Mesoscopic Numerical Analysis of Concrete Damage Based on Random Aggregate Model

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Abstract. Concrete is defined as a three-phase composite material consisting of aggregate, mortar matrix, and interfacial transition zone (ITZ). To investigate the influence of various phases on the strength and damage of concrete, this research constructs a two-dimensional random aggregate model of concrete based on mesomechanics. In this foundation, this research further applies the plastic damage model to simulate the mechanical properties and damage of concrete beams under three-point bending. Meanwhile, this research employs the contraction-expansion factor to generate aggregates with diverse shapes, thereby exploring the influence exerted by aggregate shapes on the damage of concrete beams. Relevant research findings indicate that the mechanical properties and damage forms of concrete depend on the distribution of aggregate as well as the strength of the mortar matrix, with aggregate shapes generating a significant impact on the damage of concrete.

Keywords: concrete; mesomechanics; random aggregate model; expansion factor; damage destruction

1 INTRODUCTION

Concrete, as an important building material, embodies a host of advantages, such as easy access to raw materials and low price, favorable plasticity, integrated pouring, high compressive strength, excellent durability, and fire resistance[1]. Nonetheless, the safety of concrete buildings has consistently been highly valued. Within the study of engineering example failure, the mesoscopic composition of concrete affects the change in concrete strength. Seen from a mesoscopic perspective, concrete is defined as a multi-phase composite material composed of aggregate, interface, and mortar, exhibiting a complex internal structure and distinct heterogeneity. The above characteristics lead to the randomness of crack initiation position, propagation path, and internal damage diffusion of concrete under external load[2]. In this regard, the mesoscopic modeling of concrete requires not only ensuring the statistical consistency of aggregate shape, size, and distribution position with the actual concrete but also formulating reasonable aggregate gradation, content, and the maximum content that can be put should be considered comprehensively[3]. Therefore, in this paper, irregular aggregate particles are used instead of the original round particles in order to study
the concrete damage and crack development more rationally and accurately. Compared with macroscopic numerical analysis, mesoscopic numerical simulation is prone to reflect the heterogeneity of concrete.

At a macroscopic level, the crack initiation and propagation of concrete materials under load completely depend on the direction of principal stress\[^4\]. At a mesoscopic level, however, the size and distribution of aggregates, coupled with the strength of different phases, exert a significant impact on the initiation of damage and crack propagation. Particularly, the damage of the concrete interfacial transition zone (ITZ) typically occurs in the weakest part of concrete, with ITZ acting as the weakest link in the concrete structure. Nevertheless, it can be assumed that the behavior of ITZ is similar to that of mortar matrix, with a reduced stiffness and strength.

Currently, the models for the study of concrete microscopic damage are mainly random aggregate models with circular, elliptical and polygonal shapes, using uniaxial tension, uniaxial compression, and three-point bending force to simulate the concrete fracture process and analyze the factors affecting the damage of the concrete, such as interfacial parameter, size effect, and so on. However, the current description of aggregate particles is difficult to present the irregularity of actual concrete aggregates. To this end, this research implements appropriate topological transformation based on circle, ellipse, and polygon by utilizing the contraction-expansion factor\[^5\], thereby forming pebble aggregates, crushed stone aggregates, and convex polygonal aggregates to more realistically characterize the irregularity and complexity of concrete aggregates. On these grounds, this research leverages a two-dimensional mesoscopic numerical simulation method to conduct the bending-resistance numerical analysis of concrete beams, intending to investigate the influence of aggregate shapes on the bearing capacity and damage of concrete beams.

2 CONSTITUTIVE RELATION OF CONCRETE MATERIALS WITH DIVERSE PHASES

When investigating the mesoscopic model of concrete, a heterogeneous material, it is imperative to take into account the constitutive relation of materials with diverse phases separately. As concrete aggregate is difficult to damage due to its high strength and elasticity modulus, aggregate is generally regarded as suitable for the linear elastic model, whereas interface and mortar are regarded as suitable for the elastic-plastic damage constitutive model\[^6\].

2.1 Calculation of Stress-strain

According to the current Code for Design of Concrete Structures: GB 50010-2010\[^7\], the stress-strain equation of plastic damage of concrete is determined by the following expressions:

In the case of tension, the stress-strain equation is given by:

\[
\sigma = (1 - d_e) E \varepsilon
\]  

(1)
In the case of compression, the stress-strain equation is given by:

$$\sigma = \left(1 - d_c \right) E \varepsilon$$  \hspace{1cm} (2)

Guo et al. (1982)\cite{8} divided the stress-strain curve of concrete into ascending section and descending section, thus effectively expressing the tensile-compressive behavior of concrete, with its expression depicted as follows:

In the case of compression, the stress-strain equation is given by:

$$y = \frac{\sigma}{f_c} = \begin{cases} 
\frac{nx}{n-1+x^n} & x = \frac{\varepsilon}{\varepsilon_c} \leq 1 \\
\frac{x}{\alpha_c (x-1)^2 + x} & x = \frac{\varepsilon}{\varepsilon_c} > 1 
\end{cases}$$  \hspace{1cm} (3)

where $f_c$ represents the standard value of uniaxial compressive strength of concrete; $\varepsilon_c$ represents the compression strain corresponding to $f_c$, with $\varepsilon_c = \left(700 + 172\sqrt{f_c}\right) \times 10^{-6}$ being satisfied; $\alpha_c$ represents the parameter of the descending section of the concrete compressive curve, with $\alpha_c = 0.157 f_c^{0.785} - 0.905$ being satisfied; and, $n$ denotes a variable, with $n = E \varepsilon / \left(E \varepsilon - f_c\right)$ being satisfied.

In the case of tension, the stress-strain equation is given by:

$$y = \frac{\sigma}{f_t} = \begin{cases} 
1.2x - 0.2x^6 & x = \frac{\varepsilon}{\varepsilon_t} \leq 1 \\
\frac{x}{\alpha_t (x-1)^{1.7} + x} & x = \frac{\varepsilon}{\varepsilon_t} > 1 
\end{cases}$$  \hspace{1cm} (4)

where $f_t$ represents the standard value of uniaxial tensile strength of concrete; $\varepsilon_t$ represents the tensile strain corresponding to $f_t$, with $\varepsilon_t = f_t \times 65 \times 10^{-6}$ being satisfied; and, $\alpha_t$ represents the parameter of the descending section of the concrete tensile curve, with $\alpha_t = 0.312 f_t^2$ being satisfied.

Concurrently, the tension-compression constitutive relation of concrete in *Code for Design of Concrete Structures*\cite{7} is utilized to determine the parameters of the concrete damage plasticity (CDP) model to obtain the nominal stress-strain of concrete. Furthermore, the following formula transformation is employed to determine the real stress-strain, so that the numerical model is more in line with the real model\cite{9}.

$$\sigma_{true} = \sigma_n (1 + \varepsilon_n)$$ \hspace{1cm} (5)

$$\varepsilon_{true} = \ln \left(1 + \varepsilon_n\right)$$ \hspace{1cm} (6)
where \( \sigma_{\text{ture}} \) and \( \varepsilon_{\text{ture}} \) represent real stress and strain, respectively, whereas \( \sigma_n \) and \( \varepsilon_{\text{ture}} \) represent nominal stress and strain, respectively.

### 2.2 Calculation of Damage Factor in the CDP Model

In the calculation of damage factor by the CDP model, besides stress-strain, both tensile cracking strain \( \varepsilon_t^{\text{ck}} \) and compressive inelastic strain \( \varepsilon_c^{\text{in}} \), which are beyond the elastic stress stage in tension and compression, need to be defined to describe the tension stiffening and compressive stiffening of concrete\(^{[10]}\). The aforementioned tensile cracking strain \( \varepsilon_t^{\text{ck}} \) and compressive inelastic strain \( \varepsilon_c^{\text{in}} \) are determined as:

\[
\varepsilon_t^{\text{ck}} = \varepsilon_t - \sigma_t / E_0
\]

\[
\varepsilon_c^{\text{in}} = \varepsilon_c - \sigma_c / E_0
\]

where \( \sigma_t \) and \( \sigma_c \) represent tensile elastic stress and compressive elastic stress, respectively, while \( \varepsilon_t \) and \( \varepsilon_c \) represent tensile strain and compressive strain, respectively.

Regarding the calculation of damage factor, ABAQUS will automatically convert tensile cracking strain and compressive inelastic strain into plastic strains \( \varepsilon_t^{\text{pl}} \) and \( \varepsilon_c^{\text{in}} \) upon inputting them. Drawing on the research of Liu et al. (2014)\(^{[11]}\), this paper assumes that the ratio of plastic strain \( \varepsilon_t^{\text{pl}} \) to tensile cracking strain \( \varepsilon_t^{\text{ck}} \) during tension is \( \eta_t \), and the ratio of plastic strain \( \varepsilon_c^{\text{pl}} \) to compressive inelastic strain \( \varepsilon_c^{\text{in}} \) during compression is \( \eta_c \). Accordingly, the damage factor is determined as:

\[
d_t = \frac{(1-\eta_t) \varepsilon_t^{\text{pl}} E_0}{\sigma_t + (1-\eta_t) \varepsilon_t^{\text{pl}} E_0}
\]

\[
d_c = \frac{(1-\eta_c) \varepsilon_c^{\text{pl}} E_0}{\sigma_c + (1-\eta_c) \varepsilon_c^{\text{in}} E_0}
\]

### 2.3 Other Parameters

Apart from the foregoing parameters, CDP calculation necessitates the remaining calculation parameters in the CDP model\(^{[9]}\), as described in Table 1:

Since the damage of concrete materials is caused by the formation and expansion of damage, the use of the Concrete Damage Plasticity (CDP) model to establish the ontological relationship to analyze the damage of concrete can reflect the nonlinear behavior of concrete at the time of damage, taking into account the average distribu-
tion of microdefects and micropores, and at the same time, it can describe the whole process of concrete cracking.

Table 1. Remaining parameters of the CDP model.

<table>
<thead>
<tr>
<th>Parameter</th>
<th>Value</th>
</tr>
</thead>
<tbody>
<tr>
<td>Expansion angle $\psi$</td>
<td>30°</td>
</tr>
<tr>
<td>Eccentricity ratio $\varepsilon$</td>
<td>0.1</td>
</tr>
<tr>
<td>Biaxial/uniaxial ultimate compressive strength $f_{b0}/f_{c0}$</td>
<td>1.16</td>
</tr>
<tr>
<td>Invariant ratio $K$</td>
<td>0.667</td>
</tr>
<tr>
<td>Viscosity coefficient $\mu$</td>
<td>0.0005</td>
</tr>
</tbody>
</table>

3  MESOSCOPIC DAMAGE ANALYSIS OF CONCRETE

Fig. 1. Two-dimensional mesoscopic analytic model of concrete with aggregate content of 55% under three-point bending.

Leveraging the methodology of two-dimensional mesoscopic numerical simulation, this paper investigates the influence of diverse parameters of concrete, such as aggregate shapes, particle size, distribution, volume fraction, strength of interface transmission zone, and mortar matrix strength, on the tensile properties of concrete. On the same note, the numerical simulation of this model adopts the fracture of Peterson's three-point bending notched beam\cite{12}, with the beam size being 2000x200x50 mm$^3$, and the notch depth being half the height of the beam. Notably, the 200 mm mid-span part is taken as a non-uniform part\cite{13}, whereas the rest is taken as a uniform part, with the plane stress triangular element (CPS3R) employed for grid division. Furthermore, within the random distribution of aggregate in the uneven part of concrete, the radii of coarse aggregate, medium aggregate, and fine aggregate are respectively in the range of 20 to 40 mm, 10 to 20 mm, and 2.5 to 10 mm, and the aggregate gradation is 4: 3: 3, as outlined in Figure 1. Given the limited experimental data concerning the thickness and mechanical properties of the interfacial zone, this research assumes that the thickness of the interfacial zone is 0.6 mm, with its behavior described by the CDP model proposed. In comparison with the cement mortar matrix, the interfacial zone exhibits weaker mechanical properties.
3.1 Material Mechanical Properties of Concrete's Mortar Matrix and Interface

At a mesoscopic level, the mechanical properties of each material exert a considerable influence on the overall macro-behavior of concrete. Normally, microcracks start at the weakest link. ITZ acts as the weakest link in concrete structures. Hence, determining the tensile and compressive behavior of the transition interface is one of the important tasks in predicting the overall mechanical properties of concrete composites. In this regard, this research assumes that ITZ presents weak mechanical properties, behaves similarly to the mortar matrix, and showcases uniformity in thickness[14].

Table 2. Mechanical parameters of concrete materials with different phases.

<table>
<thead>
<tr>
<th>Materials</th>
<th>Elasticity Modulus (GPa)</th>
<th>Poisson ratio</th>
<th>Tensile strength (MPa)</th>
<th>Compressive strength (MPa)</th>
</tr>
</thead>
<tbody>
<tr>
<td>Aggregate</td>
<td>80</td>
<td>0.18</td>
<td>3.72</td>
<td>37.2</td>
</tr>
<tr>
<td>Mortar</td>
<td>30</td>
<td>0.2</td>
<td>2.5</td>
<td>25</td>
</tr>
<tr>
<td>Interface</td>
<td>22</td>
<td>0.22</td>
<td></td>
<td></td>
</tr>
</tbody>
</table>

In this research, high aggregate strength and elasticity modulus make it difficult to produce cracks and plastic deformation in cases of concrete damage. It is assumed, therefore, that the damage zone occurs in the mortar aggregate and the interface layer, with the aggregate being a linear elastic material. Meanwhile, this research assumes that the relationship between compressive strength and tensile strength of aggregate and interface is 0.1 times [15-16], and the mechanical parameters of materials with diverse phases are illustrated in Table 2 [13].

3.2 Influence of Aggregate Shapes

The circle, ellipse, and polygon are typically utilized to approximately simulate pebble aggregate and crushed stone aggregate for two-dimensional concrete aggregate within the research field of concrete mesoscopic damage. In practice, however, the application of circular aggregate simulation may lead to some errors. To determine more realistic concrete characteristics, it is imperative to adopt more detailed methods.
to simulate the geometric characteristics of aggregate shapes. At this point, this paper employs the contraction-expansion factor to obtain more complex and diverse shapes of pebble aggregate, crushed stone aggregate, and convex polygonal aggregate, thereby constructing a random aggregate model of concrete.

In the case of concrete beam damage, the distribution of microcracks in concrete with different aggregate shapes is depicted in Figure 2, while the corresponding load-deflection curves are shown in Figure 3. During the loading process of concrete beams, the bending side of concrete is affected by tension, which is more likely to lead to tensile failure\(^{17}\). In the case of damage, cracks generally originate from the weak interface of the mid-span notch of concrete beams and subsequently spread along the interface, ultimately propagating from the interior of mortar when leaving the interface. Taking into account that concrete aggregate is a linear elastic material, on the other hand, cracks will not occur inside the aggregate. In addition, concrete aggregate particles exert a certain obstacle to the propagation of microcracks, rendering the diffusion of microcracks along the aggregate interface. Thus, in the case of concrete beam damage, cracks usually originate and develop upward at the weak part of the beam notch, with the development of damage stemming from a main crack. Notably, the stress concentration at the edges and corners of crushed stone aggregate and polygonal aggregate results in the concentrated emergence of cracks in these places, which in turn causes the damage unit of the pebble aggregate model to be lower than that of other aggregate models during the damage process. Moreover, the obstruction of aggregate to the development of cracks makes the softening section of the concentrated stress-deflection curve in Figure 3 show a slight floating phenomenon. Concurrently, the maximum loads of the crushed stone aggregate model and polygonal aggregate model are 665.12 N and 675.35 N, respectively, which are close to each other. Compared with other aggregates, the concrete beam with pebble aggregate shows a strong bearing capacity of 718.35 N. Accordingly, its fracture energy required in the damage process is higher than that of other aggregate shapes. Meanwhile, the damage of concrete beams in this paper is the same as the damage pattern in the references, the crack development is sprouted and upward in the more vulnerable part of the beam incision, and all of them are developed and damaged by one main crack, and their peak loads are shown in Figure 3, which is a relatively small difference.
Fig. 3. Mid-span Concentrated Stress-deflection Curve of Concrete Beams with Different Aggregate Shapes.

4 CONCLUSION

In summary, based on the pebble aggregate model generated by the contraction-expansion factor, this research leverages the three-point bending analysis method to investigate the influence of aggregate shapes on the bending-resistance performance and damage of concrete. Regardless of the limited influence of the changes in aggregate shapes on the bearing capacity of concrete, different aggregate shapes lead to varying distributions of concrete cracks. Compared with pebble aggregate and crushed stone aggregate, polygonal aggregate is more prone to stress concentration at sharp edges. Meanwhile, the mid-span bearing capacity of concrete models with pebble aggregate is slightly higher than that of models with other aggregate shapes.

However, in the course of this paper, concrete is only regarded as a composite material composed of aggregate, mortar, and interface, and the non-uniformity of the mechanical parameters of each phase of the material due to the initial defects existing inside the concrete is not taken into account, and the effect of the random distribution of its mechanical parameters should be considered in the subsequent research.

REFERENCES

1. He Z B (2021) Detailed Mechanical Study on Damage Mechanism of Basalt Fiber Concrete under Fatigue Load. Xinjiang University.