A Group Decision-making Method Based on Evidence Theory and Deng Entropy

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Abstract. As group decision-making problems become increasingly complex and the decision-making environment continues to change, a large amount of uncertain information emerges in the decision-making process. How to express and deal with the uncertainty to make the decision results more reasonable is particularly important. In this paper, a group decision-making method based on Evidence theory and Deng entropy is proposed. Firstly, the uncertainty of expert opinions is expressed using Evidence theory through the employment of basic probability assignments (BPAs). Secondly, the uncertainty degree of each BPA is measured by Deng entropy. Thirdly, the BPAs are fused by considering their uncertainty degree and converted into probability distributions through the pignistic probability transformation function. Finally, a decision is made. A case study is provided to demonstrate the effectiveness of the proposed method.

Keywords: Group decision-making, Deng entropy, Evidence theory, CREAM

1 Introduction

Group decision-making, as a comprehensive and effective decision-making method, has scientific and democratic characteristics in the decision-making process[1], while ensuring the comprehensiveness, rationality, and accuracy of the results. It is widely used in many fields such as supplier selection[2], risk analysis[3], education[4].

In the process of group decision-making, there are multiple factors that affect the contribution of each expert opinion in the information fusion process. For example, the difference in the level of expertise and experience among the experts (the reliability), the difference in the tendency of the expert opinions (the inconsistency), and the difference in the uncertainty degree in expert opinions (the amount of information). How to determine the contribution of expert opinions in the decision-making process is an important issue, which has been widely concerned by researchers. In view of the first two dimension, i.e., the difference in the level of expertise and experience among the experts (the reliability) and the difference in the tendency of the expert opinions (the inconsistency), a large number of literature studies have been reported[5][6][7]. Focusing on the third dimension, determining the uncertainty degree in

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expert opinions (the amount of information) and its application in information fusion is currently a hot topic[8].

D-S evidence theory is a generalization of probability theory, which can express uncertain information, and has been widely used in uncertain information modeling and information fusion. Deng entropy is a representative method to express uncertainty based on D-S evidence theory, which was first proposed by Deng in 2016 [9]. Due to its advantages in describing and measuring uncertain information, it is widely used in fault diagnosis, medical diagnosis and risk analysis. For example, Yuan et al. effectively addressed the conflict issue in sensor data fusion based on the weighted average combination of evidence distance and Deng entropy [10]. Wang et al. proposed a novel conflict evidence fusion method by integrating Lance distance and Deng entropy[11]. Chen et al. used Deng entropy to measure the uncertainty of expert opinions and obtained the weights based on it. The fusion result was applied to software risk assessment[12]. However, in the above studies, a positive correlation has been suggested between information entropy and its contribution in the fusion process, which is not reasonable in practical applications. The higher the entropy value of a BPA, the greater the uncertainty it contains, leading to an increased amount of information required for its determination. Therefore, the valuable information provided by the BPA is less, resulting in a smaller contribution, and it should be assigned a lower weight. Vice versa.

In this article, we propose a new group decision-making method based on Evidence theory and Deng entropy. Firstly, the tasks and objectives are determined, and domain experts are selected and asked to evaluate. Secondly, expert opinions are generated into BPAs based on evidence theory. Thirdly, the uncertainty degree of the BPA is measured by Deng entropy , which helps to determine the amount of information. Following this, the BPAs are fused and converted into probability distributions using the pignistic probability transformation function. Finally, a decision is made. A case study is presented to show that the proposed method can obtain reasonable results.

2 Preliminaries

2.1 D-S Evidence Theory

D-S evidence theory, first proposed by Dempster and further developed by Shafer[12], can deal with uncertain information flexibly. Its basic contents are described as follows [13]. Let Θ be a finite nonempty set consisting of N mutually exclusive and exhaustive elements, and denote $P(\Theta)$ as the power set composed of 2^N elements of Θ. The basic probability assignment (BPA) function assigns values ranging from 0 to 1 to elements of the power set $P(\Theta)$, denoted by m : $P(\Theta) \rightarrow$ [0, 1], and which satisfies the following equation:

$$m(\varnothing) = 0, \sum_{A \subseteq \Theta} m(A) = 1 \tag{1}$$

where the mass function $m(A)$ represents the strength that supports proposition A.

Suppose that m_1 and m_2 are two BPAs. The result after fusing two BPAs according to Dempster's combination rule is expressed as $m_{1\oplus 2}$. The formula is as follows:

$$\begin{cases} m_{1\oplus 2}(A) = \dfrac{1}{1-k}\sum_{B\cap C=A} m_1(B)m_2(C) \\ k = \sum_{B\cap C=\varnothing} m_1(B)m_2(C) \end{cases} \tag{2}$$

where k represents that the conflict coefficient between m_1 and m_2.

Let m be a BPA on Θ. Its corresponding pignistic probability transformation (PPT) function $\mathrm{BetP_m}$: $\Theta \rightarrow [0, 1]$ is defined as:

$$\begin{cases} BetP_m(\omega) = \sum_{A\subseteq\Theta,\omega\in A} \dfrac{1}{|A|}\dfrac{m(A)}{1-m(\varnothing)} \\ m(\varnothing) \neq 1 \end{cases} \tag{3}$$

where $|A|$ is the cardinality of A.

2.2 Deng Entropy

Deng entropy is a representative method used to measure the uncertainty of BPA [9]. Deng entropy can measure not only the uncertainty contained in the BPAs, but also the uncertainty contained in the probability distribution, which can be expressed as:

$$E_d(m) = -\sum_{A\subseteq\Theta} m(A)\log_2 \frac{m(A)}{2^{|A|}-1} \tag{4}$$

where m is the mass function defined on the frame of discernment Θ. A is the focal element of m, and $|A|$ is the cardinality of A.When the focal element assignment (or BPA) satisfies $m(A_i) = \frac{2^{A_i-1}}{\sum_{A\subseteq\Theta}(2^{|A|}-1)}$, the Deng entropy value reaches its maximum, and the uncertainty contained in the BPA is maximized. The maximum Deng entropy: $D_{\max}$ is calculated as follows:

$$D_{\max} = \log_2 \sum_{A\subseteq\Theta}\left(2^{|A|}-1\right) \tag{5}$$

3 Proposed Method

The flowchart of the proposed method is shown in Fig. 1, in which the determination of the weights of BPAs based on Deng entropy (*Step 5*) is the innovation of this paper.

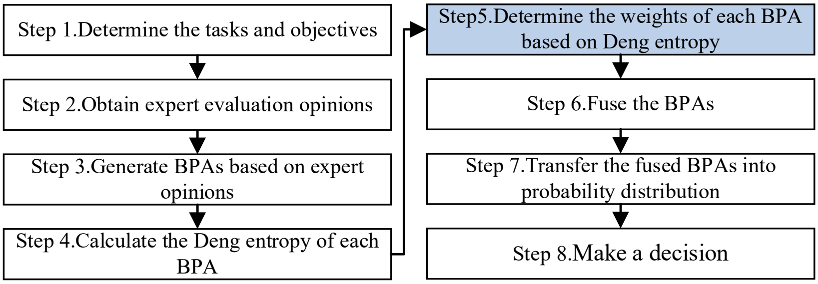

Fig. 1. The flowchart of the proposed method

Step 1: Determine the task and objectives, and select n domain experts to participate in the relevant evaluation according to the task background.

Step 2: Obtain expert evaluation opinions. Each expert evaluates the propositions concerned according to the corresponding criteria, and gives the confidence degree of their own judgment, ranging from [0, 1]. The confidence degree of 0 indicates that the expert has no confidence of his own judgment at all. The confidence degree of 1 indicates that the expert is completely confident of the judgment.

Step 3: Generate BPAs based on expert opinions. Convert expert opinions into BPAs based on evidence theory.

Step 4: Calculate the Deng entropy of each BPA. Calculate the Deng entropy values of each BPA according to Eq.(4).

Step 5: Determine the weights of each BPA based on Deng entropy. Firstly, calculate the weight index of each BPA based on Eq.(6), denoted as ω_i. Secondly, normalize ω_i to obtain the final weights of each BPA by Eq.(7), denoted as ω_{di}. The relationship between the weights and Deng entropy values is shown in Fig. 2.

$$\omega_i = -\frac{1}{D_{\max}} E_{di} + 1 \tag{6}$$

where ω_i is the weight index of the BPA generated by the i th expert opinion. E_{di} is the Deng entropy value of the BPA generated by the i th expert opinion. $D_{\max}$ is the maximum Deng entropy, at which point, the BPA has the highest uncertainty.

$$\omega_{di} = \omega_i / \sum_{i=1}^{n} \omega_i \tag{7}$$

where ω_{di} is the final weights.

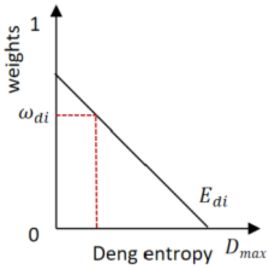

Fig. 2. The relationship between the final weights and Deng entropy values

Step 6: Fuse the BPAs. Firstly, the fused BPAs are obtained by Eq.(8), denoted as $m_P(A)$. Assuming that there are n experts, the fused BPAs are then self-fused for $n-1$ times using the classical Dempster's combination rule (see Eq.(2)) to obtain the final fused BPAs, denoted as $m'_P(A)$ [11].

$$m_P(A) = \sum_{i=1}^{n} \omega_{di} m_i(A) \tag{8}$$

Step 7: Transfer the fused BPAs into probability distribution. The final fused BPAs are converted into the corresponding probability distributions according to Eq.(3).

Step 8: Make a decision. Making decisions based on the obtained probability distributions.

4 Case Study

4.1 Cognitive Reliability and Error Analysis Method (CREAM)

CREAM, which was proposed by Hollnagel in 1998[14], is a representative human reliability analysis method to quantify the cognitive failure probability (CFP) by focusing on the influence of situational context on humans. CREAM divides human cognitive function into four categories (Observation, Interpretation, Planning and Execution). Each type of cognitive function has several failure types. Hollnagel also gives the explanation of general failure types and the nominal value of the failure probability, denoted as CFP_0 . CREAM also defines nine common performance conditions (CPC). The levels of CPCs will have an impact on the CFP. The specific formula is as follows:

$$CFP = CFP_0 \times \prod_{i=1}^{9} CPC_i \tag{9}$$

where CFP is a comprehensive cognitive failure probability that takes into account common performance conditions. CPC_i is the correction factor for the i th CPC.

4.2 Implementation Steps of the Proposed Method

Step 1: Determine the tasks and objectives. Taking CREAM as the application background, assuming that there are five domain experts participate in the evaluation. In this article, taking the failure type of "Wrong identification" under human cognitive function "Observation" as an example, the corresponding CFP_0 is 0.007 (see Table 1). Assuming that the levels of remaining eight CPCs are normal, and the correction factors are 1. This article only uses the CPC "Working conditions" to illustrate the application process of the proposed method.

Table 1. Nominal values and uncertainty bounds for cognitive function failures

Cognitive function	Generic failure type	Lower bound (0.5)	Nominal value	Upper bound (0.95)
Observation	O1. Wrong object observed	3.0E-4	1.0E-3	3.0E-3
	O2. Wrong identification	2.0E-2	7.0E-2	1.7E-2
	O3. Observation not made	2.0E-2	7.0E-2	1.7E-2
Interpretation	I1. Faulty diagnosis	9.0E-2	2.0E-1	6.0E-1
	I2. Decision error	1.0E-3	1.0E-2	1.0E-1
	I3. Delayed interpretation	1.0E-3	1.0E-2	1.0E-1
Planning	P1. Priority error	1.0E-3	1.0E-2	1.0E-1
	P2. Inadequate plan	1.0E-3	1.0E-2	1.0E-1
Execution	E1. Action of wrong type	1.0E-3	3.0E-3	9.0E-3
	E2. Action at wrong time	1.0E-3	3.0E-3	9.0E-3
	E3. Action on wrong object	1.0E-3	5.0E-4	5.0E-3
	E4. Action out of sequence	1.0E-3	3.0E-3	9.0E-3
	E5. Missed action	2.5E-2	3.0E-2	4.0E-2

Step 2: Obtain expert evaluation opinions. According to Table 2, each expert make a judgment for the levels of CPC "working conditions" and determine the corresponding confidence level. The opinions of the five experts are shown in the first four columns of Table 3.

Table 2. The levels of CPC "Working conditions" and correction factors

CPC	CPC levels	correction factor
Working conditions	Advantageous	0.8
	Compatible	1
	Incompatible	2

Table 3. Expert opinions and the corresponding BPAs constructed

CPC	Experts	levels	Confidence level	BPAs
Working conditions	Expert 1	{Advantageous}	0.6	m({Advantageous})=0.6
		{Compatible}	0.2	m({Compatible})=0.2,m({Θ})=0.2
	Expert 2	{Compatible}	0.4	m({Compatible})=0.4, m({Θ})=0.6
	Expert 3	{Advantageous, Compatible}	1	m({Advantageous, Compatible})=1
	Expert 4	{Advantageous, Compatible	0.5	m({Advantageous, Compatible})=0.5, m({Θ})=0.5
	Expert 5	{Advantageous}	1	m({Advantageous})=1

Step 3: Generate BPAs based on expert opinions. The frame of discernment of the CPC "Working conditions" is Θ={Advantageous, Compatible, Incompatible}. The BPAs constructed based on expert opinions are shown in the fifth column of Table 3.

Step 4: Calculate the Deng entropy of each BPA. The Deng entropy values (E_{di}) of the BPAs calculated by Eq.(4) are shown in the second column of Table 4.

Table 4. Deng entropy and weight values

Experts	E_{di}	ω_i	ω_{di}
Expert 1	1.9324	0.5451	0.1951
Expert 2	2.6554	0.3749	0.1342
Expert 3	1.5850	0.6269	0.2243
Expert 4	3.1962	0.2476	0.0886
Expert 5	0	1	0.3578

Step 5: Determine the weights of each BPA based on Deng entropy. The weight index of each BPA (ω_i) calculated by Eq.(6) and the final weights of each BPA (ω_{di}) obtained by Eq.(7) are shown in the third and fourth columns of Table 4.

Step 6: Fuse the BPAs. The fused BPAs ($m_P(A)$) are obtained by Eq.(8), as shown in the third column of Table 5. Then, the final fused BPA ($m'_P(A)$) obtained by Eq.(2) are shown in the forth column of Table 5.

Table 5. The final fused BPAs

CPC	BPAs	$m_P(A)$	$m'_P(A)$
Working conditions	m({Advantageous})	0.4749	0.9376
	m({Compatible})	0.0927	0.0388
	m({Advantageous, Compatible})	0.2686	0.0234

Step 7: Transfer the fused BPAs into probability distribution. The probability values calculated by Eq.(3) are as follows:

$$P\{Advantageous\} = 0.9494, P\{Compatible\} = 0.0506$$

Step 8: Obtain a comprehensive correction factor. The comprehensive correction factor of CPC "Working conditions" is the weighted fusion of the probability value of the CPC level and the correction factor, and the calculation results are as follows:

$$\omega_{workingconditions} = P\{Advantageous\} \times 0.8 + \mathrm{P}\{Compatible\} \times 1 = 0.9494 \times 0.8 + 0.0506 \times 1 = 0.81012$$

Step 9: Make a decision. The CFP calculated according to Eq.(9) is shown as follows:

$$CFP = 0.007 \times (0.81012 \times 1 \times 1 \times 1 \times 1 \times 1 \times 1 \times 1 \times 1) = 5.67 \times 10^{-3}$$

5 Conclusion

The growing complexity of group decision-making problems and the variability of decision-making contexts result in a large amount of uncertainty information in the decision-making process. In this study, a group decision-making method based on Evidence theory and Deng entropy is proposed to deal with the uncertainty, and the CREAM method for predicting the cognitive failure probability is used as an application example to demonstrate the effectiveness of the proposed method. The Deng entropy of the BPA in this paper is negatively correlated with its uncertainty degree during the information fusion process, which is more reasonable. In the future research, the specific expression of the negative correlation between Deng entropy of a BPA and its corresponding weight needs to be further explored.

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