Term Screening-Based Response Surface Method and Its Application on Reliability Prediction

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Abstract. Response surface methods are widely used in reliability predictions due to their accuracy and simplicity. However, commonly used response surface methods have the following problems: 1) The fitting result is greatly affected by the number of terms in the final expression; 2) the existence of insensitive input variables makes the corresponding computational cost expensive; 3) response surface methods including all the cross-terms are usually too complicated for practical engineering, while response surface methods with no cross-terms can be inaccurate. To this end, this paper proposes a screening-based response surface method that improves accuracy and simplicity by eliminating insensitive variables and unnecessary terms according to the corresponding sensitivity analysis. Besides, to demonstrate its validity and superiority, the proposed method is compared to the mainstream response surface method using numerical example. Finally, the application of the proposed method to the reliability assessment of an arch bridge will demonstrate its practicality.

Keywords: Response surface method; Term screening; Sensitivity; Reliability; Dimensionality reduction

1 Introduction

Obtaining the performance function (PF) that characterizes the service state of a bridge is a crucial aspect of bridge reliability analysis. Obtaining PF using finite element simulation is computationally prohibitive; hence, less resource-intensive proxy models are frequently employed to substitute the original PF in engineering. Consequently, various types of surrogate models have been proposed in recent years, including the radial basis function model, Kriging model, artificial neural network model, support vector model, and the response surface method (RSM) [1], among others. Among these, RSM is extensively utilized in bridge engineering due to its simple form, analytic expression, high accuracy, and computational efficiency [2-6].

RSM can be broadly classified into two categories: Design of Experiments (DoE)-based RSM [7] and Stochastic Sampling (SS)-based RSM [8]. SS-based RSM is distinguished by its high accuracy [9], enabling convergence analysis to achieve precise solutions and rendering it suitable for addressing low-dimensional reliability

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problems. Conversely, DoE-based RSM offers the advantage of cost-effectiveness for high-dimensional reliability problems. Additionally, SS-based RSM encounters challenges related to inadequate fitting accuracy when dealing with strongly nonlinear PF, whereas DoE-based RSM exhibits good fitting performance near the design point [10].

Given that practical problems often involve high dimensions and the computational cost of finite element stochastic simulation is prohibitively high, DoE-based RSM is commonly employed in engineering. In selecting fitting equations, models with all cross-terms [11] and models without cross-terms [12] are typically considered. Furthermore, J. A. Nelder [13] proposed an inverse polynomial model to mitigate the limitations of simple quadratic polynomials, which are unbounded and symmetric. Yaxun Yang [14] and others introduced a Gaussian function term into quadratic polynomials with all cross-terms to enhance fitting accuracy. Additionally, Han Wang [15] devised a method to ascertain the optimal fitting order of each polynomial parameter, addressing the constraints of fixed and uniform orders in conventional polynomial RSM. Conversely, DoE also plays a crucial role in influencing the accuracy of the aforementioned methods. To enhance accuracy further, numerous new DoE methods have emerged in recent years, including 3^k factorial design [16], Box-Behnken design (BBD) [17], central composite design (CCD) [18], Hoke design [19], and others.

Despite the advancements made by numerous scholars [20-21] in enhancing the RSM based on DoE, the second-order model without cross-terms and including all cross-terms remains predominantly utilized as the fitting function in engineering practice, owing to the simplicity of the fitting function. Nonetheless, the aforementioned second-order model encounters the following issues: 1) The fitting result is greatly affected by the number of terms in the final expression; 2) the existence of insensitive input variables makes the corresponding computational cost expensive; 3) response surface methods including all the cross-terms are usually too complicated for practical engineering, while response surface methods with no cross-terms can be inaccurate. Consequently, this paper introduces an improved second-order RSM based on term screening to streamline parameter and identify essential cross-terms.

2 Term screening-based response surface model

2.1 Methodology

This method initially evaluates the sensitivity of all terms in the response surface function using Sensitivity Analysis (SA) based on Quasi-Monte Carlo (QMC) sampling, obtaining the Variance-based Global Sensitivity Measure (VGSM); then utilizes the VGSM value of each term to filter those that should be included in the final response surface function, achieving parameter dimensionality reduction and identifying essential cross-terms. Compared to the traditional second-order response surface model, the improved model selectively filters out non-essential terms and retains necessary ones through quantitative indicators, ensuring high accuracy with fewer terms, thereby effectively addressing the issues in the traditional model. The process of improved the model is as follows:

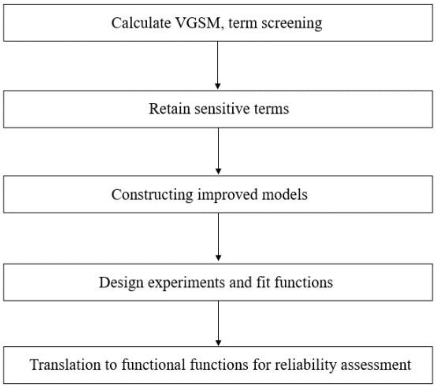

Fig. 1. Improved second-order RSM flow chart based on term screening

2.2 Numerical verification

This section uses the Morris function [22] as an example. It compares the fit and error of the improved model, the model with all cross-terms, and the model without cross-terms.

First, within the normalized [0,1] interval of each parameter, QMC sampling is employed to extract sample points and calculate each VGSM, with each parameter in the table representing a term containing only that parameter. With a sensitivity threshold of $\mu = 0.01$, terms with a VGSM equal to or greater than this value are retained. Secondly, after determining each term, an improved model is constructed, and the BBD method is used to design experiments to obtain the fitting function. Finally, the number of R_{adj}^2, AAD, terms, and the number of runs of the analytical expressions of the fitting functions obtained from the three models are calculated and compared.

The Morris function features numerous parameter cross-terms, and its analytical expression with three parameters is as follows:

$$\begin{aligned} y = {} & 40(x_1 - 0.5) + 40(x_2 - 0.5) + 40\left(\frac{1.2x_3}{x_3+1} - 0.5\right) - 60(x_1 - 0.5)(x_2 - 0.5) \\ & - 60(x_1 - 0.5)\left(\frac{1.2x_3}{x_3+1} - 0.5\right) - 80(x_1 - 0.5)(x_2 - 0.5)\left(\frac{1.2x_3}{x_3+1} - 0.5\right) \end{aligned} \tag{1}$$

Using QMC sampling, 40 sets of sample points were selected. The VGSM results for each term are presented in Table 1:

Table 1. VGSM analysis results of Morris function

Term	S_i^T	S_i	μ	Preserve or not
x_1	0.4701	0.4118		Yes
x_2	0.4757	0.4133		Yes
x_3	0.1415	0.1022		Yes
x_1, x_2	0.0409	0.0405	0.01	Yes
x_2, x_3	0.0219	0.0215		Yes
x_1, x_3	0.0178	0.0174		Yes
x_1, x_2, x_3	0.0004	0.0004		No

The improved model is obtained as follows:

$$y = \beta_0 + \beta_1 x_1 + \beta_2 x_2 + \beta_3 x_3 + \beta_{12} x_1 x_2 + \beta_{13} x_1 x_3 + \beta_{23} x_2 x_3 + \beta_{11} x_1^2 + \beta_{22} x_2^2 + \beta_{33} x_3^2 \tag{2}$$

The BBD method was employed to design 17 sets of experiments and fit the functions according to the three models. Comparative results are presented in Table 2:

Table 2. Fit and error testing

	R_{adj}^2	AAD	terms count	number of runs
Model1(revised model)	**0.9932**	**0.6669**	**7**	57
Model2(no cross-terms model)	0.8214	22.7088	7	17
Model3(Includes all cross-terms models)	0.9932	0.6669	10	17

3 Application to the reliability assessment of an arch bridge

3.1 Project information

The bridge is a top-loaded web-type reinforced concrete arch bridge designed to withstand highway-level II vehicle loads. It has a total length of 21.88m, a main arch ring clear span of 20m, a main arch ring thickness of 0.6m, a main arch ring width of 9.8m, and a net vertical height of 4m. The external load is determined based on the lane load provisions specified in the "General Specifications for Design of Highway Bridges and Culverts (JTG D60-2015)".

3.2 Method implementation

To obtain the required PF value, a finite element automatic modeling program for the arch bridge was developed using Python, as depicted in Figure 2.

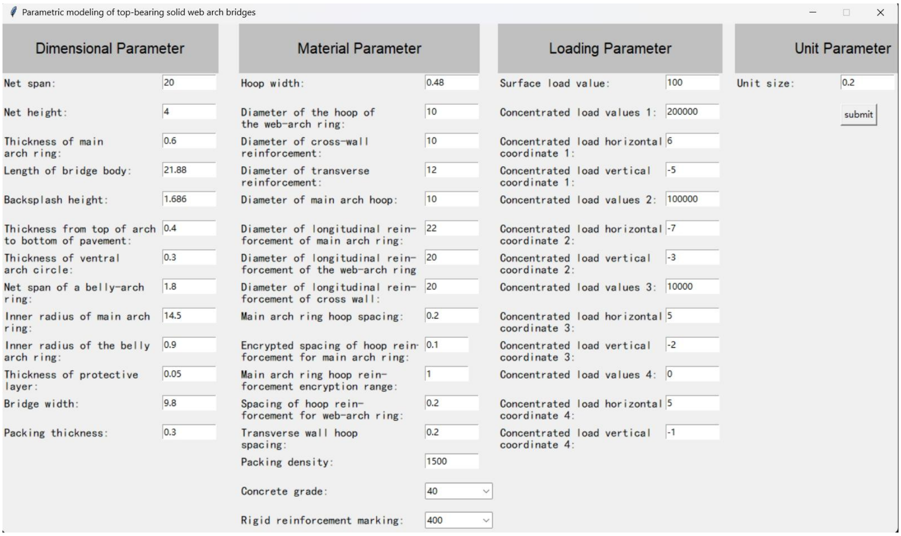

Fig. 2. Parametric modeling script

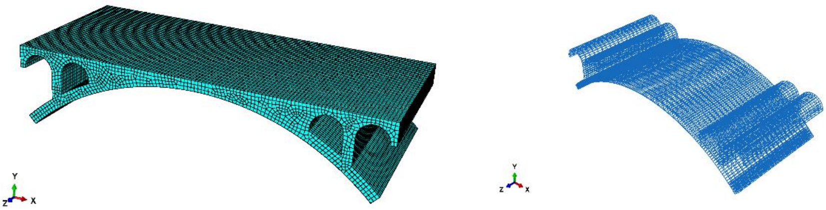

Fig. 3. Large sample of the model

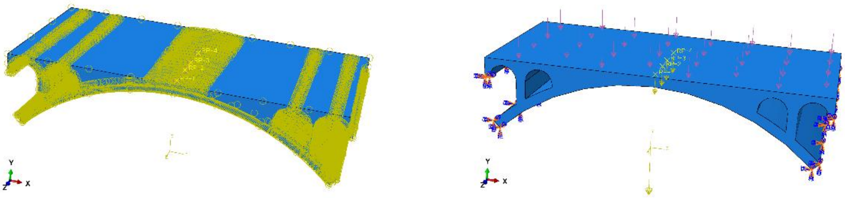

Fig. 4. Coupling, constraints and loading

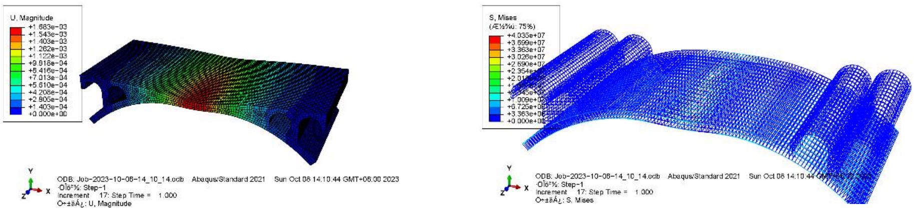

Fig. 5. Stress and strain cloud diagram

Employing the parametric finite element modeling program and following the proposed method, 190 sets of sample points were extracted using both QMC and normal sampling to derive the response surface analytical model. The sensitivity analysis results are presented in Table 3.

Table 3. VGSM results for parameters

Term	S_i^T	S_i	μ	Preserve or not
x_2 = the tensile strength of reinforcing steel	0.0000	0.0000		No
x_2 = the concrete density	0.0261	0.0261		Yes
x_3 = the rebar elastic modulus	0.0753	0.0753		Yes
x_4 = the protective layer thickness	0.1312	0.1068		Yes
x_5 = the longitudinal reinforcement diameter	0.0000	0.0000		No
x_6 = the main arch ring thickness	0.2211	0.1522		Yes
x_7 = the packing thickness	0.0006	0.0005		No
x_8 = the packing density	0.0000	0.0000	0.01	No
x_9 = the concentrated load	0.0447	0.0447		Yes
x_{10} = the surface load	0.0000	0.0000		No
x_{11} = the elastic modulus of concrete	0.1477	0.1477		Yes
x_{12} = the width of main arch ring	0.2963	0.2716		Yes
x_{13} = the main arch ring net vector height	0.1536	0.0854		Yes
x_4, x_6	0.0002	0.0002		No
x_4, x_{12}	0.0242	0.0242		Yes
x_6, x_{13}	0.0682	0.0680		Yes

The final structure of the improved model is as follows:

$$y = \beta_0 + \beta_2 x_2 + \beta_3 x_3 + \beta_4 x_4 + \beta_6 x_6 + \beta_9 x_9 + \beta_{11} x_{11} + \beta_{12} x_{12} + \beta_{13} x_{13} + \beta_{4,12} x_4 x_{12} + \beta_{6,13} x_6 x_{13} + \beta_{22} x_2^2 + \beta_{33} x_3^2 + \beta_{44} x_4^2 + \beta_{66} x_6^2 + \beta_{99} x_9^2 + \beta_{11,11} x_{11}^2 + \beta_{12,12} x_{12}^2 + \beta_{13,13} x_{13}^2 \quad (3)$$

The BBD method was employed to design 120 sets of improved models and 220 sets of experiments for the traditional model. These were fitted using the improved model (Equation 3), the second-order model without cross-terms, and the second-order model with all terms. Table 4 presents the comparative results of the analytical formulations of the aforementioned three models.

Table 4. Fit and error testing

	R_{adj}^2	AAD	terms count	number of runs
Model1(revised model)	**0.9339**	**2.3411**	**19**	310
Model2(no cross-terms model)	0.9141	2.3519	27	220
Model3(Includes all cross-terms models)	0.8934	2.3821	105	220

3.3 Reliability Assessment

Following the calculation formula outlined in the "Design Code for Highway Reinforced Concrete and Prestressed Concrete Bridges and Culverts" (JTG3362-2018) 6.4.2-6.4.3, we substitute the maximum stress y to construct the maximum crack width M, as shown in equation (40). After optimizing the interval variables, we extract 10^7 sample points using the Monte Carlo method to compute the failure probability P_f and reliability index β. The outcomes are presented in Table 5.

$$M = 0.2 - 1.5 \times \frac{y}{2 \times 10^{11}} \left(\frac{0.05 + 0.022}{0.3 + 1.4 \times 0.01} \right) \tag{4}$$

Table 5. Reliability calculation

	Failure probability P_f	Reliability indicator β
Model3 (Includes all cross-terms models)	1.5×10^{-6}	4.6708
Model2 (no cross-terms model)	9.1×10^{-7}	4.7596
Model1 (revised model)	8.8×10^{-7}	4.7792

Following the "Unified Standard for Structural Reliability Design of Highway Engineering" (GB/T50283-1999), this study adopts the reliability index $\beta = 4.2$ and the failure probability $Pf = 1.3 \times 10^{-5}$ of secondary ductile damage as thresholds. The evaluations of the three models indicate safety, affirming the safety and reliability of the bridge structure.

4 Conclusions

This paper introduces an improved second-order RSM that employs term screening. It assesses the VGSM of each term in the second-order model through SA and applies it to filter terms in the analytical equation. Numerical examples and a reliability analysis of an arch bridge illustrate the accuracy, superiority, and practicality of the proposed method. The following conclusions are drawn:

(1)The proposed improved model exhibits superior accuracy compared to both the traditional model incorporating all cross-terms and the traditional model without cross-terms. Additionally, the improved model yields the fewest analytical terms, significantly fewer than the second-order model with all cross-terms.

(2)The proposed improved model requires slightly more PF calls compared to both the model with all cross-terms and the model without cross-terms.

(3)Numerical and engineering examples demonstrate that the improved model retains its superiority when faced with complex or polynomial functions, suggesting its universality. The model with all cross-terms is preferable for fewer parameters, while the model without cross-terms suits scenarios with more parameters, albeit both being affected by parameter fluctuations. The improved model proposed in this paper consistently achieves high accuracy, indicating stability despite parameter fluctuations.

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