Exploring the Impact of DESMOS on Students' Understanding and Learning of Transformations

Dr.Anita Dani [1] and Daliya Ashok [2*]

[1 2] Manipal GlobalNxt University, Malaysia
1 anita.dani@campus.globalnxt.edu.my
*Corresponding author
Daliya Ashok
DA18025@campus.globalnxt.edu.my

Abstract. Understanding abstract or complex mathematical concepts is challenging for students. Disinterest and trouble understanding the subject can be due to teaching strategies that don't suit the student's learning style or an ineffective presentation of the content. Manipulatives, both real and virtual, are teaching aids that help students to explore and comprehend the content. Although visualization of abstract concepts can be effectively supported by manipulatives, maintaining student attention and fostering their knowledge of abstract concepts require an organized pedagogy. The 5E (Engage, Explore, Explain, Elaborate, Evaluate) framework provides a foundation for teachers to develop a structured pedagogy. DESMOS is an online graphing calculator and a virtual manipulative tool. It enables educators to create inquiry-based lessons that facilitate students' deeper understanding of mathematical expressions.

This paper presents research on the integration of technology in secondary mathematics classes. The focus of this study is to investigate how secondary students' understanding of Geometrical transformations is impacted by the web-based graphing tool DESMOS. Lessons were planned using the 5E mode as a part of a design-based research methodology. This study was conducted in a secondary school. Students used the tool and graph paper to solve questions on geometrical transformations. Pre and post-assessment questionnaires helped to examine the effectiveness of the pedagogy. The data shows that most students' ability to analyze transformations and validate their results through visualization improved with DESMOS. Accompanied by proper pedagogy and assessment, integrating DESMOS into the classroom has the potential to impact student learning significantly.

Keywords: Geometrical transformations, DESMOS, learning experience, visualization, secondary mathematics, 5 E model

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1 INTRODUCTION

Mathematics education has undergone significant transformations in recent years. Mathematics serves as a foundation for problem-solving, critical thinking, and quantitative analysis. It is hard to retain knowledge and develop skills without implementing the newly learned concepts. It is consistently found that the students struggle to visualize mathematical ideas, particularly those related to shapes, graphs, and transformations. Manipulatives are helpful in the mathematics classroom as they support experiential learning and facilitate visualization and abstraction. As virtual manipulatives, computers, tablets, and other electronic devices are used by students to have digital representations of the real world, they are preferred over physical manipulatives.

As technology's influence on our lives grows, its incorporation into educational procedures is crucial, especially in the field of mathematics. Incorporating technology to address diversity in the learning needs of students will make the learning experience more inclusive and reflective of the real world [1]. Technology integration equips students with digital and computational literacy, which prepares them for future world opportunities, including entrepreneurship and technology-driven employment. The way mathematics is taught and learned has drastically changed due to the incorporation of technology.

Technology offers a range of tools and resources that can enhance engagement, provide personalized learning experiences, and facilitate a deeper understanding of mathematical concepts. Technology helps students manipulate mathematical concepts visually, especially in geometry. This paper is research on the integration of technology in secondary mathematics classes. This research aims to design interactive learning activities based on the 5E framework and assess their effectiveness. The findings of this research will provide practical recommendations and guidelines for educators, curriculum developers, and policymakers to decide the appropriate methods for successful implementation to maximize the benefits of this integration.

The paper is organized into five sections: a literature review, which provides a review of the chosen manipulative tool, and the pedagogical model 5E. This section is then followed by the research methodology, data analysis, discussion, and conclusion. The paper ends with identifying the limitations and directions for future work.

2 LITERATURE REVIEW

This literature review examines critical research on the use of DESMOS with 5 E strategy using the principles of the Design-Based approach

2.1 Use of Manipulatives in Teaching Mathematics

To enhance student learning, different types of tools are incorporated into mathematics education. Each tool has its advantages and disadvantages. Compass and straightedge, strings, paper folding,3D figures, reflective devices, and geometric software are some tools used to enhance student engagement and deeper understanding [2]. After the Covid-19 pandemic, most students have shown reduced interest in pen and paper-based worksheets [3]. To enhance student's interest in mathematics, dynamic geometry software is beneficial and should be incorporated into lessons due to its positive impact on student learning by providing real-time feedback and developing interactive lessons.[4]. These tools help students to manipulate and represent visual representations of mathematical concepts [5]. To accommodate individual learning styles and preferences, most technological tools can be customized and modified to accommodate individual learning styles and preferences. This helps to include the interests of different groups of students by providing multiple ways to express their understanding. [1]

A graphing calculator (also graphics calculator or graphic display calculator) is a handheld computer capable of plotting graphs, solving simultaneous equations, and performing other tasks with variables. The use of graphing calculators had a positive impact on students' ability to solve problems.[6] Most popular graphing calculators are programmable, allowing users to create customized programs, typically for scientific, engineering, or education applications. A graphing calculator can be used as a computational tool, transformational tool, data collection and analysis tool, visualizing tool, and checking tool.

Many researchers have provided empirical evidence of the effectiveness of graphing calculators in teaching geometry concepts [7,8,6]. Some schools may permit their use during regular class activities and assessments, while others may restrict their usage or provide guidelines on acceptable models. Exam boards and educational authorities may also set regulations regarding the usage of calculators in standardized tests and board examinations. Incorporation of the graphing calculators solely depends on the school curriculum, policies, and cost factors. This barrier can be overcome using free web-based software for classroom activities, but it is also essential to develop students' cognitive ability to solve questions with or without the tool.

2.2 DESMOS.

Schools are focusing on implementing technology more purposefully in classrooms. Technology is an essential component in teaching mathematics. It helps to understand the different ways in which mathematics is learned. According to [9] using computer tools, especially for geometry, makes students move objects and change to other representations (i.e., transformations).

Desmos is a free online graphing calculator that runs as a browser application and a mobile app. Just like a hand-held graphing calculator, Desmos can graph equations and inequalities and perform a wide range of other computational functions such as lists, plots, graph restriction, geometrical transformations, and simultaneous graphing tasks. The most obvious advantage of Desmos is that the screen is as large as the screen of the device on which it runs and, therefore, much bigger than that of a graphing calculator. Students can manipulate graphs, change parameters, and observe how the changes affect the plotted functions. In addition, DESMOS is much faster at graphing, thus providing immediate feedback to the students who can see the function change in real-time as they modify parameters. It is a tool that can engage students innovatively in mathematics. Drawing graphs manually can be tedious and can discourage some students. Instead interactive tools like DESMOS, give a clear representation of the nature of the graphs [10]. This interactivity is incredibly useful for learning and teaching.

Geometry concepts such as transformations are easier to visualize with some form of representation, which cannot be given in printed form. Desmos Graphing Calculator helps students to improve their conceptual understanding, especially in geometry concepts [11]. DESMOS is a user-friendly tool that helps to develop conceptual understanding through visualization [12]. By using such manipulatives, students can save time and focus more on meaningful discussions and applying mathematics in real-world situations. In addition, for a meaningful implementation of DESMOS, an appropriate combination of technology and pedagogy is required [8].

2.3 The 5E Instructional Model

The 5E instructional model is a commonly used framework in education to guide and implement effective learning experiences [13]. The model consists of five phases, each beginning with the letter "E," representing various stages of the learning process. These phases are as follows: Engage, Explore, Explain, Elaborate, and Evaluate. According to [14] students' learning outcomes in mathematics can be enhanced using a 5E instructional approach.

The 5E model provides a framework for creating engaging and interactive learning experiences, which promotes conceptual understanding and enhances problem-solving skills and mathematical reasoning [15]. It is flexible and adaptable, allowing teachers to tailor it to the needs and interests of their students [14]. The instructions designed based on this framework foster constructivist learning and increase students' creativity.

A pedagogy based on this framework is student-centered and leads to constructivist learning. Learning activities can be designed to foster problem-solving skills among students following a constructivist approach. Constructivism is a theory based on observation which suggests that people understand the world through experiences and reflections. According to this theory, students learn by doing experiments and solving real-world problems, while the teacher acts as a facilitator and guides students to build on their existing knowledge[16]. Figure 1 shows the five phases and other parameters that influence the implementation of each phase.

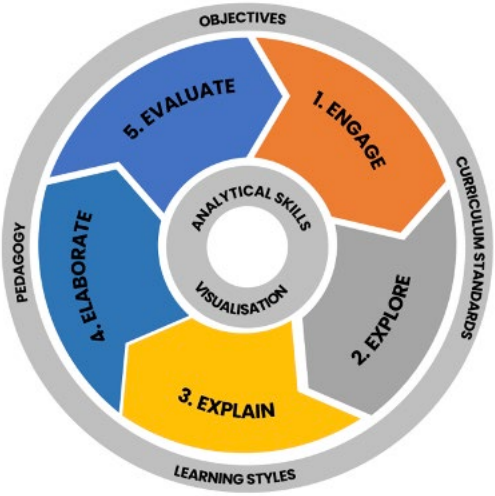

Fig. 1. 5E Instructional model adopted from [14]

The teacher's focus or aim is to improve students' analytical skills,which is represented in the centre and the process of achieving the aim is facilitated by fostering visualization.Teachers can implement learning activities sequentially to have a structured lesson as shown in Figure -1. The starting point of developing a lesson plan is to set the learning objectives for each lesson. These activities should be guided by curriculum standards and supported by an appropriate pedagogy that can cater to the diversity of students' learning styles.

The following table describes each phase and expected activities for teachers and students.

Table 1. Teacher-student activity in the different phases of the 5E model.

5E Phase	Teacher activity	Student Activity
Engage	Capture students' interest and curiosity, sparking their initial engagement with the topic.	Students engage in the activity and recall their previous knowledge and connect to the new concepts.
Explore	Provides a learning environment and supports the exploration process to formulate their findings.	Students actively explore the topic through hands-on activities, experiments, or investigations and share their findings with the class.
Explain	During this phase, teachers provide explanations, introduce new concepts, and clarify misunderstandings based on students' exploration in the previous phase.	With the help of the teacher, students use their findings and experiences to explain the new concepts.
Elaborate	The teacher extends the concepts to new situations.	Students extend their understanding by applying the concepts in new contexts, solving problems, or engaging in further exploration.
Evaluate	Assess the students' understanding and learning outcomes.	Students reflect on their understanding of new concepts and develop next steps for improvement.

Researchers have examined the effectiveness of this framework in teaching various concepts in mathematics, and they have found that students' attitudes toward mathematics, performance in summative exams, and attainment of higher-order thinking skills improved significantly [17,18,19].

2.4 Design-Based Approach

Design-based research is a process that allows researchers to generate pedagogical interventions and assess their effectiveness by scientifically integrating design and research methods. Design-based research methodology is concerned with both the development of learning environments and a systematic study of forms of learning generated in these learning environments [20]. Design-based research methods use both qualitative and quantitative methods and are suited to studies of technological intervention.

These methods have been applied successfully in research studies in the field of science and mathematics education [21]. Various stages of this approach are illustrated by [22] in Figure 2.

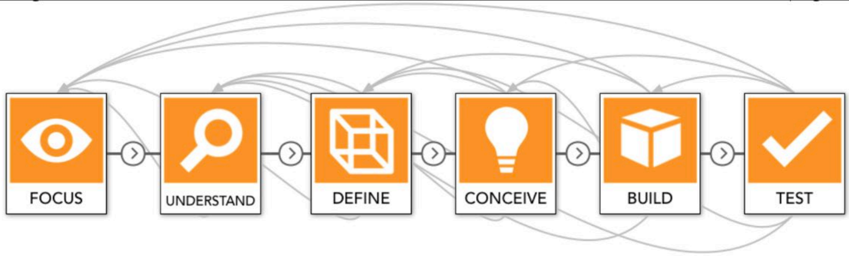

Fig. 2. Design-based research process by [22]

This research approach is applicable in studying the learning environment, which can be designed and systematically changed by the researcher as necessary. It is possible to conduct the research in naturalistic settings, and the findings can lead to the creation of theoretically-inspired innovative learning environments to address a locally isolated problem. Using the results of the findings at each iteration, a new theory can be generated [23].

METHODOLOGY.

In this section, the aim of the research and a description of the research methodology is presented.

Ethical Consideration.

Ethical approval was obtained from the school according to the school policy before conducting the study.

Research Aim.

This research aims to design interactive learning activities based on the 5E framework and assess their effectiveness. The effectiveness of this learning design is measured at each iteration based on the design-based research approach

Research Approach.

This research adopts mixed-method research. Qualitative data is collected through students' reflections, and quantitative data is collected using closed-ended surveys before and after each class. Teaching experiments and data collection procedures were based on the principles of design-based research. Actions taken at each stage are described below.

- Problem identification: Visualization skills play a significant role in understanding geometric transformations, such as translations, reflections, rotations, and dilations. This includes difficulties in mentally manipulating shapes, understanding the effects of transformations on geometric figures, or visualizing transformations in different orientations or dimensions. To check the prerequisite knowledge of the students, a pretest was conducted, which showed the gap that the majority of students' lacked visualization of the geometric transformations.
- Conceptualization: DESMOS was incorporated to enhance the visualization of geometric transformations. Students were allowed to design different shapes and transform them in different orientations, which allowed them to deepen their understanding of the concepts.
- Design development: A detailed lesson plan for each topic was developed, incorporating 5E instructional model for active student participation.
- Pilot testing: To assess the effectiveness of the teaching strategies and identify any areas that need adjustment, a pilot test was done with a small group of students. The teacher gathered the feedback from students and reflected on the pilot test results to modify the teaching strategy. Areas where students did not demonstrate clear understanding were identified from the pilot test study, and the teaching strategy was updated to focus on these areas.
- Data collection: During this phase, the teacher collected data on students' learning outcomes, engagement levels, and progress through various assessment methods to evaluate the effectiveness of the teaching strategies.
- Iteration: After each session, necessary modifications were made, and the implementation was done in the next session. This was done for all four lessons conducted on Geometrical transformations.

Research context.

This research was carried out in a secondary school in the United Arab Emirates, where the use of handheld graphing calculators is not permitted. A purposive sample of this study is a class of 18 students, including boys and girls aged 15 to 16. The primary purpose of the lesson was to teach how to identify a transformation as a function that inputs an object and outputs the same object. As specified in the course learning outcomes, students are expected to understand the connection between the image and pre-image of the shape in transformation and recognize that they are congruent. They should be able to represent transformations as geometric functions. At a higher cognitive level, they should develop the ability to compare two types of transformations; the first type of transformation preserves distance and angle measures, while the other type of transformation does not preserve them. In the first type of transformations the original shape is congruent with its transformed image. In the second type of transformation, students should detect and specify a sequence of transformations that will transform a given figure onto another. The scope of the lesson activities presented in this paper is limited to the first type of transformation.

Experimental Procedure.

Based on the 5E model, a lesson plan was implemented as described below.

To understand the basic knowledge of transformations, a starter was given to students, which helped the teacher to identify the gaps. On the first day of class, students took a pre-test. This pre-test included short answer questions regarding a brief description of the transformations they can remember from previous grade mathematics and the concept of congruency of two shapes. Students recalled translations, rotations, and reflections. The majority of students were able to name three transformations and perform the simplest transformation, such as the translation of the objects. This pre-test activity helped them prepare for the next level of discussion on the same topic. The lesson focused on students exploring these transformations and coming up with a concrete understanding of what each transformation does in the coordinate plane. The following figure shows the screenshot of the pre-session assessment question.

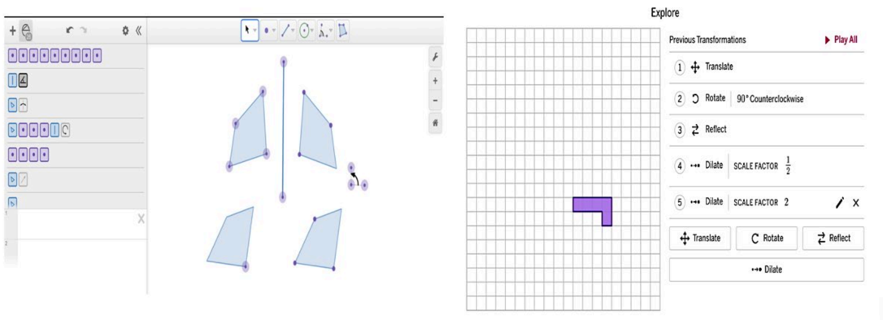

Fig. 3. DESMOS exploration activities

During the engage and explore part of the 5E model, students were put in pairs to consult their friends if they needed any help. The students were asked to translate an object. They were given a worksheet to complete, which assessed their understanding.

During the explain phase of the 5E lesson plan, students participated in a discussion about three different transformations such as reflection, translation, and rotation. They completed similar tasks using traditional methods. The results were analyzed, and it was observed that there needs to be more clarity in analyzing and identifying the coordinates of the transformed image and mapping the shape to the image. The translation was much easier for students than the other transformations. Rotation was the most challenging topic, as the angle, center, and orientation were confusing, and it was difficult to visualize rotation. A sample of students' pen paperwork is shown in Figure 4.

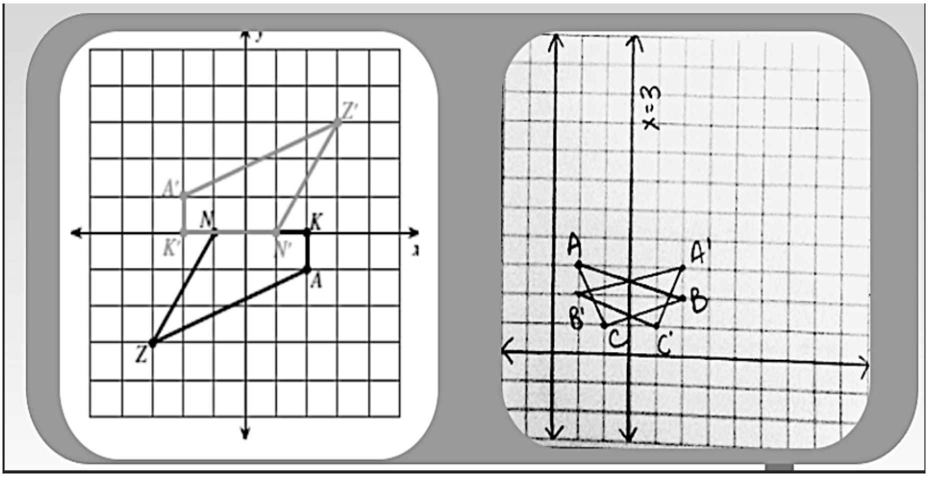

Fig. 4. Transformation work done by students on pen and paper

These points were noted, and in the next stage, the visualization tool DESMOS was used in teaching. Using their laptops, students explored transformation on geometric figures, such as triangles and quadrilaterals, to complete an online task on DESMOS. This task demonstrated how this transformation works and its various properties. Based on their level, students completed the activities in groups or individually. It was found from their interactions with the teacher and peers that they could complete the task with more confidence and in less time. Throughout the activity, students were making conjectures about their explorations. Following the activity, students shared their conjectures and drew connections in a class discussion. During this discussion, students were encouraged to critique their peer's reasoning to ensure everyone gained valuable understanding from the activity. Clearing of misconceptions and further discussions were done during the elaborate part with the teacher's help. Students extended the knowledge of transformations to the real world in an elaborate part of 5E. An assessment was done at the end to check the level of understanding of transformations.

DATA ANALYSIS.

During class activity, students solved formative assessment questions using DESMOS. Samples of those questions are shown in Fig-5.

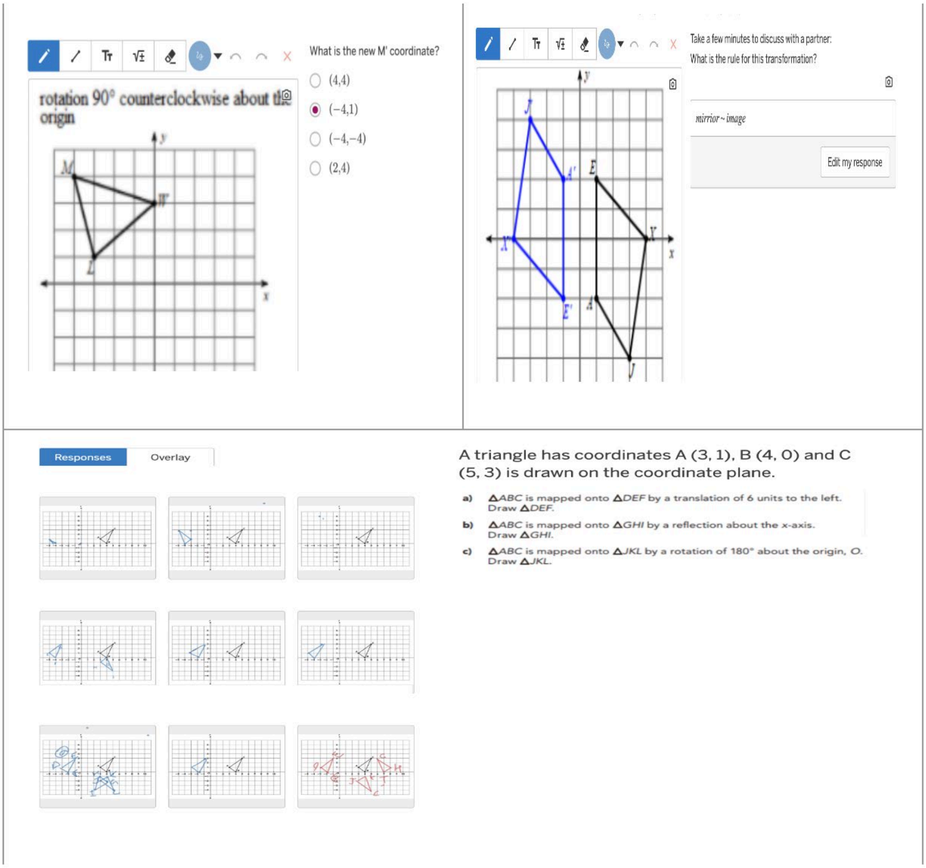

Fig. 5. Student responses for the assessment done to understand the gaps in the topic transformations.

Students' understanding was assessed using formative assessment. DESMOS-generated test reports were used as the source of quantitative data. Each formative assessment included questions of the type of analysis and application. The following table shows the assessment questions, their cognitive level, and completion rate during session 1.

Table 2. Session 1 - Test Questions

Q No	Test question	Cognitive level	Number and (%) of correct responses
1.	Triangle A is rotated 90 clockwise with the origin as the center of rotation to create a new figure. Which triangle shows the new location?	Application	17 (94%)
2.	Which point shows the reflection of the point Y (-5, -7) over the y-axis?	Analysis	17(94%)
3.	Which translation will move the blue shape to the green shape?	Application	18(100%)
4.	Reflect over x-axis. Which point would be at (3, -6)?	Application	18(100%)

The qualitative data was collected from informal follow-up discussions with students. The student who got the answer wrong was encouraged to discuss her thoughts. It was revealed that the mistake in understanding the first question was the orientation of rotation. It is a common mistake to overlook the instruction that specifies the center of rotation. In the second question, instead of reflecting over the y-axis, students reflected the point over the x-axis. This was identified and rectified in the session 2. So similar types of questions were given in the second session after clearing the misconception in which the students completed the task without error. Session 2 was planned in the same way as session 1, but the emphasis was given to clearing common misconceptions through rigorous discussion and more hands-on activities.

The following questions were asked at the end of the session.

Q No	Test question	Cognitive level	Number and (%) of correct responses
1.	Triangle A is rotated 180 anti-clockwise with the origin as the center of rotation to create a new figure. Which triangle shows the new location?	Application	17(100%)
2.	The point M (3, -4) is reflected over the x-axis. Which point shows the reflected image?	Analysis	17(100%)

The average number of correct responses in the first session is 96% and the same in the second session is 100%. The performance of the students in the summative assessments was commendable, as four sessions were conducted using DESMOS with the 5E framework. Students' confidence levels had improved a lot for the topic of transformations due to the incorporation of DESMOS. There were fewer than 15% wrong answers to the higher-order thinking questions from the topic of geometrical transformations. Students' perceptions about the use of tools and its effect on their understanding were assessed through an anonymous opinion poll. The following figure shows the student feedback for the poll. A feedback form was used to check their understanding of concepts and how DESMOS helped improve the understanding of transformation by visualization. Data collected from these feedback forms was analyzed using Excel. A summary of this analysis is presented below.

About 75% of the students completed the task without the teacher's support, and the rest completed it with peer support. As DESMOS is easier to use in certain aspects of math, students who use this calculator experience less stress when solving math problems. Out of 17 students, 95% of the students gave the opinion that DESMOS-based activities were helpful and their transformation skills were enhanced using the tool. The student responses for the assessment are shown in Fig 6.

Fig. 6. Student feedback on the implementation of DESMOS for learning transformations

From the poll results, it was found that although all students agreed that the tool was easy to use and helped them understand the topic of transformations, only 97% of students were willing to use it for reinforcement. 10% of students were still not certain if the tool could help them understand it more deeply. There was one student who missed the DESMOS sessions and he was not as confident as the remaining students while taking the poll.

DISCUSSION.

The data shows that most of the students understood the concepts clearly with the use of DESMOS. Their ability to analyze transformations and check their answers through visualization was better when using DESMOS [7]. This was difficult when working with paper and pencil.

The observation indicates the challenges faced by students in visualizing and understanding concepts of reflection and rotation. These topics require more time and practice for mastery. Pen and paper method limits opportunities for redoing and reflection leading to fewer attempted questions which in turn hinder comprehension skills. After introducing DESMOS, more opportunities were provided to students to analyze their mistakes, experiment with different approaches, and gain a deeper understanding through multiple trials in a shorter time frame. This learning experience improved problem-solving confidence [24] and enhanced students' ability to visualize and explain concepts of rotation which was the most difficult in the pen-paper method which fostered greater confidence and proficiency in the topic transformation.

As per the principles of design-based research, the outcome of this research is not a verification of any hypothesis, but the validation of an instructional design. As stated by [25], when designed instructional tools, such as a lesson plan based on 5E framework, are implemented in the classroom, a learning ecology is created, that improves student learning. This was verified by analyzing data from two sources: students' responses to formative assessments and their feedback. The feedback was gathered qualitatively from classroom discussions and quantitatively from opinion polls. This can lead to the following implications for teachers. The first implication is to incorporate tools that are interactive and allow collaboration. The second implication is to design lesson activities iteratively in which the current topic is connected to the previous topic and provides optimum scaffolding to students to create curiosity about the next topic.

Educators can use DESMOS to promote inquiry-based learning. DESMOS helps educators take a trial-and-error approach to learning, where immediate feedback can be given to the students to analyze their results and make improvements. Additionally, DESMOS promotes a personalized learning approach that accommodates different learning styles where students can learn more deeply about the concepts according to their needs.

CONCLUSION AND FUTURE WORK.

This research confirms that using DESMOS helps students to concretely explore abstract mathematical concepts. Interacting with visual representations enables students to understand these concepts better and encourages greater involvement, which in turn motivates them and improves learning. With DESMOS, students can make and test hypotheses, link ideas, engage in dynamic activities, and interpret the results by changing various parameters.

This research presented empirical evidence of the effectiveness of 5 frameworks for integrating DESMOS. Such evidence is scarce in the context of secondary-level schools where graphing calculators are not permitted.

This research was carried out on a small sample, but the principles of design-based research were followed rigorously. Students' participation and engagement levels were closely monitored. Qualitative data collected from these observations and quantitative data from the opinion poll and the formative assessment responses support our claim about the usefulness of DESMOS. As the lessons were planned using the 5E framework, all lessons were consistent and facilitated active learning, which is very important in understanding abstract mathematical concepts.

While these benefits are significant, it is essential to consider the study's potential limitations as it was conducted in a small cohort. Implementing DESMOS in a larger group would address potential concerns in different contexts.

When evaluating DESMOS, it is valuable to conduct a comparative study with other mathematics tools to understand its pros and cons. As students with learning disabilities need modified instructions, it is important to explore DESMOS implementation and special education to understand its potential impact on these areas.

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