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Bi-objective Hospital Bed Assignment Problem in Emergencies

Hela Jedidi[1*], Hajer Ben-Romdhane[1], Issam Nouaouri[2], and Saoussen Krichen[1]

[1] Université de Tunis, Institut Superieur de Gestion de Tunis, LARODEC laboratory, Tunis, Tunisia.
[*] hela.jedidi@esprit.tn
[2] Université de l'Artois, UR 3926, LGI2A laboratory, Bethune, 62400, France

Abstract. During emergencies and pandemics, such as COVID-19 or Mpox, the massive demand for critical resources, including hospital beds, has significantly challenged healthcare systems. Reducing the pressure on these systems is crucial and requires an efficient strategy for patient allocation. This paper proposes a bi-objective optimization model to allocate patients to hospital beds during emergencies, including pandemics. The problem is formulated as a linear programming model. The first objective is to minimize the cost of transporting patients to the nearest hospitals, while the second objective is to minimize the cost of assigning patients to available beds, guaranteeing that bed capacity is fully utilized. We applied the weighted-sum method to combine objectives into a single-objective instance to solve this problem. We tested the proposed model using benchmarks from the literature with IBM Cplex Studio. The findings provide healthcare analysts with valuable insights for creating flexible and effective allocation plans during pandemics.

Keywords: Emergencies, hospital allocation, optimization, patient allocation, bed assignment.

1 Introduction

One of the key challenges for public health systems during emergencies or pandemics is to allocate patients to the nearest and most appropriate hospital and assign them to available beds. Therefore, it is necessary to determine the locations of healthcare facilities, the position of patients, and the volume of demand. This task is part of a larger class of allocation problems in healthcare systems, which deal with the process of allocating patients or resources including beds, medical personnel, and equipment to existing healthcare facilities [4].
Hospital Allocation Problems (HAP) focus on assigning patients to hospitals while considering several factors such as hospital capacity, patient needs, and transportation logistics. The main goal is to minimize the total travel distance for patients through a multi-objective optimization problem formulation. Santibanez et al. [9] in 2006, developed a mathematical model to optimize Fraser Health's inpatient hospital network in Canada, proposing two objectives using

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the weighted-sum method: (1) minimization of the total travel patient distance, and (2) minimization of the total disruption cost to the existing system.
Mitropoulos et al. [10] employed constraint methods to distribute healthcare facilities effectively while minimizing travel distance for patients.
Furthermore, Beheshtifar et al. [11] introduced a multiobjective model integrating GIS analysis with a genetic algorithm, which addresses multiple objectives, including travel costs and equitable access. In urban settings, Zghan et al [12] tackled HAP using genetic algorithms to generate Pareto solutions, allowing planners to evaluate trade-offs effectively. These studies highlight the significant progress in understanding hospital allocation challenges, although many approach these problems in isolation.

In contrast, a few studies focused on HAP in pandemics. For example, Li et al. [13] in 2014, optimized patient access by minimizing travel distance. Their mathematical models aim to predict resource shortages and optimize the distribution of additional resources. The authors validated the proposed model using a real case study from hospitals in Kentucky.
More recently, Devi et al. [14] proposed a mixed-integer linear programming model for optimizing the location and allocation of temporary testing laboratories during influenza pandemic scenarios, including COVID-19. The model aimed to minimize both the total cost and maximum travel time for testing, with a case study in Maharashtra, India, demonstrating its practical application for decision-makers.
In 2023, Tsai et al. [15] proposed single and multiple objective linear programming models during a dengue fever outbreak. Focusing on minimizing patient travel distance and additional resources like ICU beds, the model's applicability was tested through an empirical study in Ciudad del Este, Paraguay.

Allocation problems in healthcare systems involve assigning patients to hospitals and allocating resources to those patients. Transitioning to the Patient Bed Assignment Problem (PBAP), which specifically addresses the assignment of patients to available beds as a single-objective optimization problem, these issues have developed through several extensions, with an emphasis on both static and dynamic situations [4][18].
Elective patients are allocated to beds in the static version for a predetermined amount of time. Depending on their requirements, constraints in this context are often classified as either soft (flexible) or hard (required) [5] [6]. The dynamic version takes into account real-world uncertainties while rescheduling daily appointments for urgent patient assignments [1][2][3]. Due to PBAP's NP-hardness [8], several techniques have been developed, including exact algorithms and heuristics like simulated annealing with local search and hybrid tabu-search [7][19]. Moreover, other works have combined machine learning techniques with optimizing algorithms to deal with the PBAP complexity [20].

Despite the advances in both HAP and PBAP, researchers have often explored these as two separate issues [15]. Tsai et al. proposed combining these two problems into a multi-objective allocation problem, aiming to minimize the total distance traveled by each patient and the cost of locating new hospitals.

In summary, while significant strides have been made to deal with HAP and PBAP, gaps still exist in integrating these issues, particularly in emergency contexts.

Our paper introduces a bi-objective hospital bed assignment model (HBAP) that integrates HAP and PBAP for the first time during emergencies. Our approach provides a comprehensive framework for optimal patient assignment to beds based on their locations. Tsai et al.'s [15] work differs from our proposal in that it addresses PBAP as a set of constraints and focuses on minimizing both objectives related to travel distances, whereas our model introduces distinct objective functions and considers PBAP as one of these objectives. Specifically, we focus on minimizing the total travel distance between the patient's location and the hospital, as well as minimizing the total assignment costs, which include both the assignment and transfer costs from one bed to another.
We develop a mathematical programming model and employ a weighted-sum method to determine a representative set of optimal Pareto solutions for the static variant of PBAP.

The remainder of this paper is organized as follows: section 2 details the problem description. Then, section 3 presents the problem formulation. Section 4 proposes the solution methodology. Section 5 reports the computational results. The conclusion with some future work is given in section 6.

2 Problem description

In emergencies, hospitals aim to respond to the massive demand for medical resources, particularly hospital beds. The main goal is to allocate the available beds efficiently to ensure that all patients receive appropriate care. Suppose there are two hospitals, H1 and H2, in the area where patient demand exists. Three patients A, B, and C require critical and emergent care. Hospital H1 has one available bed (B11), and Hospital H2 has two available beds (B21 and B22). In such a situation, it is crucial to know the exact location of each patient to determine the nearest hospital. Therefore, in our work, we address not only the PBAP but also the HAP by identifying the closest hospital for each patient and allocating them to an available bed in that hospital. We introduce a bi-objective linear programming model to determine the optimal allocation. For instance, if Hospital H2 is nearest to patients A and C, and Hospital H1 is nearest to patient B, then the optimal assignment would be as follows: Patient A and Patient C would be assigned to beds B21 and B22 in Hospital H2, respectively, while Patient B would be allocated to bed B11 in Hospital H1. Our goal is to minimize the total travel cost, which includes the distance between the patient's location and the hospital, as well as the total assignment cost for patients in inappropriate beds, including the transfer cost between beds.

To address the complexity of the problem [8], we make the following assumptions:

- Homogeneous Bed Types: All hospitals are considered to have the same type of beds, there is no difference between ICU beds and regular ones.
- Fixed Hospital sites: During an emergency, no new hospitals are built. The locations of the existing hospitals are predetermined and fixed.
- Patient Arrival Rates: Patient arrival rates are known and constant.
- Single Assignment Rule: Each patient can be assigned to only one hospital bed.
- Non-Transferability of Patients: Patients cannot be transferred from one bed to another.
- Capacity Constraints: Each hospital has a maximum number of available beds that it can accommodate.
- Mixed Rooms: In emergencies, the primary focus is on efficiently assigning beds to patients; therefore, distinctions between male and female patients are not considered in our model.

3 Problem formulation

We formulate the HBAP as a bi-objective linear programming problem. Table 1 reports the problem notations proposed in this study.

Optimizing this problem involves minimizing two objectives: (1) The first objective is to minimize the cost of the travel distance from patients to hospital locations. (2) The second objective is to minimize two components: The cost of assigning patients to inappropriate beds and the cost associated with transferring patients.

$$Min \quad Z_1 = \sum_{p \in P} \sum_{h \in H} \sum_{b \in B_h} C_{ph} * d_{ph} * X_{pbh}; \tag{1}$$

$$Min \quad Z_2 = \sum_{p \in P} \sum_{h \in H} \sum_{b \in B_h} Cost_{pbh} * X_{pbh} + \sum_{p \in P} \sum_{h \in H} \sum_{b \in B_h} \sum_{k \in B_h} w_{pbh} \times T_{pbkh}; \tag{2}$$

S/C

$$\sum_{h \in H} \sum_{b \in B_h} X_{pbh} = 1 \qquad \forall p \in P \tag{3}$$

$$\sum_{p \in P} X_{pbh} = 1 \qquad \forall h \in H, b \in B_h \tag{4}$$

Table 1: Problem notations

Parameters:	
P	Set of patients
B_h	Set of Beds in hospital h, indexed by b
H	Set of hospitals
d_{ph}	Distance between patient p location and hospital h;
$Cost_{pbh}$	Cost of assigning patient p to bed b in hospital h;
C_{ph}	Travelling cost from patient p location to hospital h;
w_{pbh}	Cost of transferring patient p from bed b in hospital h

Decision Variables:

$$X_{pbh} = \begin{cases} 1 \; if \; patient \; p \; is \; assigned \; to \; bed \; b \; in \; hospital \; h; \\ 0 \; Otherwise. \end{cases}$$

$$T_{pbkh} = \begin{cases} 1 \; if \; patient \; p \; is \; transferred \; from \; bed \; b \; to \; bed \; k; \\ 0 \; Otherwise. \end{cases}$$

$$\sum_{p \in P} X_{pbh} \leq B_h \quad \forall h \in H, b \in B[h] \tag{5}$$

$$\sum_{k \in B_h} T_{pbkh} \leq X_{pbh} \quad \forall p \in P, b \in B_h, h \in H \tag{6}$$

$$\sum_{b \in B_h} T_{pbkh} \leq X_{pkh} \quad \forall p \in P, k \in B_h, h \in H \tag{7}$$

The objective function aims to minimize the distance to the closest hospital (1) and minimize the cost of Non-appropriate bed assignments(2), and Transfers (3). Constraints (4) specify that each patient must be assigned to exactly one hospital. Constraints (5) ensure that each patient is assigned to exactly one bed. Constraints (6) guarantee that the total number of assigned patients does not exceed the capacity of each hospital in terms of available beds. Constraints (7) and (8) define the transfer constraints if a patient is transferred from one bed to another.

4 Solution methodology

In emergency scenarios where rapid decision-making is critical, we utilize the popular weighted-sum (WS) method to address the HBAP due to its speed and clarity, making it particularly well-suited for real-time applications. By assigning a weighting coefficient w_i to each objective function, the WS technique converts the initial bi-objective issue into a single objective problem. Then, by altering the weights of the goals, it resolves a sequence of single-objective HBAPs [16] [17]. Due to its ease of implementation and comprehension, the WS approach is widely utilized [16]. For all $i = 1, ..., k$, we assume that the weighting coefficients w_i are non-negative and normalized. The normalization of weights ensures that $\sum_{i=1}^{k} w_i = 1$.

The HBAP is then transformed into the following problem:

$$\begin{aligned} &minimize \quad Z = w_1 Z_1(x) + w_2 Z_2(x) \\ &subject\ to \qquad\quad (4) - (8) \end{aligned} \tag{8}$$

We obtain a set of Pareto solutions by solving a sequence of single-objective HBAP (WS) problems using various combinations of w_1 and w_2 values. We note that the values of w_1 and w_2 are modified at each iteration to determine the best solution.

5 Computational results

In this section, we present six benchmark instances from the literature [15] to solve the Bi-objective HBAP, as given in Table 2a. These instances contain more than 200 patients. These instances can be categorized into two classes. The first class consists of small instances (1 to 3), which have 84 beds. The second class comprises instances 4 to 6, which are larger sets composed of more complex test problems with a number beds ranging from 84 to 141 and a greater number of patients.

This benchmark investigation was conducted with data from a Ciudad del Este-based privately operated healthcare cooperative in the Paraguay Oriental Region. This data instance includes a total of six hospitals. Table 2b lists the number of beds available in each of the six hospitals during an outbreak.

Based on an estimated population per sector, Ciudad del Este's entire area can be divided into nine parts. Table 2c presents the number of demands per area for each instance, while Table 2d displays the distances between the polygon centroids of the six hospitals and the nine population regions.

Experiments in this paper were performed using Cplex 12.10 running on 2.40GHz Intel(R) Core(TM) i5 with Windows 10. Table 3 presents the Pareto-optimal solutions identified for both small and large instances. Using the CPLEX Studio optimizer, the model provides an optimal solution for each objective function, Z_1, and Z_2, in each instance, with an execution time of around 5 minutes for the largest instances. For the bi-objective model Z, decision-makers can choose a

Table 2: Benchmark dataset

(a) data set instances

	Instance	Patients	Hospitals	BEDS
Small instances	1	81	6	84
	2	125	6	84
	4	145	6	84
Large instances	3	156	6	141
	5	251	6	141
	6	251	6	141

(b) Hospital beds capacity

Hospitals	H1	H2	H3	H4	H5	H6
Beds	24	20	21	5	8	6
Extension	0	0	7	27	0	23

(c) Patients demands per area

Area	ins.1	ins.2	ins.3	ins.4	ins.5	ins.6
A1	12	5	17	23	33	36
A2	19	17	31	28	39	44
A3	14	25	23	24	37	35
A4	3	16	11	14	21	22
A5	11	12	21	29	38	37
A6	10	24	17	15	30	27
A7	5	14	11	8	19	18
A8	0	7	5	7	12	10
A9	7	5	9	8	22	22

(d) Distance between hospital location and patient area per Km

	Hospitals Location (Km)					
Area	H1	H2	H3	H4	H5	H6
A1	7.37	6.89	7.19	6.80	8.08	9.19
A2	4.99	3.99	3.43	4.93	4.99	5.82
A3	3.19	2.30	2.28	2.95	3.50	4.52
A4	2.56	2.17	1.56	3.18	1.63	1.56
A5	7.11	7.59	8.56	6.47	8.11	8.93
A6	1.47	1.60	2.57	0.79	2.39	3.44
A7	2.49	2.78	2.77	3.19	1.60	0.50
A8	2.02	2.96	3.95	1.78	2.83	3.39
A9	3.77	4.40	4.59	4.39	3.18	2.20

solution by assigning different weight combinations based on their specific priorities. We discuss three different weight combinations for the two objectives, w_1, and w_2, to reflect varying preferences. In the first scenario, the decision-maker places greater emphasis on minimizing travel distance, with weights of 0.7 for w_1 and 0.3 for w_2, indicating a higher concern for service quality. In the second scenario, the objectives are weighted equally ($w_1 = 0.5$, $w_2 = 0.5$), reflecting a balanced consideration of both service quality (traveled distance) and assignment cost. In the third scenario, the decision-maker prioritizes cost-efficiency, assigning weights of 0.3 to w_1 and 0.7 to w_2, thereby signaling a stronger focus on minimizing assignment costs.

Figure 1 displays the optimal solutions for the three scenarios. Firstly, solutions 1, 2, and 3 demonstrate a reduced operational cost by the proposed model, with an average reduction of approximately 27.76%. Secondly, solutions 2 and 3, are dominated by solution 1, where $w_1 = 0.7$ and $w_2 = 0.3$, indicating that the travel distance cost is prioritized over the assignment cost. However, in instances 5 and 6, the costs are identical, as they involve the same number of patients and available beds. The only difference is in the distribution of demands per area. The identical solution implies that not all patients are admitted to hospitals, and given that the type of beds is the same, costs remain unchanged. This highlights the importance of further investigating HBAP with specific bed types, particularly to address the outcomes observed in instances 5 and 6. These results indicate that our integrated decision model and solution methods are effective only for small instances.

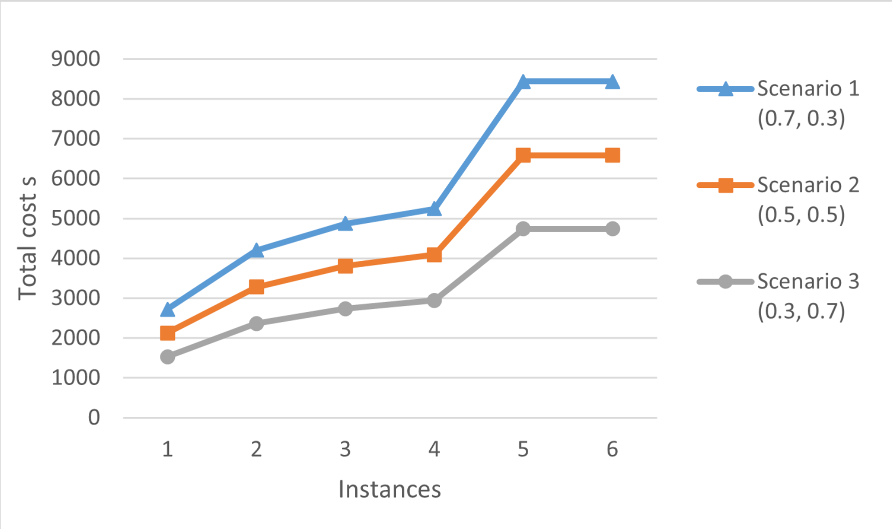

Fig. 1: Solutions obtained by WS

Table 3: Experimental results

	Instance	Patients	Hospitals	Beds	Cost scenario 1 (0.7, 0.3)	Cost scenario 2 (0.5, 0.5)	Cost scenario 3 (0.3, 0.7)	CPU time (min)
Small instances	1	81	6	84	2722,572	2125,98	1529,388	3,63
	2	125	6	84	4201,375	3280,625	2359,875	4,98
	4	145	6	84	4873,595	3805,525	2737,455	3,16
Large instances	3	156	6	141	5243,316	4094,22	2945,124	4,56
	5	251	6	141	8436,361	6587,495	4738,629	7,51
	6	251	6	141	8436,361	6587,495	4738,629	4,82

6 Conclusion

In this paper, we propose a bi-objective HBAP for emergencies that uniquely integrates both HAP and PBAP. The presented model is formulated as a linear programming problem, with the first objective aimed to assign patients to the closest hospital in terms of distance, while the second objective focuses on assigning patients to the available beds within the selected hospital.

To validate this proposal, we conducted 6 benchmark instances from the literature, aiming to solve the bi-objective model as a single objective HBAP using the weighted sum method. Through our experiments, We explored three different scenarios with varying weights to assess the relative importance of each objective function.

The results demonstrated that the model performs effectively with small instances; however, for the largest instances, we observed similar outcomes, indicating a need for further enhancement.

Our findings suggest several future directions for research. Specifically, we propose extending the model to incorporate different types of beds and other relevant resources to ensure better assignments that meet patient demands during emergencies. Additionally, exploring the dynamic version of the bi-objective HBAP could provide insight into real-time decision-making processes in healthcare settings. Furthermore, integrating alternative solution methods, such as epsilon-constraint techniques, or metaheuristic approaches, could enhance the robustness of our proposal.

In summary, our integrated approach not only contributes to the optimization of hospital bed allocation but also opens avenues for future research, enhancing the capacity of healthcare systems to respond effectively during emergencies.

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