A Design Method for Tensegrity Structures Based on Adaptive Force Density Method and SQP

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Abstract. Tensegrity structures, characterized by self-stabilization through internal prestress, are widely employed in disciplines such as architecture, aerospace, and robotics. The form-finding problem of tensegrity structures has received considerable attention in research. This paper proposes a novel design approach for tensegrity structures utilizing the Adaptive Force Density Method (AFDM) in conjunction with the Sequential Quadratic Programming (SQP) algorithm. The objective is to optimize nodal coordinates to achieve force equilibrium while closely approximating the desired geometric configuration. Firstly, the basic concepts of tensegrity structures and current design methods are introduced. Next, it is a detailed explanation of the AFDM and SQP-based optimization algorithm, including the definition of the objective function, calculation of gradients and Hessian matrices, line search, and updates of Lagrange multipliers. Two numerical examples are presented to verify the feasibility and effectiveness of the method: one involves designing a 6-strut-24-cable tensegrity structure, and the other is based on a diamond structure derived from regular octahedral units. Results from these simulations demonstrate the method's efficacy in shape control, highlighting its robustness and convergence properties.

Keywords: Tensegrity design, Adaptive Force Density Method, SQP.

1 Introduction

Tensegrity, as being proposed by Richard Buckminster Fuller [1] in 1959, has gained attention for decades. Tensegrity structures are composed of isolated components (referred to as struts) in compression and a network of continuous tension elements (referred to as cables), this formation offers unique advantages including lightweight and high strength. With their ability to achieve self-stabilization through internal prestress, tensegrity structures have garnered significant interest in fields such as architecture, aerospace, and robotics. However, the design and optimization of

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tensegrity structures pose challenges due to maintaining force equilibrium while achieving the desired geometric configuration.

The form-finding process, crucial in the design of tensegrity structures, elucidates the self-equilibrium state. This step involves shaping the structure, defining its topology, and calculating internal forces. Several techniques have been devised to tackle this challenge, encompassing the dynamic relaxation method [2], force density method [3], finite-element method [4], and stiffness matrix-based approach [5]. The advent of the force density method marked a significant progress in form-finding methodologies. Tran and Lee [6] leveraged double singular value decomposition to improve the form-finding process, offering a sturdy approach for ascertaining equilibrium shapes of tensegrity structures. Cai et al. [7] presented a form-finding technique for multi-mode tensegrity structures by extending the force density method through element grouping, enabling broader design possibilities. Wang et al. [8] investigated the global stability criteria in the topology determination of tensegrity structures, underscoring the necessity of maintaining the stiffness matrix's positive definiteness for stable configurations. However, these techniques encounter difficulties in accurately controlling the eventual geometry of tensegrity structures. Furthermore, many methods typically necessitate supplementary input of material properties for components during computations [9].

Zhang and Ohsaki [10] introduced the Adaptive Force Density Method (AFDM), showcasing its effectiveness in achieving stable configurations. AFDM distinguishes itself by guaranteeing a non-degenerate configuration and adhering to conditions for super stability, thus alleviating worries regarding improper material selections and overly high prestress levels typical in traditional force density methods. Nonetheless, AFDM necessitates a specific set of nodal coordinates to obtain a unique and non-degenerate structure configuration, which may not consistently match the desired geometric layout.

In this context, this paper introduces a novel approach that integrates AFDM with SQP for the design and optimization of tensegrity structures. The proposed method aims to optimize the nodal coordinates by solving a nonlinear optimization problem, ensuring the structure's force equilibrium and closely approximating the target geometric configuration. This approach leverages the AFDM to calculate the force density matrix and utilizes SQP to iteratively update the nodal positions, achieving a stable and efficient design.

The paper is organized as follows: Section 2 introduces the design method, including the introduction of AFDM and SQP as the fundamental algorithm of the method. Section 3 presents two numerical examples to illustrate the method's effectiveness: the design of a 6-strut-24-cable tensegrity structure and the design of a diamond structure derived from regular octahedral tensegrity units. Finally, Section 4 concludes the study and provides discussions on future research directions.

2 Adaptive Force Density Method

The force density of a member is its internal force divided by its length [11]. A column vector $\mathbf{q} = [q_1, q_2, \cdots, q_m]^T \in \mathbf{R}^{m+1}$ is used to represent m members' force density, positive for tension and negative for compression. In this paper, AFDM is employed to determine a feasible set of force densities.

Explaining the practical physical meaning of force density is challenging, as it represents the ratio between the force and length of a member. However, by multiplying force density and coordinate difference of the member in a direction, we can get the component of the internal force in the direction. The self-equilibrium analysis is based on the process. It's defined in Eq. (1) that a force density matrix $\mathbf{E} \in \mathbf{R}^{m+1}$ is composed of the m force densities, of which n is the number of nodes.

$$\mathbf{E}_{i,j} = \begin{cases} \sum q_\alpha & i = j, \text{member } \alpha \text{ is linked to node } i \\ 0 & i \neq j, \text{no member is linked to node } i \text{ or } j \\ -q_\beta & i \neq j, \text{member } \beta \text{ links node } i \text{ and } j \end{cases} \tag{1}$$

E is a symmetric matrix, in which the ith row represent the self-equilibrium equation of the ith node, if it is multiplied by a column vector of nodal coordinates in a direction, the result will be the sum of the force (i.e. resultant force) in the direction of the node. Then the equilibrium equations for all *d.o.f.s* can be listed as below, for the structure without external forces:

$$\begin{cases} \mathbf{Ex} = \mathbf{0} \\ \mathbf{Ey} = \mathbf{0} \\ \mathbf{Ez} = \mathbf{0} \end{cases} \tag{2}$$

where column vectors **x**, **y**, and **z** are coordinates of all nodes in 3 directions, respectively. To ensure the structure stays 3-dimension, **x**, **y**, and **z** should be linearly independent. While $\mathbf{E}\{\mathbf{1}\} = \mathbf{0}$, where $\{\mathbf{1}\}$ is the column vector with all the elements in it are 1, **E** is supposed to have more than 4 of all eigenvalues equal to 0, according to linear algebra. In addition, **E** should be positive semi-definite to meet one of the sufficient conditions of the tensegrity structure to be super-stable [12].

The AFDM process comprises two distinct stages. Initially, eigenvalue analysis is executed on the force density matrix, with the four smallest eigenvalues constrained to zero. This triggers iterative computations to determine a viable force density distribution. Subsequently, an independent set of nodal coordinates is designated to solve the equilibrium equation as depicted in Eq. (2). For comprehensive insights into the formulation and procedural intricacies of AFDM, readers are directed to the study conducted by Zhang and Ohsaki [13].

The feature of AFDM is its ability to guarantee a non-degenerate configuration and the necessary conditions for super stability, thus minimizing the risks associated with inappropriate material selection and excessive prestress levels in traditional force density methods.

3 Design Method Based on AFDM and SQP

Although AFDM is effective in obtaining a stable configuration, it may not always closely match the desired geometric configuration. To address this limitation, this paper employs SQP to solve for the final configuration. This algorithm is particularly effective for problems where the objective function and constraints are nonlinear.

To solve this problem, the optimization model is proposed: to find the configuration closest to the target structure configuration $\mathbf{X}_{obj}$ while satisfying the equilibrium equation related to the nodal coordinates. Therefore, the design variable $\mathbf{X}$ is the nodal coordinate vector, and the objective is the distance between $\mathbf{X}$ and $\mathbf{X}_{obj}$.

$$\begin{aligned} &\min_{\mathbf{X}} \left\| \mathbf{X} - \mathbf{X}_{obj} \right\| \\ &s.t.\, \mathbf{K}_G \mathbf{X} = \mathbf{0} \end{aligned} \tag{3}$$

where

$$\mathbf{K}_G = \begin{pmatrix} \mathbf{E} & \mathbf{0} & \mathbf{0} \\ \mathbf{0} & \mathbf{E} & \mathbf{0} \\ \mathbf{0} & \mathbf{0} & \mathbf{E} \end{pmatrix} \tag{4}$$

where $\mathbf{E}$ is the force density matrix obtained from AFDM.

The optimization problem aims to find the configuration closest to the target structure while satisfying the equilibrium equation related to the nodal coordinates. SQP is ideal for this due to its effectiveness in handling nonlinear objective functions and constraints. This flexibility ensures the final configuration meets all necessary physical and mechanical conditions. By employing SQP, we can achieve a configuration that closely matches the target geometry while satisfying equilibrium conditions, leading to a stable and feasible design.

Designer who is based on their experience and design requirements, provide the node coordinates $\mathbf{X}_{obj}$ and nodal connection relationships $\mathbf{C}$ (typically referenced to spherical tensegrity structures [14]) of the structure at the beginning of the design process.

However, unlike truss structures, arbitrarily designed geometric configurations in tensegrity structures often result in non-equilibrium or instability. This paper proposes a design method that takes the configuration meeting the design intent as the target, while satisfying self-equilibrium and the necessary conditions for super-stability of a tensegrity structure. By integrating SQP with AFDM, the proposed method iteratively updates the nodal positions to closely approximate the target geometric configuration, ensuring both force equilibrium and geometric accuracy.

4 Numerical Examples

This section introduces two numerical examples that demonstrate the design method described in Section 2. Optimization stopped because the relative changes in all elements of **X** are less than options and the relative maximum constraint violation is less than options. The resulting structures are self-balanced and meet the necessary conditions for super-stability. Example 1 involves deriving a tensegrity structure configuration that closely approximates an initial spatial geometric shape. Example 2 involves modifying the configuration of a known tensegrity structure to obtain the final tensegrity structure.

4.1 Example 1: Design of a 6-strut-24-cable Tensegrity Structure

The frame structure shown in Fig. 1 can not be a tensegrity structure becuase this pure geometrical model can't hold a set of self-balanced internal forces. We aim to derive a stable tensegrity structure based on this geometric shape. Therefore, we treat the geometric shape as the target configuration, with its coordinates designated as $\mathbf{X}_{\mathrm{obj}}$, and use the design method to realize the form-finding design.

The result configuration shown in Fig. 2 satisfies the self-equilibrium equations and meets the conditions for super stability [13]. The structure maintains force equilibrium and achieves the intended geometric layout, confirming the effectiveness of the design method. The coordinates of each node of the resulting structure are listed in Table 2 and Table 3, along with the axial forces of each element.

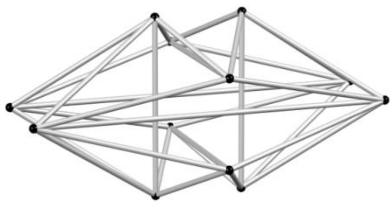


Fig. 1. Spatial geometric shape formed by nodes listed in Table 1 and known connectivity.

Table 1. Node coordinates of the target geometric shape.

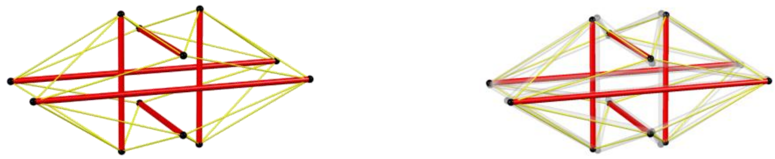
node	1	2	3	4	5	6	7	8	9	10	11	12
x_{obj}	0	0	0	0	10	10	-10	-10	5	-5	5	-5
y_{obj}	-5	5	-5	5	0	0	0	0	-20	-20	20	20
z_{obj}	-10	-10	10	10	-5	5	-5	5	0	0	0	0

Table 2. Node coordinates of the calculated structure.

node	1	2	3	4	5	6	7	8	9	10	11	12
x	0	0	0	0	9.30	9.30	-9.30	-9.30	6.07	-6.07	6.07	-6.07
y	-5.72	5.72	-5.72	5.72	0	0	0	0	-19.79	-19.79	19.79	19.79
z	-9.71	-9.71	9.71	9.710	-5.51	5.511	-5.51	5.51	0	0	0	0

Table 3. Axial forces of elements in the calculated structure.

element	1	2	3	4	5	6	7	8	9	10
s	-8.13	-8.13	-11.74	-11.74	-20.75	-20.75	5.84	5.84	3.68	3.68
element	11	12	13	14	15	16	17	18	19	20
s	5.84	5.84	3.68	3.68	5.84	5.84	3.68	3.68	5.84	5.84
element	21	22	23	24	25	26	27	28	29	30
s	3.68	3.68	7.90	7.90	7.90	7.90	7.90	7.904	7.90	7.90



(a) (b)

Fig. 2. (a) Result configuration. (b) Result configuration comparing with Target configuration (shadow).

4.2 Example2: Design of a Diamond Tensegrity Structure Based on Regular Octahedral Tensegrity Structure Units

The 2nd design is based on a regular truncated octahedral tensegrity structure. The geometry of the diamond framework shown in Fig. 3 is the design tareget, after the design method, a tensegrity structure that approximates the diamond framework is obtained.The resulting structure, derived from the diamond framework, satisfies the self-equilibrium conditions and adheres to the super stability criteria described in [13]. This confirms the design method's ability to obtain the desired geometric form while ensuring stability.

The regular truncated octahedral tensegrity structure [15] is a common basic unit of tensegrity structures, known in Fig. 4. To derive the diamond-shaped tensegrity structure, the nodal position of the regular truncated octahedral tensegrity structure are modified to achieve the geometric configuration of the diamond framework. But this leads to an unreasonable distribution of internal forces, rendering the design ineffective. As shown in Fig. 5, the design method proposed in this paper identifies a process that obtains the result configuration that meets the equilibrium conditions and closely approximates the target configuration, thus avoiding issues of unequilibrium or instability. By starting with regular or truncated polyhedrons and then modifying the node positions, people can design diverse tensegrity structures that go beyond spherical forms. the method in this paper offers a systematic way to explore and create a wide range of tensegrity configurations.

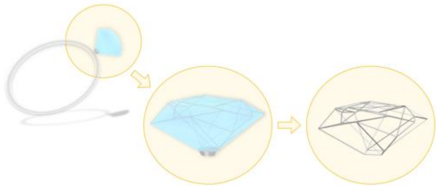


Fig. 3. Transformation from an artistically crafted diamond to an abstracted geometric model.

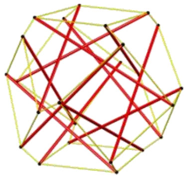


Fig. 4. The regular truncated octahedral tensegrity structure.

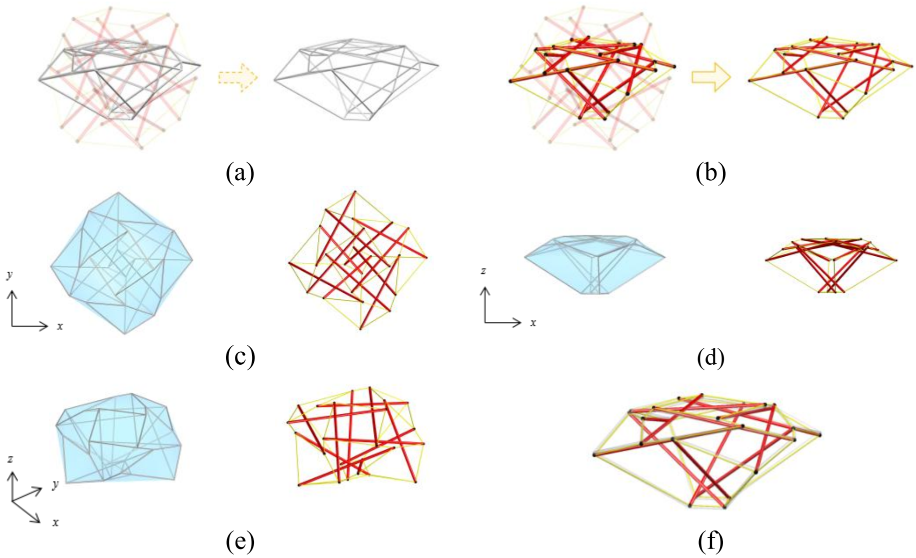


Fig. 5. Design process illustration. (a) The target configuration is based on the regular truncated octahedral tensegrity structure; (b) The actual result is compared with the regular truncated octahedral tensegrity structure; (c)–(e) The comparison of the final calculated structure with the target structure model from different views; (f) The comparison showing the structural overlap to observe the degree of approximation to the target configuration.

Conclusion

In conclusion, the design method for tensegrity structures based on AFDM and SQP effectively achieves form-finding and optimization, ensuring the resulting configurations satisfy self-equilibrium and super stability conditions. However, the initial topology assignment can be challenging and may affect the final structure.

Therefore, combining this method with topology-finding techniques could enhance its effectiveness.

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