



Maximizing Mutual Information: Optimal Keypoint Selection using Golf Optimizer for Improved Multimodal Image Registration

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Abstract. One of the eminent image processing tools for performing tasks including classification, detection, recognition, and other analysis tasks is the image registration process. This technique is useful for solving a huge variety of real-world problems like surveillance, medical image processing, geophysics, computer vision, remote sensing, etc. The application of optimization techniques has acquired considerable attention in the last decades in the multimodal image registration domain. Thus, this work focuses on suggesting a new optimizer for promoting the performance of multimodal image registration to aid the medical industry. Initially, the multimodal medical images are gathered from the standard datasets to carry out the research. As the medical images are acquired at different locations and environments, it is required to perform the pre-processing stage for enhancing the image registration efficiency. Here, the Histogram Equalization (HE) is applied for preprocessing and then the difference of Gaussian (DoG) is employed for identifying the key points in the pre-processed images. As an innovation to this concept, the percentage of key points is optimized using a recently suggested Golf Optimization Algorithm (GOA). Therefore, it is required to optimize and determine the optimal percentage of key points for promoting the performance of image registration in terms of similarity measures. Thus, this work adopts mutual information as a similarity measure for evaluating the improvements in multimodal image registration.

Keywords: Multimodal Image Registration; Histogram Equalization; Difference of Gaussian; Keypoint Selection; Golf Optimization Algorithm; Mutual Information;

1 Introduction

Over the decades, clinical experts and physicians have employed a single imaging modality for the identification of the abnormal and normal structure of the human body [1]. Though, medical professionals suffer from precise analysis and examination of the information from a single imaging modality as it has only limited information. It can be solved by adopting a multimodal technique for maximizing the quantitative

and qualitative medical information to simpler detection of diseases in earlier stages [2]. Medical imaging modalities like Computed Tomography (CT), Magnetic Resonance Imaging (MRI), Single Photon Emission Computed Tomography (SPECT), Positron Emission Tomography (PET), and X-rays are vastly applied by pathologists for medical analysis [3]. Hence, multimodal medical image analysis helps to provide required functional, anatomical, useful, and unique information while comparing with single modality image analysis. Multimodal medical imaging further helps to identify tumours, brain haemorrhages, cancerous cells, fractures, and lesions [4]. Nevertheless, robust and rapid multimodal image registration remains an unresolved challenge, primarily due to significant appearance discrepancies between different modalities (like CT and MRI) [5].

Generally, image registration can have several types like affine registration, rigid registration, feature-based registration, intensity-based registration, non-rigid registration, hierarchical registration, and local registration [6]. Affine registration involves estimating a 2D or 3D affine transformation (scaling, rotation, translation, and shearing) to align images whereas rigid registration focuses on finding a transformation with rotation and translation alone while non-rigid registration aims to align images with the spatial deformations [7]. In addition, feature-based registration involves finding and matching different features (key points) among images and then estimating the transformation parameters while on the contrary, intensity-based registration compares pixel intensities between images directly for evaluating the transformation parameters [8]. Next, the registration is performed at multiple levels of image resolution in hierarchical registration and the specific region of interest within the images rather than the whole images is aligned in the local registration [9]. This paper focuses on intensity-based registration [10], with the computation of transformation among the two images based on the pixel values together with the adoption of optimization and similarity measure computation.

Meta-heuristic algorithms provide recourse to acquire promising solutions within optimization quandaries. The intricate search space will be explored via these algorithms using trial-and-error mechanisms and stochastic operators. The optimization has the objective of finding the suitable alignment among the images by minimizing or maximizing the similarity function for evaluating the alignment [11]. Thus, the registration quality is based on the utilized search technique for optimizing the similarity measure function. Image registration aims to uncover the transformation that aligns one image, designated as the moving image, with the image space of another image, identified as the fixed image to the greatest extent possible with the aim of achieving precise medical results [12]. The keypoint optimization in image registration gives more advantages to improve the medical image registration process [13]. The suitable balance among utilizing required key points is determined by the optimization of the percentage of key points, which reduces computational overhead and maximization of precise registration [14]. This optimization can be performed via optimization algorithms like genetic algorithms, grid search, Particle Swarm Optimization (PSO), parameter tuning, etc. [15]. Though, the effectiveness of optimizing the percentage of key points depends on several factors, such as the complexity of the registration task,

image characteristics, and the choice of optimization algorithm. Therefore, this search work adopts GOA to increase the performance of image registration.

The highlight of this research is to introduce an innovative approach to optimize the percentage of key points using the GOA, aiming to improve image registration performance in terms of mutual information. The upcoming sections of this work are ordered as follows. Section 2 shows the literature survey. Section 3 describes the architecture and processing steps of the proposed multimodal image registration. Section 4 implements GOA-based optimal keypoint selection for medical image registration. Section 5 devises the experimental analysis on multimodal image registration whereas Section 6 concludes this research work.

2 Literature survey

2.1 Existing works

In 2019, Wang *et al.* [16] Have recommended Shift Particles Observation Unscented Kalman Filter Particle Swarm Optimizer (SPO-UKFPSO) by improving the existing PSO for maximizing the overall optimization accuracy, lowering the possibility of premature convergence and increasing the convergence speed. They have assigned an appropriate weighting to every measure and diverse feature and then, normalized to combine them. The optimization was guided by allowing several numbers of similarity measures or features. They have also shown a capability of aligning images with motion-related artifacts when compared with other registration methods.

In 2020, Xinyuet *et al.* [17] have implemented a Chaotic Brain Storm Optimization Algorithm in Objective Space (CBSO-OS) for maximizing their ability of global exploitation through the gauss map by replacing a random function. The data for the experimentation was acquired from the standard dataset and analysed over measures like translation error and root mean square error and they were also reduced to increase the performance. This model has maximized the accuracy and reproducibility.

In 2020, Daly *et al.* [18] have presented a novel method for a multimodal image registration process with the help of a hybrid optimization technique through a specific development of a genetic algorithm integrated with the Gradient Descent technique IRE database for collecting multimodal brain images. The collected reference and moving images were initially fed to affine registration and then, forwarded to non-rigid registration, where in both stages the hybrid optimizer was applied to perform the precise image registration. The registration accuracy was enhanced and shown in the suggested model.

In 2020, Dida *et al.* [19] have adopted a new Grey Wolf Optimizer (GWO) using CT and MRI images for performing the multimodal image registration. The analysis was performed on two datasets of brain images for performing image registration. Initially, a transformation was found through the alignment of the sensed and the reference images, and then, evaluated by a similarity measure. This model has found the opti-

mal transformation parameters through reducing the similarity measure called Normalized Mutual Information (NMI). Thus, this technique was more accurate and faster than others.

In 2022, Babaheninet *al.* [20] have proposed two optimization techniques like Butterfly Optimization Algorithm (BOA) and the Flower Pollination Algorithm (FPA). The simulation was conducted on the Whole Brain Atlas Database. This work was designed by the following contributions. The impacts of FOA/BOA were estimated and then, performance was analyzed in registered images. The performance of the FOA/BOA was compared with the transformed image to the fixed image through evaluating several similarity measures like mutual information.

2.2 Specifications of the traditional works

The description of existing works with features and challenges is listed in Table I. These methods aim to enhance registration accuracy, reduce errors, and improve convergence speed. However, they also face challenges such as the evaluation of medical images, time consumption, parameter balancing, and convergence speed variability. Overall, continuous exploration and innovation in optimization algorithms hold promise for enhancing image registration processes in medical domains.

Author[citation]	Methodology	Features	Challenges
Wang <i>et al.</i> [16]	SPO-UKFPSO	It maximizes the overall optimization accuracy, lowers the possibility of premature convergence and increases the convergence speed.	It is not evaluated on the medical images.
Xinyuet <i>al.</i> [17]	CBSO-OS	This model reduces registration errors, translation errors, and root mean square errors to enhance the image registration performance.	It suffers from the average time consumption for execution.
Daly <i>et al.</i> [18]	Hybrid optimizer	- It increases the registration accuracy and is robust to noise. - It gives precise image registration results.	One challenge for this research would involve implementing the registration mode for

			atlas-based segmentation of anatomical organs.
Dida <i>et al.</i> [19]	GWO	• It maximizes the convergence rate and thus, reaches the optimal solutions in a fast way. • It gives a more accurate image registration process.	• GWO's convergence speed may vary depending on the complexity of the registration problem.
Babahenini <i>et al.</i> [20]	FPA/BOA	• The FPA technique performs faster. • BOA gives better registration results.	• However, this method suffers from the right balance of several parameters in the hybrid technique, which may pose a challenge.

3Architecture and processing steps of proposed multimodal image registration

3.1 Proposed Model and Description

Fig. 1 depicts the processing diagram of the implemented multimodal image registration using the GOA technique.

The existing methods of multimodal medical image registration have several features that also have their complications while processing the mutual information for getting precise registration accuracy. The mutual information may lead to misregistration while images exhibit small regions of overlap, have limited information, or have lower resolution. Another major complication is their speed, when time is a critical factor, this method may not be feasible. Despite these drawbacks, mutual information and entropy remain among the suitable techniques for image registration [21]. Thus, this model considers the GOA for performing the image registration with the keypoint detection.

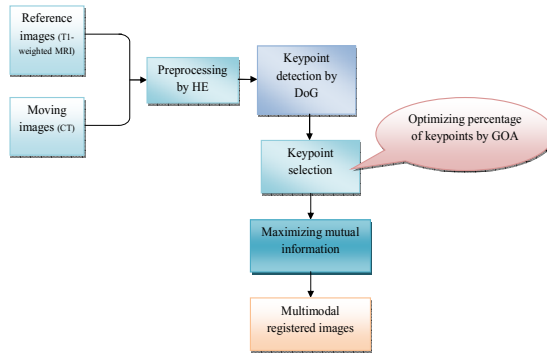


Fig. 1. Processing model of the suggested multimodal image registration using the GOA technique

The proposed registration model is initiated by collecting multimodal medical images from a suitable dataset. Preprocessing is essential in medical image registration regarding the noises and low-quality resolution in images, in which HE is applied for performing the contrast enhancement, which is further helpful for identifying the key points in images. Key points, also known as interest points or feature points, play a crucial role in image registration by identifying distinctive regions in images that can be matched between the reference and target (moving) images. Initially, the DoG is applied to the pre-processed images for detecting the key points in images. The optimal percentage of key points may vary depending on factors such as image resolution, noise level, and the presence of geometric distortions or occlusions. Therefore, it is required to optimize and determine the required percentage of key points for promoting the performance of image registration in terms of similarity measures, which is carried out by the recently suggested GOA technique. Optimizing the percentage of key points involves balancing registration accuracy with computational efficiency. Increasing the number of keypoints typically leads to longer computation times, so it's essential to consider the optimal keypoints among images for acquiring better-registered images, which is focused on this research for giving better registration.

3.2 Data acquired for processing

The dataset for this study is sourced from “<https://heidata.uni-heidelberg.de/dataset.xhtml?persistentId=doi:10.11588/data/ICSFUS>”, containing multimodal abdominal medical images. Specifically, T1-weighted MRI and CT images are chosen for registration purposes, where 1000 images are considered for the registration process.

3.3 Pre-processing

Multimodal medical images acquired from different imaging modalities (e.g., PET, CT, MRI,) often exhibit variations in intensity levels, spatial resolution, and noise characteristics. Pre-processing helps to standardize these image characteristics, making them more compatible for registration. It is carried out with the HE technique. HE [22] is applied for improving the brightness and contrast of the input image. The HE is described as given in Eq. (1).

$$S_j = \frac{j_k}{j} \quad (1)$$

Here, the complete count of pixels in images is denoted as j , and the count of pixels with intensity is shown as j_k , where $k = 0, 1, \dots, N-1$ and $N-1$ specifies the range of gray level values. It computes the histogram of the input image and then, determines a Cumulative Distribution Function (CDF) for providing the information about the intensity distribution of the images. Then, the pixel intensity values are redistributed via an intensity transformation function. The preprocessed output images are obtained by applying the pixel mappings determined by the intensity transformation function. The result is an image with improved contrast and brightness, where details in both dark and bright regions are also enhanced.

3.4 Keypoint detection

DoG[23] is a popular method for keypoint detection in image registration due to its effectiveness in identifying distinctive features. Hence, this paper adopts DoG for keypoint detection by creating a scale-space representation of the pre-processed images by convolving them with Gaussian kernels of varying scales. The difference between adjacent scales in the scale-space representation is computed. Next, key points are detected as local extrema in the DoG images, where a pixel has higher intensity than its neighboring pixels in both scale and spatial dimensions. After keypoint detection, descriptors are computed around each keypoint to capture the local image information. The detected key points are fed to the GOA for optimizing the percentage of key points used for image registration.

4 Golf Optimization-based Optimal Keypoint Selection for Medical Image Registration

4.1 GOA-based keypoint selection

The key points detected in the multimodal medical images are further fed to the GOA [24] technique to optimize the percentage of key points used for image registration. The selection of key points significantly impacts the performance of image registration algorithms, as it determines the features that are used to align images. Optimizing the percentage of key points can lead to better registration accuracy, computational efficiency, and robustness of the registration algorithm, which is further evaluated by the mutual information.

GOA is a game-based meta-heuristic approach that works in two diverse stages like exploration and exploitation, with the innovation from the sport of golf with the observation of player conduct and strategic dynamics. It works by simulating the behavior and rules of players in the golf game. In golf, the swing motion involves a combination of controlled movements aimed at striking the ball accurately towards the target. Similarly, in the GOA, the optimization process involves iteratively adjusting the solution candidates to converge towards the optimal solution. GOA initializes a population of potential solutions within the solution space and then, the fitness function is formulated by evaluating every solution candidate with the quantification of results regarding the optimization objective. Golfers continuously use their swing method by considering the feedback from earlier attempts to improve their performance. Likewise, in the GOA, solutions are refined iteratively through a combination of exploration and exploitation to converge towards the optimal solution.

Exploration: The players have to make their first swing to the destination called a hole, which is represented by the exploration stage, where GOA members search different areas of the solution space. GOA members are updated by computing new positions depending on the formulation of the strongest shot by the player toward the ball. The previous location of the GOA member is updated while a new position is improved by the objective function value. It is mathematically derived as follows.

$$G_a^{B1} : g_{a,m}^{B1} = g_{a,m} + h \times (BM_m - Q \times g_{a,m}) \quad (2)$$

$$G_a = \begin{cases} G_a^{B1}, & F_a^{B1} < F_a \\ G_a, & \text{else} \end{cases} \quad (3)$$

In Eq. (2), a random number in the interval [0-1] is denoted as h , the best member of GOA, and its m^{th} dimensions are correspondingly specified as BM , and BM_m . Here, a random number selected randomly from the set $\{1, 2\}$ is denoted as Q . The

parameter " Q " in the update equation determines whether the shot approaches the hole ($Q=1$) or explores several regions of the solution space ($Q=2$), enhancing the algorithm's global search capability. The updated position of GOA member following the exploration phase is denoted as G_a^{B1} , its m^{th} dimension is devised as $g_{a,m}^{B1}$, and their objective function value is shown as F_a^{B1} .

Exploitation: In golf, the area surrounding the hole is called the green, where players execute precise shots. Hence, these controlled shots are performed with the minimum force for ensuring the presence of the ball within the green area around the hole. It is performed with the exploration of every GOA member's location, demonstrating the ability of ability to exploit local search spaces. The following equations are utilized for updating the exploitation stage.

$$G_a^{B2} : g_{a,m}^{B2} = g_{a,m} + (1 - 2h) \times \frac{l_a + h \times (u_a - l_a)}{t} \quad (4)$$

$$G_a = \begin{cases} G_a^{B2}, & F_a^{B2} < F_a \\ G_a, & else \end{cases} \quad (5)$$

Here, new positions are computed for every member by considering gentle shots toward the ball. The iteration counter is derived as t , the updated position of GOA member following the exploitation phase is denoted as G_a^{B2} where its m^{th} dimension is devised as $g_{a,m}^{B2}$ and their objective function value is shown as F_a^{B2} , and the upper bound and lower bound of the m^{th} variable are respectively formulated as u_a and l_a . Finally, the optimal solution is acquired and retained for getting optimal solutions. Henceforth, GOA optimizes the percentage of key points by getting the optimal solutions, which are used for image registration by identifying distinctive regions in images that can be matched between the reference and moving images. Once the optimal percentage of key points is determined, mutual information can be determined with the aim of promoting the registration quality.

4.2 Maximizing Mutual Information

This recommended medical multimodal image registration model considers the objective of maximizing mutual information. Mutual information evaluates the statistical dependency between corresponding pixels in two images. Maximizing mutual information signifies that the registered images exhibit the highest degree of shared information, implying a strong alignment or similarity between them. Hence, the objective is to fine-tune the registration parameters to maximize mutual information, reflecting an optimal alignment between the images.

Mutual information is formulated with their marginal entropies $\beta(\mathfrak{R})$, and $\beta(\eta)$, and joint entropy $\beta(\mathfrak{R}, \eta)$ for registering a reference image $\mathfrak{R}$ and a moving image η as devised in Eq. (6).

$$MI(\mathfrak{R}, \eta) = \beta(\mathfrak{R}) + \beta(\eta) - \beta(\mathfrak{R}, \eta) \tag{6}$$

This similarity measure ensures the registration performance for aiding the medical industry.

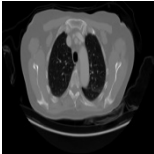
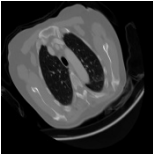
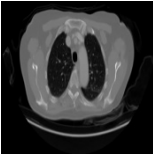
5 Experimental analysis of multimodal image registration

5.1 Simulation Setup

The implementation was carried out using Python. For the experiments, 80% of the input images from three test cases were allocated for training, while 30% of the remaining samples from the same test cases were reserved for testing. The execution parameters were set to 25 iterations and a population size of 10. In the training process, the CT images were considered fixed, while the MRI images were rotated. Subsequently, the rotated MRI images served as the moving images for the training phase.

5.2 Image results

Sample experimental images with reference, moving, and registered images are shown in Fig. 2.

Images	Reference image	Moving image	Registered images using GOA
Image 1			

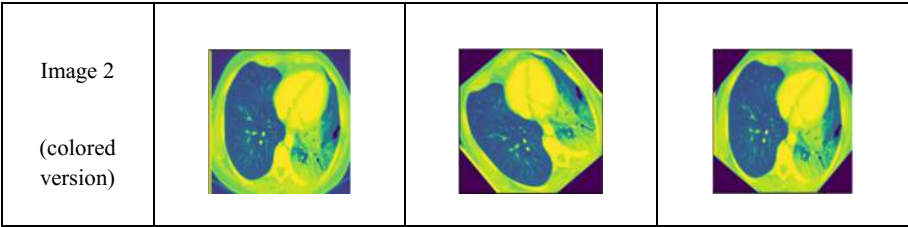
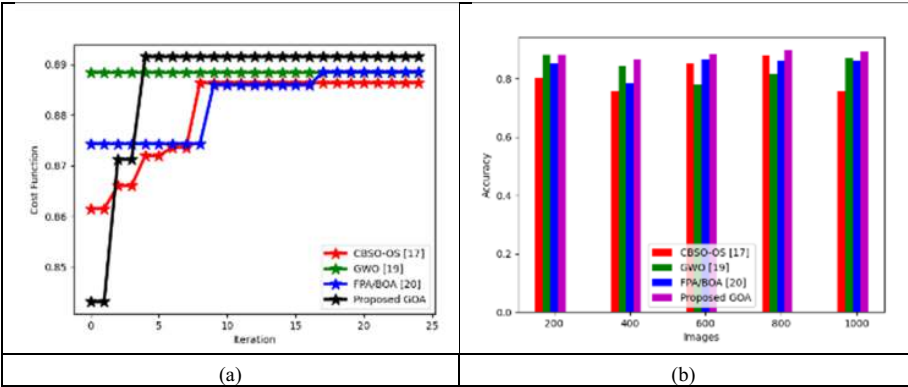


Fig.2. Experimental image results on multimodal image registration

5.3 Experimental analysis

Fig. 3 demonstrates the analysis of the multimodal image registration to show how much the efficacy of the implemented method is increased while estimating with conventional methods. The testing is conducted by varying the count of trained and testing images. For image registration, convergence analysis in the context of maximizing mutual information involves assessing how the registration algorithm progresses over iterations toward an optimal solution (percentage of keypoint optimization). This model further increases the mutual information and registration accuracy of the multimodal image registration.



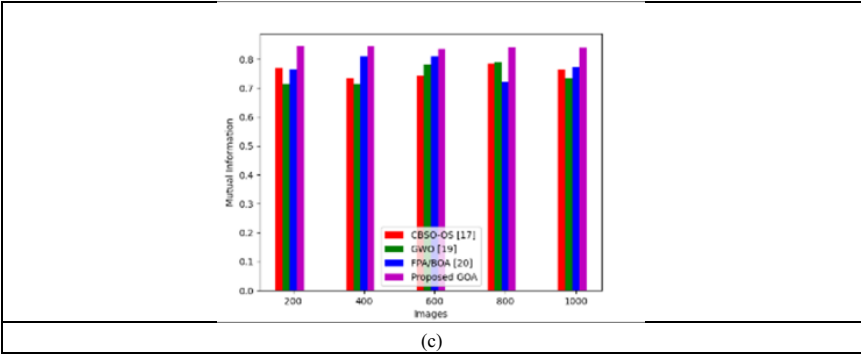


Fig. 3.Analysis of the multimodal image registration regarding (a) Convergence analysis(b) Mutual information, and(c) Accuracy

5.4 Statistical analysis

Statistical evaluation of the proposed image registration technique over traditional image registration techniques regardingmutual information as given in Table II.

TABLE 1. STATISTICAL EVALUATION OF THE PROPOSED IMAGE REGISTRATION TECHNIQUE REGARDING MUTUAL INFORMATION

Measures/Algorithms	CBSO-OS	GWO	FPA/BOA	Proposed GOA
BEST	0.9426	0.9449	0.9433	0.9499
WORST	0.8003	0.8031	0.8016	0.8329
MEAN	0.8652	0.8674	0.8667	0.9266
MEDIAN	0.8628	0.8684	0.8622	0.9337
STANDARD DEVIATION	0.0369	0.0384	0.0369	0.0236

By evaluating these statistical metrics, researchers can understand the overall performance of theregistration algorithm. Thus, from Table 1, the GOA-based percentage of keypoint optimization promotes the performance of image registration.

6 Conclusion

In conclusion, this research aimed to propose a new optimizer to enhance multimodal image registration performance for medical applications. An innovative aspect of this work has involved optimizing the percentage of key points using the GOA to improve image registration efficiency. The success of the recommended model was then evaluated using mutual information by comparing it with traditional techniques. One future challenge for this topic could involve extending the research to address the com-

plexity of real-world medical imaging scenarios. Additionally, enhancing multimodal image registration performance could be a promising direction for future research by exploring the potential of combining multiple optimization algorithms or incorporating machine learning techniques.

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