



Study on Material Utilization of Stepped Pile under Horizontal Load

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Abstract. Compared with traditional uniform piles, stepped piles increase the load transferred to the upper soil layer, which is consistent with the distribution characteristics of the internal force of the pile, which is larger in the upper part and smaller in the lower part. It has been reported that the vertical bearing capacity per volume of stepped piles is higher than that of uniform piles. To investigate the horizontal load-bearing performance per volume of stepped piles, the field test and finite element method are used to analyze the variation law of horizontal bearing capacity and efficiency of material utilization per volume of stepped piles with different expansion ratio and length of enlarged segment. The results indicate that increasing the expansion ratio can significantly improve the horizontal bearing capacity of stepped piles. When the ratio of the length of enlarged segment to the total pile length is less than 25%, increasing the length of the enlarged segment can slightly improve the horizontal bearing capacity of stepped piles. However, increasing the expansion ratio and length of enlarged segment reduces the efficiency of material utilization. Taking the uniform piles as a reference, the efficiency of material utilization of stepped piles is a problem concerning the cross-section optimization of the expansion ratio and the length ratio of the enlarged segment.

Keywords: Stepped piles, Horizontal bearing capacity, Numerical simulation, Material utilization.

1 Introduction

Pile foundations have been extensively used in various types of important buildings (structures) for high bearing capacity, small differential settlement, stability, and seismic capacity. In recent years, many new forms of pile foundations, such as helical piles, wedge piles, static drilled rooted piles, variable section piles, and oversized diameter hollow piles, etc., have emerged in pile foundation engineering to meet the rapidly growing demand for infrastructure construction and the challenges

faced by large-scale engineering foundations [1-6]. Early research on new pile foundations focused on wedge piles, Kong et al [7] simulated the fresh wedge piles numerically using FLAC3D to analyse the load carrying capacity and this' load transfer behavior pile sort, and the new wedge piles performed optimally in terms of compressive load carrying performance in comparison with the results of other pile types. Maria et al [8] derived the stiffness matrix and the expression for the load components of the response of variable section circular pile in non-homogeneous soil, and the results of the arithmetic analysis show that conical piles support lateral loads more efficiently than equal section piles. Zhao et al [9] researched the vertical dynamic characteristics of conical piles with defects in laminated soils and analysed the effect of defects on the dynamic response of the piles. The bearing capacity of conical piles with different geometrical shapes varies considerably. Shafaghat et al [10] presented an empirical equation for the optimum taper angle considering the nonlinearity of the soil and the interaction between conical piles and surrounding soil under axial load. Hu et al [11] established a semi-analytical solution for the vertical dynamic impedance of conical piles, which is of great significance for evaluating the influence of the taper angle on the vertical dynamic characteristics. This method is important for the optimized design of conical piles. The above study of conical piles is instructive for the study of stepped piles.

The stepped pile is a kind of heterogeneous pile with a step-shaped change in pile cross-section, which makes use of the characteristics of the pile foundation internal force of the upper big and lower small, and gives full play to the material properties and bearing capacity of the shallow foundation soil, which is cost-effective compared with the traditional equal cross-section piles [12]. In soft soil composite foundation treatment, the stepped pile as a new type of composite foundation also shows the advantages of a higher stability coefficient and smaller settlement than the traditional straight pile [13, 14]. Zhao et al [15] investigated the load transfer mechanism and settlement behaviour of T-shaped deep mixing piles when supporting the roadbed, and found that reasonable selection of the parameters of the T-shape deep mixing piles can effectively reduce the foundation settlement. Huang et al [16] and Jiang et al [17] proposed a load transfer model and theoretical resolution for the subside of large-scale stepped conical hollow core piles. The model can better describe the nonlinearity of the pile-soil interface, and the latter introduces load transfer model with the consideration of time-varying, and the relevant theoretical solution can better reflect the subside of the pile-soil interface, which is of larger implication for the application of stepped conical hollow core piles in soft soil foundation. In addition, the shear damage of pile foundation by horizontal load or seismic action mainly occurs in the shallow layer of foundation soil, and enlarging radius of the upper region of pile foundation is favorable to improve the horizontal resistance of pile foundation [18]. Xiong et al [19] analysed the effect of geometric parameters of stepped conical piles on the bearing properties of piles under different load forms by numerical simulation technique, and the results indicated that, by enlarging the diameter of upper section of conical piles in a certain range, the horizontal and vertical ultimate load carrying capacity is improved. Ghazavi et al [20] found that the enlargement of the pile top section in non-saturated soil can significantly reduce the

lateral deflection of the pile top by analysing the influence of heterogeneity of the pile section on its lateral response. Stepped piles have been used to a certain extent in bridge engineering and composite foundation treatment, but there is a lack of sufficient theoretical and experimental research on their horizontal bearing mechanism.

Complex pile-soil interaction exists in the variable cross-section of the stepped pile, and the traditional pile foundation theory cannot accurately describe its working properties, which brings great difficulties to the design, construction, and application of stepped piles and greatly restricts its development. For example, early scholars such as Zhang Lianyang and Cha Jinxing from Tongji University [18, 21] have theoretically analysed the horizontal force performance of variable section piles, but this pile type is less applied in engineering, resulting in the lack of its all-round performance analysis. In current years, with the growth of infrastructure and special geological problems, some domestic bridge piles have begun to use stepped variable section piles again. Foreign scholar Ismael [12] studied the vertical and lateral bearing characteristics of stepped pile in sandy soil throughout field trials, and the findings show that stepped piles have higher lateral bearing capacity and smaller deflection than ordinary piles. The model test results of Fang et al [4] show that the vertical bearing capacity characteristics of the stepped piles were significantly influenced by the variable section ratio, and the stepped piles are loaded uniformly under transverse loading. Fang et al [22] investigated the horizontal deformation and bearing characteristics of stepped conical piles in clay soil throughout model trials, and the results showed that stepped piles had better lateral bearing performance, and the best effect was achieved when the expanded section accounted for the ratio of pile length between 33.3% and 46.7%. Huang et al and Hu et al [5, 23] established the vertical and horizontal bearing capacity calculation models of oversized diameter variable section hollow core piles by improving the existing theoretical calculation method of bearing capacity and demonstrated feasibility of the model by comparing them with the numerical solutions. Wang et al [6] studied the vertical vibration of stepped piles through theoretical derivation, and the analysis showed that the increasing the upper flare section can appreciably enhance the dynamic stiffness of pile top. dynamic stiffness. Few scholars have carried out studies on the material efficiency of piles. Fang et al [24] set up a set of field static load tests on stepped piles and found that the efficiency of material play per unit volume of first-order stepped piles was increased by about 8 % compared with that of piles with an equal cross-section through the test data. However, the tests involved a single size of stepped pile. Hamid et al [25] conducted a detailed study on the load carrying capacity and material efficiency of conical spiral piles. The results indicated that the relative material efficiency factor of helically optimized piles can be up to 2.5 times that of the non-optimized piles. However, this factor only applies to axial forces. Whether in theory or test, the research of step-type variable section piles focuses on the horizontal bearing performance, and there are fewer studies on the material utilisation per unit volume, in some practical engineering problems, the size of the material utilisation per unit volume of the pile may better reflect the goodness of the pile type.

In this paper, for the shortcomings of the single geometry of the stepped pile in the field trial, combining the field test and ABAQUS simulation means, we analyse various factors such as the location of the variable cross-section of the stepped pile and the change of the variation ratio, and then determine the effect of the different expansion ratio and length of the expansion section on the horizontal load bearing capacity of the stepped pile and the efficiency of the unit volume of the material bearing capacity, to further reveal the load bearing performance of the stepped pile. This study contributes to the optimal design of stepped piles, which in turn improves their horizontal load carrying capacity, which is particularly important for structures requiring large horizontal loads (offshore platforms, jetties, etc.).

2 Field Test

2.1 Experimental Parameters

The test pile is composed of a stepped pile and an uniform pile, as shown in Fig. 1. To ensure the quality of the pile shaft, the pile forming method of manual hole digging and pouring is adopted. Twelve HRB400 bars are adopted in the main reinforcement bars of the pile shaft. HRB400 bars are adopted in the stirrups. The diameters are 14mm and 8mm respectively. The thickness of the concrete protective layer is 60 mm. The soil surrounding the pile is mainly the Holocene alluvial floodplain, including silty clay, clay, and muddy silty clay. The basic physical parameters of the soil layer are shown in Table 1.

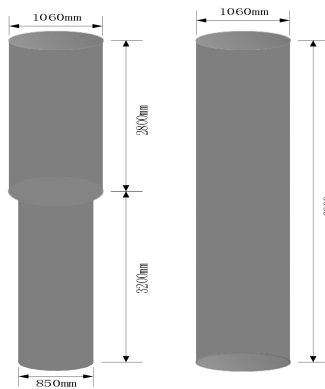


Fig. 1. Size of test piles.

Table 1. Physical and mechanical parameters of soil.

Stratum	The thickness of the layer/m	Cohesion/kPa	Internal friction angle/(°)	Density/(g·cm ⁻³)
Silty clay	0~1	18.64	30.3	2.00
Clay	1~2	23.21	32.5	1.89
Silty clay	2~4	43.88	26.4	1.93

Muddy silty clay	<4	27.60	24.2	1.96
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Fig. 2. Construction of stepped pile.

2.2 Test Scheme

The test equipment comprises three components: loading system, reaction force system, and reference system. The device diagram is as follows Fig. 3.

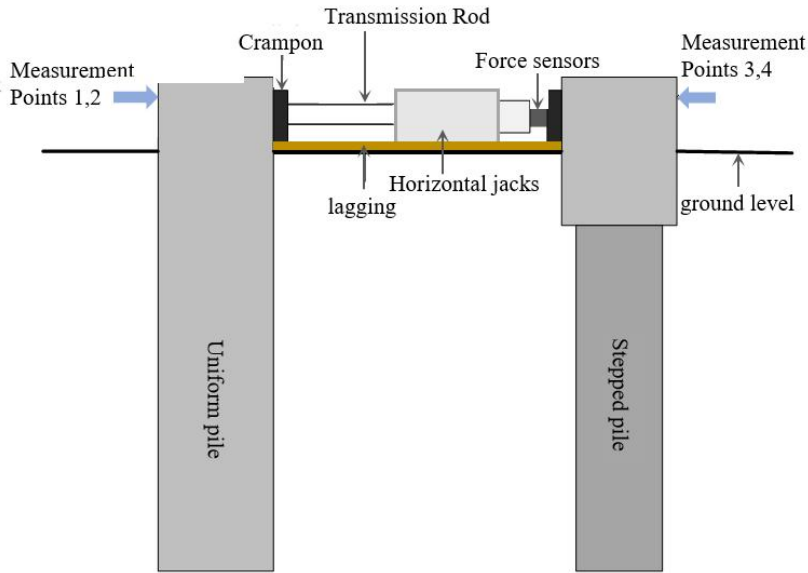


Fig. 3. Sketch of horizontal force apparatus.

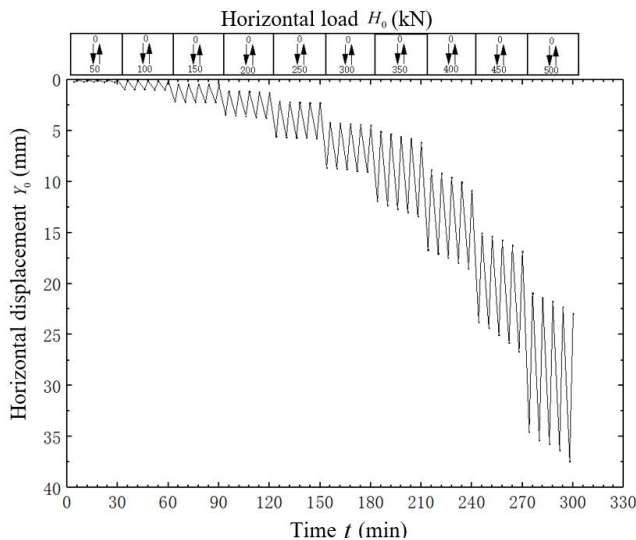
The loading system contains an oil pump, a 50-ton horizontal jack, a load cell, and dowel bar. The test oil pump working pressure is 0~80 MPa, the rated flow capacity is 0.8 L/min-20 L/min, motor power is 0.75 kW-30 kW. The oil pump is connected to the jack through the hydraulic oil pipe.

The test pile is in the form of a free pile top. The distance between the loading force and the pile top is 50 cm, loading form is unidirectional multi-cycle loading. The predicted maximum horizontal test load was 500kN, and the load was carried in 10 stages of 50kN each. When loading, each stage load is first loaded to the specified load, test read after stabilization for 4 min, then unloaded to 0, stopping 2 min before reading, this is a cycle, each stage load cycles 5 times, then enters the next load. The test is terminated when the test satisfies any of the following conditions: 1) the pile shaft breaks; 2) the horizontal displacement exceeds 40 mm.

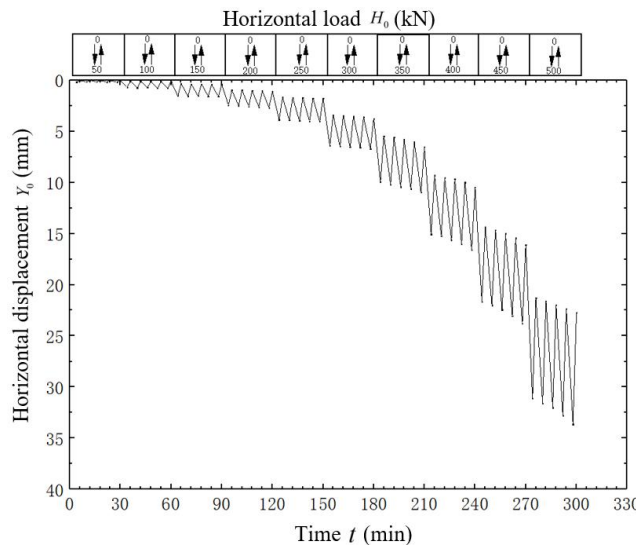
In the test, the horizontal critical load and horizontal ultimate bearing capacity of single pile are determined depending to the following conditions. Horizontal critical load: 1) Take previous level horizontal load with an inflection point in the H_0-t-Y_0 curve (load-displacement curve) of the unidirectional multi-cycle loading method; 2) Obtain the horizontal load corresponding to the first inflection point of the H_0-t-Y_0 curve (load-displacement gradient curve). Horizontal ultimate bearing capacity: 1) In the one-way multi-cycle loading method, obtain the horizontal load corresponding to the stage before the H_0-t-Y_0 curve shows an obvious steep drop; 2) Obtain the horizontal load corresponding to the second inflection point of the H_0-t-Y_0 curve; 3) Obtain the horizontal load corresponding to the stage before the pile body ruptures or tensile reinforcement yields.

2.3 Experimental Result

Fig. 4 shows that when the load is small, the lateral displacement at the top of the pile (Y_0) rises slowly with the increase of load. For the stepped pile, the horizontal residual displacement approaches 0 when unloading in each stage of the load range of 0 kN~100 kN. When the load is further increased, the gradient of the average displacement curve becomes greater, and the maximum displacement of the top of the test pile ($Y_{\max}$) is 37.42 mm when the maximum load of 500 kN is reached. Comparing to stepped piles, the uniform pile has a smaller Y_0 for the same load, while the magnitude of displacement change in uniform piles is also smaller in each level of cycling. Because both test piles belong to rigid short piles, the horizontal bearing capability is controlled through the soil strength on the pile side, the inflection points of H_0-t-Y_0 curve of the two types of piles are not obvious from an overall point of view.



(a) Stepped pile

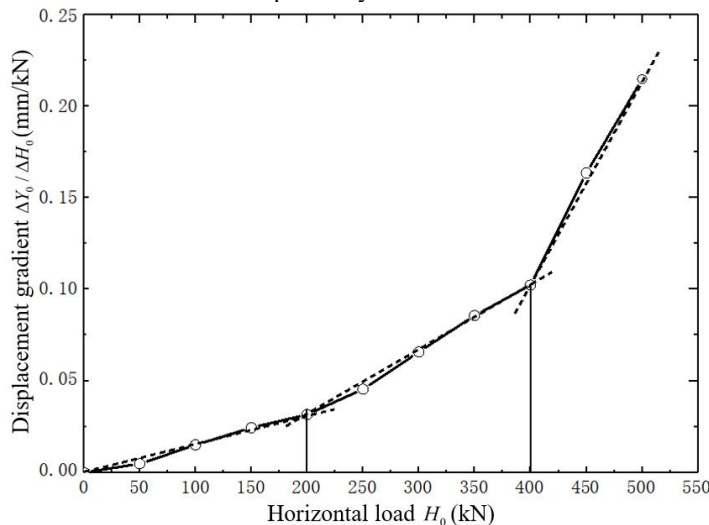


(b) Uniform pile

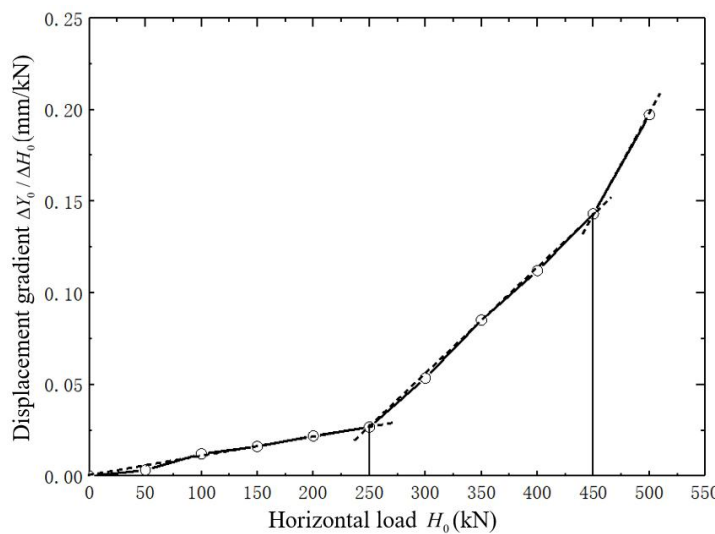
Fig. 4. $H_0 - t - Y_0$ curves of piles.

Fig. 5 shows that the displacement gradient of trial pile basically presents three stages. At the stage of 0 kN~200 kN, Y_0 is relatively stable under the cyclic horizontal thrust, and Y_0 is small after unloading. It can be considered that the pile and soil are in elastic state at this stage, and the soil surrounding the pile is basically elastic deformation. When the horizontal thrust is 200 kN~400 kN, with the gradual increase of load, Y_0 increases clearly, which enters the second stage. At this time, the pile and soil are in

elastic-plastic state, and the horizontal ultimate bearing capacity of the stepped pile to be 400 kN. When the force-displacement gradient curve enters the third stage, the displacement gradient increases significantly with increasing load, and the corresponding $H_0 - t - Y_0$ curve increases each cycle displacement increment, and the cracks appear in the soil around the piles during the field test. Based on the same method, the horizontal critical load and ultimate load of equal section pile can be determined as 250kN and 450kN respectively.



(a) Stepped pile



(b) Uniform pile

Fig. 5. $H_0 - \Delta Y_0 / \Delta H_0$ curves of pile.

Fig. 6 illustrates the $Y_{\max}$ for the two piles at each load level. $Y_{\max}$ presents typical nonlinear variation with the load. The $Y_{\max}$ of both two test piles are smaller under smaller horizontal load. The $Y_{\max}$ of the stepped pile is always larger than uniform pile, and the displacement difference gradually increases with the increase of load. When the horizontal load is 500 kN, the displacement difference between stepped pile and uniform pile is 3.77 mm.

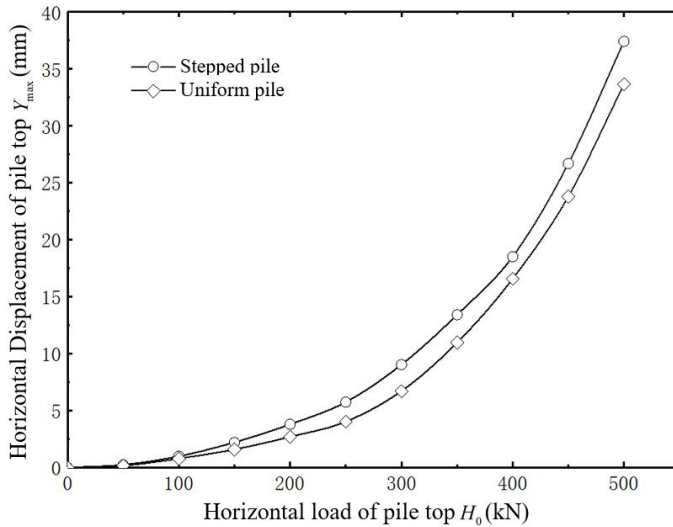


Fig. 6. A force-displacement curves on pile top.

According to the H_0-t-Y_0 curve analysis results of test piles, the horizontal critical loads of 200 kN and 250 kN for stepped pile and uniform pile respectively, respectively. The corresponding horizontal ultimate load is 400 kN and 450 kN, respectively. The critical load of pile type and the ratio of ultimate load to pile volume are used as comparison factors to analyze the material performance of stepped pile and uniform pile under horizontal load. According to the pile parameters and test results, the following are shown in Table 2.

Table 2. Horizontal bearing capacity per volume of pile.

Pile type	Voume of pile shaft/m ³	Critical load/kN	Ultimate load/kN	Unit volume bearing capacity under critical load(kN/m ³)	Unit volume bearing capacity under ultimate load(kN/m ³)
Stepped pile	4.28	200	400	46.73	93.46
Uniform pile	5.29	250	450	47.26	85.07

The top section of the pile and the length of the pile were the same in the experiment. The test results show that the horizontal bearing capacity of the stepped rigid short pile is less than the uniform rigid short pile. However, the stepped rigid short piles

have a better material capacity per unit volume than uniform piles, with an improvement of about 10 % in reaching the ultimate load. To analyze the horizontal bearing capacity of unit volume material of stepped piles in more detail, more working conditions will be analyzed by finite element simulation below.

3 Establishment of Model

3.1 Model Scale

ABAQUS finite element software was used for modeling to carry out further analyses. The model of medium uniform pile length is 12 m, the diameter is 0.8 m, stepped pile based on expanding diameter, expansion ratio of 1.25, 1.50, 1.75 three conditions, expanding section is five conditions, stepped pile geometric parameters under each condition are shown in Table 3. The thickness of soil layer at the bottom of the pile is 18 m. The height of the soil model is 30 m, and its length and width are both 30 m.

Table 3. Geometric parameters of stepped piles under different working cases

Case	Proportion of expanding section to pile length	Expanded diameter ratio	Pile length of expanding section / m	Pile diameter of expanding section / m
1	1/12	1.25	1	1.0
2	1/6	1.25	2	1.0
3	1/4	1.25	3	1.0
4	1/3	1.25	4	1.0
5	5/12	1.25	5	1.0
6	1/12	1.50	1	1.2
7	1/6	1.50	2	1.2
8	1/4	1.50	3	1.2
9	1/3	1.50	4	1.2
10	5/12	1.50	5	1.2
11	1/12	1.75	1	1.4
12	1/6	1.75	2	1.4
13	1/4	1.75	3	1.4
14	1/3	1.75	4	1.4
15	5/12	1.75	5	1.4

3.2 Constitutive Model and Physical Parameters

Uniform pile and stepped pile, the pile shaft using strength grade C30 concrete, steel model HRB400. Assuming that the pile stress is less than the compressive yield stress of concrete under horizontal load, the linear elastic model should be used. The reinforcement ratio of the pile is 0.65 %. Considering the effect of steel reinforcement, the equivalent compressive stiffness of the pile is set as

$$EA = E_z A_z + E_g A_g \tag{1}$$

where E is equivalent elastic modulus, A is the cross-sectional area, E_z and E_g are elastic modulus of concrete and steel respectively, A_z and A_g are the sectional areas of concrete and steel respectively. The soil model was modelled using Mohr-Coulomb elastic-plastic constitutive model. To simplify calculation and control of variables, the soil surrounding the pile and at the bottom of the pile is assumed to be clay with the same physical parameters. The physical parameters of the pile-soil system are shown in Table 4.

Table 4. Physical parameters of pile-soil system.

Category	Elastic modulus/Pa	Poisson ratio	Gravity/(kN/m ³)	Cohesion/kPa	Angle of internal friction/(°)
Pile	3.11×10^{10}	0.2	25	—	—
Clay	6.38×10^6	0.3	19	30	25

3.3 Interface, Element, and Boundary Condition Setting of Model

Considering the characteristics of pile-soil interaction, the pile-soil contact is set as hard contact, and the pile shaft surface was selected as the primary surface. The friction coefficient of the pile-soil contact surface is 0.4. The model adopts 3D solid model, and the pile and soil adopt three-dimensional eight-node linear reduction integral element (C3D8R). In addition, when the pile is affected by horizontal thrust, the soil surrounding the pile is affected by extrusion deformation, and the influence on the soil gradually decreases as the radial distance increases. Therefore, the around the pile is finely divided, and the model mesh is shown in Fig. 7. Displacement constraints in X, Y, and Z directions at the bottom of the soil model and radial displacement constraints at the side of the model.

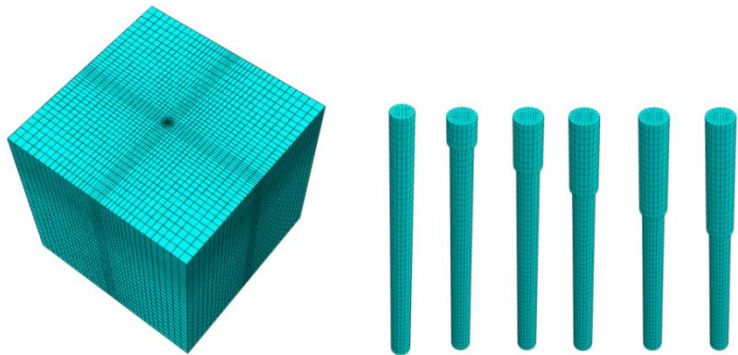


Fig. 7. Schematic diagram of meshing of pile-soil model.

4 Numerical Results and Analysis

According to the relevant specification, for cast-in-place piles with reinforcement ratio not less than 0.65%, 0.75 times of the corresponding load of Y_0 can be taken as the characteristic value of horizontal bearing capacity of single pile (Q), and the horizontal displacement to which the building is sensitive is 6 mm. In this paper, the Q of piles in various working scenarios are extracted according to the aforementioned rules, as shown in Table 5.

Table 5. Q of stepped piles under different working cases.

Cases	Proportion of enlarged segment to pile length	Expansion ratio	Volume of pile shaft V/m^3	Horizontal load at 6 mm/kN	Characteristic value of horizontal bearing capacity Q/kN	Material capacity per unit volume $Q/V/(\text{kN}/\text{m}^3)$
1	1/12	1.25	6.31	210.67	158.00	25.02
2	1/6	1.25	6.60	212.43	159.32	24.15
3	1/4	1.25	6.88	214.00	160.50	23.33
4	1/3	1.25	7.16	222.33	166.75	23.28
5	5/12	1.25	7.45	229.29	171.97	23.10
6	1/12	1.50	6.66	222.86	167.15	25.10
7	1/6	1.50	7.29	225.71	169.28	23.22
8	1/4	1.50	7.92	231.07	173.30	21.89
9	1/3	1.50	8.55	246.92	185.19	21.67
10	5/12	1.50	9.17	250.00	187.50	20.44
11	1/12	1.75	7.07	243.20	182.40	25.80
12	1/6	1.75	8.11	244.40	183.30	22.62
13	1/4	1.75	9.14	250.38	187.79	20.54
14	1/3	1.75	10.18	267.50	200.63	19.71
15	5/12	1.75	11.22	281.60	211.20	18.83
Uniform pile		/	6.03	186.78	140.09	23.23

Fig. 8 illustrates the effect curve of the length change of the expanding section on the Q of the stepped pile. The Q of the uniform pile is 140 kN. When the enlarged segment accounts for 1/12 of the pile length, the bearing capacity of the three stepped piles with the expansion ratio of 1.25, 1.50, and 1.75 is 12.9 %, 19.4 %, and 30.3 % higher than that of the equal-section pile. It can be seen that the expansion can greatly

improve the horizontal bearing capacity of the pile. However, it can also be seen from the diagram that the length of the enlarged segment increases from $1/12$ pile length to $1/4$ pile length, and the horizontal bearing capacity increases little. The expansion ratio increases by 1.6 %, 3.7 %, and 3.0 % respectively. When the length of the enlarged segment exceeds $1/4$ pile length, the increasing rate of horizontal bearing capacity increases obviously, but the cost performance may be greatly reduced. Therefore, in terms of increasing the horizontal bearing capacity, it is more practical to enhance the diameter of the stepped pile than the length of the enlarged section.

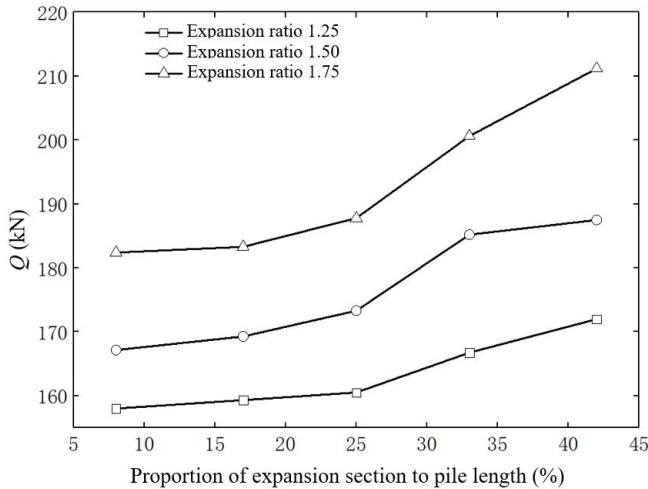
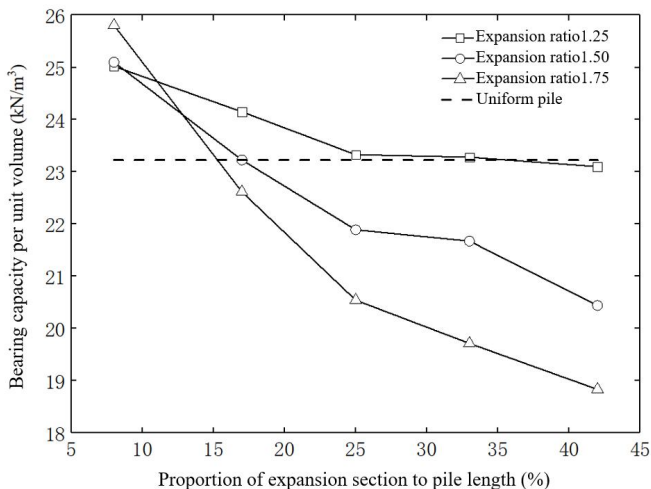


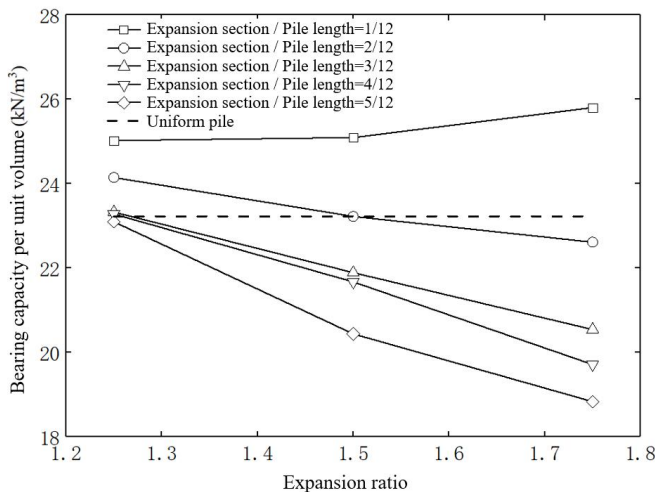
Fig. 8. Influence of stepped pile geometry on Q

Fig. 9 shows the bearing capacity curves of unit volume materials of different stepped piles, which can be used to further analyze the cost performance of stepped piles in horizontal bearing, namely, the material efficiency of unit volume. From Fig. 9(a), it can be distinctly that the larger the length of expanding section, the smaller the bearing capacity of the unit volume material, that is, increasing the length of expansion section reduces the cost performance of the stepped pile. The material efficiency of the stepped pile with the expansion ratio of 1.25 is basically greater than or equal to the uniform pile. When the length of enlarged segment reaches $5/12$ of the total pile length, it is slightly less than the uniform pile. For stepped piles with expansion ratios of 1.50 and 1.75, the material efficiency is equivalent to that of equal section only when the length of the enlarged segment is less than 2 m. As shown in Fig. 9(b), a more intuitive, smaller length of the enlarged segment can better play to the utilization of materials. This phenomenon is tightly correlated to the horizontal bearing resistance of piles mainly supplied by the upper soil.

The above calculation shows that when designing stepped piles with horizontal bearing in harder strata, the scheme with a slightly larger expansion ratio and a shorter enlarged segment should be adopted, which is more conducive to the performance of materials. For example, in this paper, it is suggested to control the length of the expanding section within 2 m.



(a) Influence of length of expansion section



(b) Influence of expansion ratio

Fig. 9. Horizontal bearing capacity per volume of stepped piles in different cases.

5 Conclusion

Field tests of the two pile types were carried out, and the differences in horizontal bearing capacity were initially analysed. Based on FEM, the changing law of horizontal bearing capacity and unit volume material play efficiency of stepped piles were analysed in detail under different expansion ratio and length of expansion section The main conclusions of this study are summarized as follows:

- (1) The diameter of the stepped pile expansion section in the test is the same as that of the contrast pile, and the diameter of the lower non-expansion section is smaller than that of the contrast pile. In this case, although the Q was smaller than that of the comparison uniform pile, the Q/V was about 10 % higher than that of the uniform pile.
- (2) Through numerical analysis of several working cases, it is found that the diameter of the expansion section of the stepped pile has an obvious influence on the horizontal bearing capacity. Increasing the diameter of the expansion section can significantly improve the Q of stepped piles. However, when the length of the expansion section is greater than 1/6 of the pile length, increasing the expansion ratio under the same length of the expansion section will lead to a decrease in Q/V , which will reduce the efficiency of the material.
- (3) Increasing the length of the expansion section increases the Q of the stepped pile with the same expansion ratio, but the increase rate is small when the length of the expansion section is less than 25 % of the total pile length. Accordingly, increasing the length of the expansion section will reduce Q/V of the stepped pile, which will reduce the efficiency of the material. but only marginally increases the horizontal load capacity when the ratio of the expansion section length to the pile length is less than 25 %. Increasing the length of the expansion section to pile length ratio of less than 25 % reduces the horizontal capacity per unit volume of the stepped pile, which will reduce the efficiency of material.
- (4) From the above two points, it can be seen that the material efficiency of stepped piles is an optimal solution problem of the expansion ratio and the length ratio of the expansion section. Therefore, the two parameters need to be pre-analyzed in the pile foundation design. For example, in this example, the better choice is to control the length of the expansion section within 2 m, and to consider the improvement of horizontal bearing capacity and material utilization rate, the expansion ratio can be selected below 1.50.

The reference values of the expansion section length and expansion ratio obtained in this study are only applicable to limited engineering conditions, and more field tests and numerical analyses are needed to determine them for a wider range of engineering conditions in order to meet the load carrying capacity requirements. In the future, advanced optimisation algorithms can also be used to optimise the parameters of variable section piles to achieve the best bearing performance and economic benefits.

Acknowledgments

The work was supported by the National Natural Science Foundation of China with grant number 52168047.

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