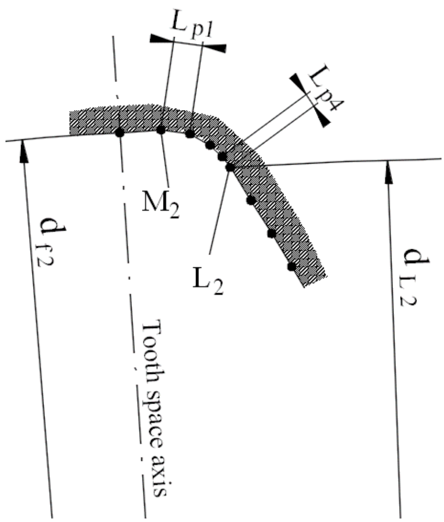

Fig. 9. Non-equidistant spacing

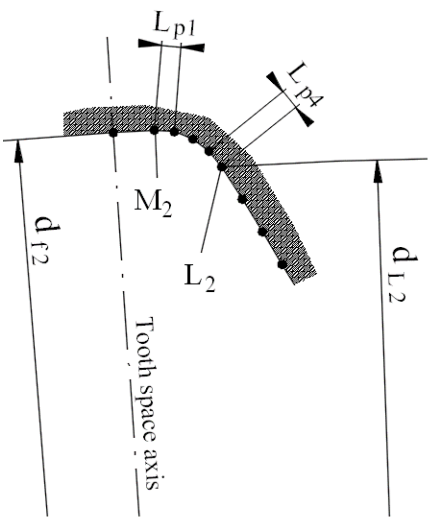

Fig. 10. Equidistant spacing

For next calculations a system of coordinate origin x, y is in the center of a ring gear. Calculation of the fillet length L_p (it does not matter which direction – from the root circle or to the root circle), as a sum of n_p-1 segments with unequal lengths (Fig. 9) –

$$L_p = \sum_{i=1}^{n_p-1} \Delta L_{p\,i} \tag{19}$$

$$L_p = \sum_{i=1}^{n_p-1} \sqrt{(x_i - x_{i+1})^2 + (y_i - y_{i+1})^2} \tag{20}$$

Determination of the equidistant spacing of adjoining points of the fillet –

$$\Delta L_p = \frac{L_p}{n_p - 1} \tag{21}$$

Calculation of new coordinates for this equidistant spacing of adjoining points is more complicated. The starting point is the first point M_2 of the fillet (it is on the root circle d_{f2}) with coordinates [x_1, y_1] and the end point is L_2 – the end point of the involute (it is on the end of the involute circle d_{L2}) with coordinates [x_p, y_p]. These coordinates (starting and ending ones) are fixed for the both cases, they are calculated using method from the foregoing part. The next coordinates of equidistantly distant points (ΔL_p) must be solved numerically. The principal formula for a numerical solution has this form –

$$\Delta L_p - \sqrt{(x_j - x_{j+1})^2 + (y_j - y_{j+1})^2} = 0 \tag{22}$$

Where coordinates [x_i, y_i] have been already known and [x_{i+1}, y_{i+1}] are calculated. A calculation is repeated this way for all points. The principal formula is added with formulas from 9) to 18) which ensure that the newly found points will lie exactly on the fillet. In these equations subscript i is substituting by subscript j+1. The last two equation are shown in a modification for j+1 –

$$x_2[j+1] = -0{,}5 \cdot d_{Y0}[j+1] \cdot sin(\gamma[j+1]) - a_{w0} \cdot sin(\varphi_2[j+1]) \tag{23}$$

$$y_2[j+1] = 0{,}5 \cdot d_{Y0}[j+1] \cdot cos(\gamma[j+1]) - a_{w0} \cdot cos(\varphi_2[j+1]) \tag{24}$$

This system of transcendent equations (9÷18 and 22) is being numerically completed. It is the best to enter the required accuracy when zeroizing the principal formula (22). When the fillet is highly divisive (n_p>50) the calculation is instable sometimes. A suitable change of a numerical calculation step will prevent it. Everything of course depends on the numerical method used. Boundaries for φ_0 were chosen from minus 1 to 2. It is enough for any shape of a tooth and any addendum modification coefficient. Single new values of an angle φ_0, (φ_2) are not necessary to save (if they will not be utilized elsewhere). The result is displayed in fig. 10.

It is good to enter one more point (or more points) which lies on the root circle in the center between initials points of fillets (for a good link between them) of adjoining teeth for creating good mesh for the next FEM calculation of more teeth.
The solution for the fillet of a ring gear is seen in Fig. 9 and 10 for 5 points. Number of segments is always one less than points number. The starting point is the first point M_2 of the fillet on the root circle and the end point L_2 is its end point. The point L_2 is the end point of the involute at once. In the figure 9 is the solution with non-equidistant spacings ($L_{p1} > L_{p2} > L_{p3} > L_{p4}$). There are new points of the fillet in the figure 10 which fulfill an equidistancy condition ($L_{p1} = L_{p2} = L_{p3} = L_{p4}$)

4 Conclusion

In the case of solving a fillet for a helical gear, the calculation is performed on a virtual spur gear. Finally, it is necessary to recalculate the coordinates to the transverse section of the solved ring gear. Here, with the increasing helix angle, a small inaccuracy typical of this conversion occurs. Elimination of this inaccuracy can be made by refining the calculation that is usual for these recalculations

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