The Calculation Method of Surge Height Under the Resonance Effect Between Earthquakes and Water Bodies

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Abstract. This paper investigates a method for calculating surge height under the resonance effect between earthquakes and water bodies, focusing on the disturbances caused by seismic waves in large water bodies such as lakes, oceans, and reservoirs, and the resulting resonance phenomenon. Through theoretical analysis and wave generation simulation experiments using a shake table, the study examines the influence of factors such as seismic wave frequency and water depth on surge height, revealing the significant amplification effect of resonance on surges. The proposed calculation formula is specifically designed for small lakes and reservoirs under resonance conditions and has been validated with real-world cases. However, it is important to note that the formula's accuracy may decrease in larger water bodies, where complex boundary conditions and nonlinear wave interactions are more prominent. The results demonstrate the method's significant application value in the prevention of secondary disasters caused by earthquakes and in water body safety assessments, particularly for small to medium-sized water bodies.

Keywords: seismic surge, water body resonance, surge height calculation, experimental study, risk assessment

1 Introduction

Large water bodies, such as oceans, lakes, and reservoirs, are vulnerable to disturbances during earthquakes, which can cause waves known as surges. These surges may lead to serious secondary disasters, including flooding, damage to coastal structures, and even the capsizing of boats. A significant example is the massive tsunami triggered by the 2011 Tohoku earthquake in Japan, which caused widespread destruction and loss of life [1,2].

When seismic waves pass through the ground beneath a water body, they can disturb the water, causing it to oscillate [3]. If the frequency of these waves closely matches the natural frequency of the water, resonance occurs, which dramatically increases the size of the surges [4]. This effect is particularly strong in enclosed water bodies like lakes and reservoirs, where wave energy can accumulate [5].

G. Zhao et al. (eds.), *Proceedings of the 2024 6th International Conference on Civil Architecture and Urban Engineering (ICCAUE 2024)*, Advances in Engineering Research 262,
https://doi.org/10.2991/978-94-6463-688-8_17

Previous studies have explored how seismic waves interact with water, but predicting the exact height of the resulting surges remains a challenge [6]. This is due to the variety of factors involved, such as the earthquake's frequency, the water's depth, and the shape of the water body [7-8]. While numerical models have advanced in recent years, further improvements are needed to capture the complexity of these interactions, especially when dealing with uneven terrain and nonlinear wave effects [9].

In this study, we conducted shake table experiments to simulate seismic-induced surges. Based on the experimental data, we developed a new formula to predict surge height in small lakes and reservoirs under resonance conditions. This model provides an important tool for assessing the risks associated with earthquake-triggered surges, helping to improve disaster prevention and safety assessments.

2 Experimental Design

To investigate the spectral characteristics of seismic surges and their resonance phenomena, this study employed a wave generation simulation experiment using a shake table. The key elements of the experimental design include the size of the model water tank, seismic wave parameters, water depth variations, and data collection methods, all of which are detailed to ensure the reproducibility of the experiment.

2.1 Model Water Tank Dimensions

The water body model was placed on a large electro-hydraulic servo-driven shake table located in the Key Laboratory of High-Speed Railway Engineering at the Ministry of Education, Southwest Jiaotong University. The water tank model used for the experiment had the following dimensions: a height of 151 cm, a length of 376 cm, and a width of 176 cm. The water tank was equipped with four high-precision wave gauges with a maximum sampling frequency of 100 Hz and a measurement range of 1 meter, achieving a precision of 0.0001 mm. Data from wave gauge #1, located near the center of the tank, were primarily used to record wave height，as shown in Fig.1. Additionally, a high-definition digital camera was placed at one end of the tank to synchronize visual data with wave height measurements and capture the surge waveforms and motion dynamics.

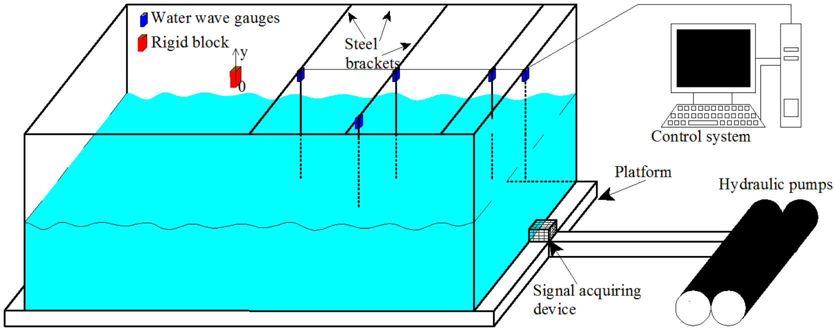

Fig. 1. Schematic diagram of the shaking table experiment

2.2 Experimental Conditions

The experiment was divided into groups based on varying water depths and seismic wave inputs. The specific parameters of the seismic waves were obtained from real-world earthquake data, including the Wolong wave (from the 2008 Wenchuan earthquake) and the Mexicali wave (from the 2010 Mexicali earthquake). Both waves are characterized by distinct frequency components and peak ground accelerations (PGA) that are critical to the resonance conditions tested.

The seismic waves were applied with the following parameters:

Wolong wave: PGA ranging from 0.05g to 0.50g, with dominant frequencies between 0.20 Hz and 0.30 Hz.

Mexicali wave: PGA ranging from 0.05g to 0.50g, with dominant frequencies between 0.15 Hz and 0.25 Hz.

For each seismic wave, five groups of water depths were tested: 10 cm, 20 cm, 30 cm, 40 cm, and 50 cm, with incremental increases of 10 cm between groups. For each water depth, the PGA was varied incrementally from 0.05g to 0.50g in steps of 0.05g, resulting in a total of 60 unique experimental setups to fully assess the influence of both water depth and seismic wave intensity.

2.3 Data Collection Methods

Pulsating water pressure sensors were used to record the changes in water pressure during the experiment, which reflected wave fluctuation patterns. The sampling frequency of the pressure sensors was set to 50 Hz to capture the high-speed wave fluctuations accurately. Additionally, the camera system captured real-time water surface dynamics. The recorded data were used for subsequent spectral analysis, applying both wavelet and Fourier transforms to examine the frequency domain characteristics of the surge waves under seismic excitation.

3 Result Analysis

The natural frequency of the rectangular water body can be calculated by simplifying the problem to that of wave motion in a rectangular tank. Typically, shallow water wave theory from fluid dynamics is used to solve for the natural frequency of the water body. The following are the basic steps of the calculation:

(1) Assumptions: The rectangular water body is considered a two-dimensional water tank, ignoring viscosity and surface tension effects. The water depth is denoted as *h*, and the length and width are represented as *Lx* and *Ly*, respectively.

(2) Wave Equation: According to shallow water wave theory, the small oscillations of the fluid surface can be described using the Laplace equation. For the free surface of a rectangular tank, the wave equation is:

$$\nabla^2\eta = \frac{1}{g}\frac{\partial^2\eta}{\partial t^2} \tag{1}$$

where η(x,y,t) represents the surface displacement, g is the gravitational acceleration.

(3) Boundary Conditions: Assume the water body is fixed at the four edges of the rectangular tank, so the following boundary conditions can be used:

$$\begin{gathered}\eta(x=0,y,t)=\eta(x=L_x,y,t)=0\\ \eta(x,y=0,t)=\eta\left(x,y=L_y,t\right)=0\end{gathered} \tag{2}$$

(4) Characteristic Frequency Solution: By using the method of separation of variables, assume the solution takes the form:

$$\eta(x,y,t) = X(x)Y(y)\cos(\omega t) \tag{3}$$

Substituting this into the wave equation and solving yields the characteristic frequency of the rectangular water body:

$$\omega_{mn} = \pi\sqrt{gh}\sqrt{\left(\frac{m}{L_x}\right)^2 + \left(\frac{n}{L_y}\right)^2} \tag{4}$$

where m and n are positive integers corresponding to different modes in the rectangular water body.

(5) Natural Frequency: The natural frequency f_{mn} is related to the angular frequency ω_{mn} by:

$$f_{mn} = \frac{\omega_{mn}}{2\pi} \tag{5}$$

Thus, the natural frequency of the rectangular water body for the m-th and n-th mode is:

$$f_{mn} = \frac{1}{2L_x}\sqrt{gh}\sqrt{m^2 + \left(\frac{Lx}{Ly}\right)^2 n^2} \tag{6}$$

Through this formula, combined with the length, width, and depth of the water body, the natural frequencies of different modes in a rectangular water body can be calculated. By substituting the corresponding dimensions, the natural frequency of this rectangular water body is calculated to be 0.23 Hz.

Now, a vibration table experiment using Time curve of Mexicali seismic acceleration (As shown in Fig. 2) and seismic surge waves is conducted to verify this theoretical value. The amplitude spectrum is obtained from the recorded sloshing time-history curve using Fast Fourier Transform (FFT), as shown in Fig. 3.

From the figure, it can be seen that the natural frequency is 0.22 Hz. This value is very close to the result calculated from equation (1), indicating that equation (6) can be used to estimate the natural frequency of an approximately rectangular water body.

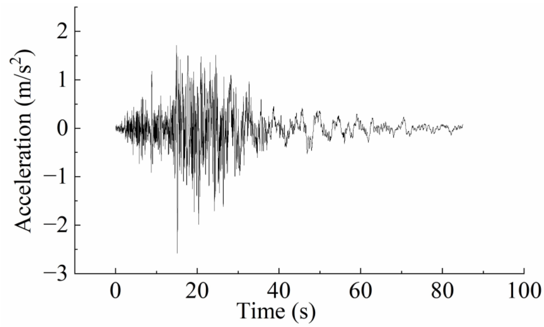

Fig. 2. Time curve of Mexicali seismic acceleration and seismic surge waves

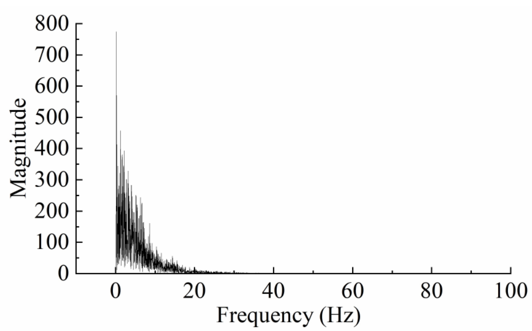

Fig. 3. Fourier amplitude spectrum of 30 cm depth

After obtaining the experimental data, wavelet transform and Fourier transform were applied to the surge time-history curves under different water depth conditions for spectral analysis. As mentioned in the introduction, the main factors influencing the maximum height of seismic surges include water depth, horizontal seismic amplitude, and the dominant frequency of the earthquake. Since resonance conditions require the earthquake's dominant frequency to be close to the natural frequency of the water body, this study treats the water depth (H) and the peak ground acceleration (PGA) as variables.

For the resonance surge under the excitation of the E2 seismic wave, the model tank water depth was set to various values. As the PGA of the seismic wave increased from 0.05g to 0.30g, the maximum surge height (h) was recorded, as shown in Figure 4. The figure indicates that under different water depth conditions, the maximum surge height under resonance approximately increases linearly with the increase in PGA. Additionally, in the vertical axis, it is observed that for a constant PGA, as the water depth increases from 20 cm to 40 cm, the maximum surge height shows a monotonous increase with the increase in H. Therefore, the maximum height of the seismic resonance surge shows a positive correlation with both water depth and seismic wave PGA.

Thus, the function for the maximum height of the seismic resonance surge can be expressed as a function of water depth and PGA. By applying the Buckingham π theorem for dimensional analysis, the dimensionless surge height and dimensionless acceleration are derived. This yields a dimensionless functional relationship for the maximum surge height. Fitting the experimental data from Figure 5 results in the expression for the maximum surge height under resonance conditions, as shown in Equation (7), and the fitted curve is displayed in Fig.4.

$$\eta = 1.2021 A_0^{0.8029}, R^2 = 0.9952 \tag{7}$$

The maximum wave height of seismic resonance surges under different PGA (Peak Ground Acceleration) conditions as it varies with water depth.

In this section, we validate the proposed Equation (6) using the case of a swimming pool surge during the Mexicali earthquake, as mentioned in the introduction. During the earthquake, the peak ground acceleration (PGA) recorded in the north-south direction at the Mexicali City fire station was 0.264g, and the dominant frequency of the seismic wave was 0.17 Hz. Post-earthquake investigations revealed that the pool had a north-south dimension of approximately 15 meters and a water depth of about 1.8 meters. Based on surveillance footage, the maximum surge height was estimated to range between 0.70 and 1.00 meters.

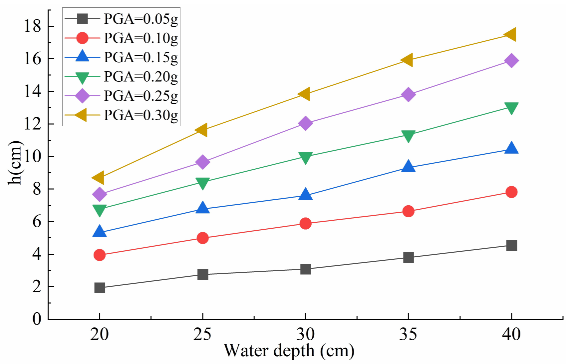

Fig. 4. Variations of maximum wave height under different PGA at different water depths.

Using Equation (7), the natural frequency of the pool's water body was calculated to be 0.18 Hz, which is close to the dominant frequency of the seismic wave (0.17 Hz), indicating that resonance occurred between the water body and the seismic wave. Substituting the relevant parameters into Equation (7), the calculated maximum surge height is 0.81 meters, which aligns well with the video estimate, the dimensionless maximum wave height fitting curve were shown in fig. 5.

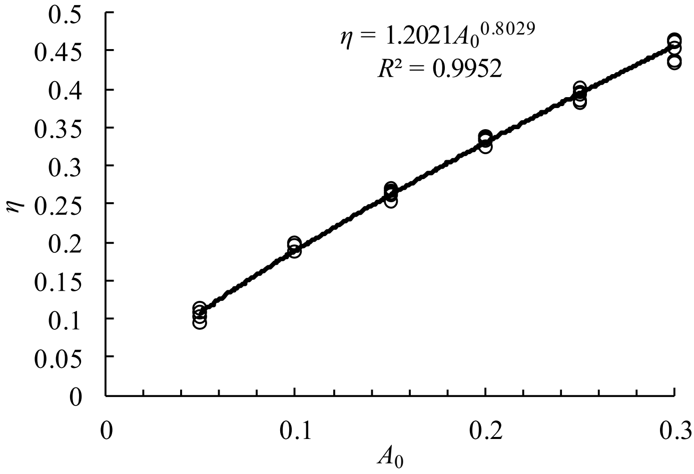

Fig. 5. Dimensionless maximum wave height fitting curve

It is worth noting that using the formula for maximum surge height under non-resonant conditions from reference [10], the estimated maximum surge height of the pool would be only about 0.40 meters, significantly smaller than the actual observed value. Therefore, Equation (2) can be effectively used to estimate the maximum surge height under resonant conditions in small reservoirs and lakes.

4 Conclusion and Discussion

4.1 Conclusion

This study investigates the effect of resonance between seismic waves and water bodies on surge height and proposes a new method for predicting surge height under resonant conditions in small lakes and reservoirs. The key conclusions are as follows:

Significant Resonance Amplification Effect: Experimental results demonstrate that when the dominant frequency of seismic waves approaches the natural frequency of the water body, significant resonance occurs, leading to a substantial increase in surge height. This effect is particularly pronounced in shallow water conditions, confirming the existence of the resonance amplification effect.

Analysis of Influencing Factors: Surge height is positively correlated with water depth and the peak ground acceleration (PGA) of seismic waves. As water depth and PGA increase, the maximum surge height exhibits an approximately linear growth trend.

Proposed Calculation Formula and Validation: Based on experimental data, a new formula for calculating surge height under resonance conditions is proposed. The formula was validated using the real-world case of the Mexicali earthquake swimming pool surge. The validation results show that the proposed formula accurately predicts surge height under resonance conditions, closely matching observed data.

Practical Application Value: The findings provide a scientific basis for mitigating secondary disasters caused by earthquakes, with particular relevance to the seismic risk assessment and dam failure warning systems for small lakes and barrier lakes.

4.2 Discussion

The surge height calculation formula proposed in this study demonstrates high accuracy under resonance conditions for small lakes and reservoirs. However, a key aspect of its significance lies in its comparison with existing models, which often lack the precision or adaptability necessary for specific seismic-induced wave conditions.

Comparison with Existing Linear Models.

Traditional methods for calculating surge height in water bodies typically rely on linear wave theory, which assumes small amplitude waves and neglects the resonance amplification effect. These models, such as the basic shallow water wave theory or simplified numerical methods (e.g., finite element or finite difference models), offer general surge predictions based primarily on the water body's physical dimensions and basic seismic wave parameters. However, they fail to adequately address the interaction between the seismic wave frequency and the natural oscillation frequency of the water body. As a result, they often underestimate surge height, particularly under resonance conditions.

For example, models derived from Du et al. [10] predict maximum surge heights based on linear assumptions without considering resonance. In cases like the Mexicali earthquake swimming pool surge, such models estimated a maximum surge height of only 0.40 meters, whereas the actual observed surge reached approximately 1.00 meter. The proposed model, which incorporates the resonance effect, produced a much closer estimate of 0.81 meters, demonstrating its superior accuracy under such conditions. This key advantage makes the proposed method far more reliable for predicting surge heights in small water bodies where resonance amplification plays a significant role.

Incorporation of Resonance Effects.

What sets the proposed method apart from these existing approaches is its ability to incorporate resonance effects between seismic waves and the natural oscillation frequencies of water bodies. By specifically focusing on the amplification of surge height under resonant conditions, the model provides more accurate predictions, particularly in smaller, enclosed water bodies like lakes and reservoirs, where resonance is more likely to occur.

Existing models often treat the seismic wave as an external force acting on the water body without accounting for the matching of wave frequencies that lead to resonance.

The innovative aspect of the current study is the integration of experimental shake table data to derive a predictive formula that directly considers both the dominant seismic wave frequency and the natural frequency of the water body. This approach offers a more detailed and realistic depiction of how seismic energy transfers into the water, amplifying surge heights.

Application of Experimental Data for Model Validation.

Another innovation of the proposed method is its reliance on extensive experimental data gathered through shake table experiments. Unlike traditional models that are purely theoretical or based on simplified assumptions, the method presented here is grounded in physical tests that simulate real-world seismic conditions. The shake table experiments allowed for precise control over variables like water depth, seismic wave frequency, and peak ground acceleration (PGA), providing a rich dataset for validating the model. This data-driven approach enhances the reliability of the formula, especially when applied to scenarios involving small lakes or reservoirs.

In contrast, existing models often lack this level of experimental validation, relying instead on theoretical assumptions that may not fully capture the complexity of real-world seismic interactions. By integrating both experimental and theoretical insights, the proposed method represents a significant advancement in the field.

Adaptability and Practical Application.

While existing models may provide rough estimates of surge height, they often fail to address the nuanced effects of variable water depths and changing seismic wave characteristics in real-time seismic events. The proposed method offers flexibility in its application by allowing users to input specific parameters, such as water depth and PGA, for different seismic waves, thereby customizing the predictions to match local conditions.

Moreover, the method's emphasis on resonance conditions adds practical value for dam safety assessments and early warning systems, particularly in regions where small to medium-sized reservoirs or barrier lakes are at risk of seismic activity. This adaptability and focus on resonance make the proposed model a powerful tool for real-world applications.

Future Directions for Comparison.

Although the proposed method demonstrates clear advantages in predicting surge height under resonance conditions, further research is necessary to extend its applicability to larger water bodies and complex topographies. Nonlinear wave effects, such as energy dissipation and wave breaking, play a significant role in larger water bodies, where boundary conditions are more irregular. Integrating these nonlinear dynamics into future models will enhance their accuracy and generality, allowing for more comprehensive comparisons with existing methods in a broader range of water body types.

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