Efficient Multi-Resolution Haptic Rendering for Real-Time Applications

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Abstract. Haptic rendering is essential for developing immersive virtual environments and providing tactile feedback using computational techniques. This study assesses five haptic rendering techniques multiresolution, image-based, edge detection, Fourier transform, and height map utilizing a varied dataset of surface textures. Performance parameters, including computing time, memory use, signal entropy, and signal to-noise ratio (SNR), were evaluated to benchmark the methodologies. The multi-resolution technique proved to be the most efficient, attaining the minimal calculation time of 0.7 milliseconds and a low memory consumption of 0.3 KB while preserving a strong equilibrium of signal entropy (4.75) and SNR (5.7 dB). Visual representations of haptic signals emphasize the multi-resolution method's capacity to harmonize computing efficiency with precise and distinct tactile feedback, rendering it exceptionally appropriate for real-time applications in virtual reality, teleoperation, and medical simulations. This study offers a mathematical basis for the selection and optimization of haptic rendering techniques in practical, performance-sensitive systems.

Keywords: Haptic Rendering, Multi-Resolution, Real-Time Feedback, Tactile Interaction , Haptic Signal Processing

1   Introduction

Haptic rendering is essential for providing users with tactile input, hence augmenting immersive and interactive experiences in virtual reality, augmented reality, robotics, and teleoperation applications. Over the past decade, significant advancements have occurred in this domain, characterized by an increased focus on instantaneous tactile response, diverse resolution techniques, and the integration of different feedback mechanisms. These breakthroughs are transforming how individuals view and participate with digital surroundings, across industries such as healthcare, education, industrial automation, and entertainment.

Multi-resolution haptic rendering is a very promising topic within this research because it enables systems to capture both intricate and rough aspects of an image or 3D object,

delivering a more detailed and genuine tactile feel. Brown et al. [22] and Ye et al. [16] proposed approaches demonstrate to us how haptic rendering systems can improve texture accuracy and detail at various scales. Real-time performance is still important in large-scale systems due to the challenge of maintaining computational efficiency.

Another recent topic is non-contact haptic rendering in which leveraging methods like focused ultrasound to offer tactile sensations without any actual touch. Jones et al. [19] examined how electrostatic actuation can be applied in AR systems to offer tactile feedback without physical contact for broadening the range of ways users can engage. Furthermore a study conducted by Zhang et al. [20] underlines the significance of force feedback in medical simulators because accurate surgical training necessitates accuracy and speedy reactions.

The application of machine learning (ML) in haptic rendering systems is developing with goal to improve feedback mechanisms and boost flexibility. Schneider et al. [21] noted in their study how machine learning can increase the precision and expandability of force feedback systems in teleoperation. In the same line, Lee et al. [15] shown in their study how machine learning may increase various haptic feedback to improving their flexibility to user actions and surrounding alterations. These versatile technologies are able to tailor input for each user's requirements fast also offer tremendous potential for eventual haptic applications.

Although these developments have been realized, there are still significant issues that remain in terms of scalability, real-time operation, and merging feedback from multiple modalities. J. Howard et al. discovered for tasks such as teleoperation and VR where Instantaneous performance is vital as even a slight delay in reaction can severely degrade the user's enjoyment [23]. Additionally, earlier haptic systems frequently concentrate on either force feedback or tactile sensations but not on both at the same time. E. Martin et al. [24] poin out that the absence of connection among diverse technologies restrains the possibility to design totally immersive surroundings.

In this work, we perpose a novel haptic rendering method that tackles these problems by increasing speed and adding various types of input. Our method focusses on establishing a better balance between signal complexity and computer efficiency, which keeps the system responsive while producing more accurate and detailed haptic feedback. We present an adaptable technique for several applications, including VR/AR, teleoperation, and medical simulations, by utilising machine learning to modify feedback in real-time in response to user input and contextual conditions.

2   Literature Review

Advances in augmented reality, virtual reality, and robotics have dramatically altered haptic rendering techniques. Important developments in signal processing, tactile

feedback, multi-resolution haptic rendering, and their applications across a range of domains are covered in the below overview.

2.1   Multi-Resolution Haptic Rendering

High-performance haptic systems now need to handle images with different levels of detail due to multi-resolution images. The Elastowave system was created by G. Reardon et al. [1] employing concentrated elastic waves to provide tactile feedback through a flexible interface that offers exceptional resolution and versatility for tasks including augmented reality and creative computing. FuseSR, a superresolution technology developed by Zhong et al. [2] is a notable advancement that highlights the value of multi-resolution fusion and provides consistent reconstructions in real-time rendering. To improve realism in virtual settings, Cho et al. [3] later created a data-based framework for multi-modal haptic rendering to unified temperature, force, and tactile responses. These results illustrate the potential for multi-resolution approaches to improve haptic input's accuracy and effectiveness. The deployment of haptic rendering in resource-intensive and complicated environments such as VR and AR has been made possible by these methods.

2.2   Tactile Feedback and Signal Processing

Tactile feedback remains an essential component of immersive haptic experiences. Gao et al. interactions [4] devised visual-tactile synthesis method which enables for high-quality tactile depiction based on visual input which considerably enhancing the realism of virtual . Another option was to use Huang et al.'s [5] wearable vibrotactile feedback device, AeroHaptix, for UAV teleoperation. It enhances obstacle awareness via directional tactile feedback. improvements in signal processing also encompassing image-based haptic creation and Fourier transforms are major research issues. Numerous studies like D. Chakraborty et al. [11] and M. Green et al. [10] have showed how frequency-domain processing may provide essential information about an object's texture and structure thereby leading in more lifelike haptic experiences.

2.3   Applications in Virtual and Augmented Reality

Particularly for tasks requiring object manipulation and spatial interaction, haptic rendering techniques have found widespread application in virtual and augmented reality systems. To enhance the real-time rendering of multi-resolution models for augmented reality scenarios, Li et al. [6] developed a view-dependent adaptive hierarchical level of detail (HLOD) technique. These techniques were improved by Boutsi et al. [7] to achieve significant rendering speed increases without sacrificing visual quality in web-based augmented reality applications . Accurate haptic feedback is essential for virtual reality

applications like remote surgery and medical simulations. Gil et al.'s [12] real-time force feedback devices are expanding the realm of what is possible in simulated environments. These gadgets allow users to interact with 3D things while gaining rapid and realistic haptic input.

2.4   Challenges and Future Directions

Despite tremendous development, haptic rendering still faces various hurdles. High processing demands and the necessity for real-time performance hamper the large-scale implementation of multi-resolution rendering systems. Morisaki et al. [8] studied the use of focused ultrasound for noncontact haptic rendering, which presents options to minimize these problems. However, achieving low-latency answers, especially in dynamic situations like teleoperation and AR, remains a crucial issue. Optimizing the balance between signal complexity and computational efficiency continues to be a crucial challenge, As proven by Battaglia et al. [9], who applied optimization approaches to minimize processing delays in large-scale simulations. According to Lee et al [15], incorporating machine learning offers considerable potential for the future development of haptic systems. Haptic feedback systems that are adaptive and able to change feedback based on user behavior have the potential to improve the user experience shown by J. Hong et al. [13]. In the opinion of S. Maheshwari et al., Machine learning has proven potential in boosting the scalability of multi-resolution systems [27], rendering them more feasible for real-world usage in sectors like industrial automation and healthcare shown by K. Kwon et al. [14].

In order to improve haptic rendering, focus needs to be redirected towards overcoming scalability and real-time processing constraints. This will allow these systems to be employed in more intricate contexts, bringing up new possibilities in teleoperation, medical simulations, and industrial monitoring. Future studies should dive into utilizing machine learning further to improve adaptability and enhance performance in these changing applications.

3   Dataset

The TUM LMT Haptic Dataset 108 is a detailed tool intended to improve studies in haptic feedback and surface analysis. It features 108 deliberately picked products showing varied surface textures and material properties, divided into nine categories for in-depth investigation. In a dataset published by Sress et al. [28] at Technische Universität München (TUM), there are two primary sets of data: one collected through ten free-hand recordings (five linear and five circular movements) and another obtained under controlled conditions with constant parameters aside from force and scanning velocity. The collection comprises images, three-axis acceleration signals, sound signals, and

friction signals, with a special emphasis on images and audio in this work. The texture images are grouped by types including mesh, stones, glossy surfaces, wood, rubbers, fibers, foams, foils, and fabrics. The layout of the files provides for easy access, with methodically ordered high-resolution photos (320 × 480 pixels) and voice recordings for each item.

In this research, the dataset is applied to investigate the link between visual and aural information received from surface textures. Using image sonification techniques, the 20 pictures of each object are turned into audio signals, providing Mel spectrograms for both the images and audio recordings. The study tries to find key links between surface images and audio created by linear movements utilizing quantitative metrics such as Euclidean distance, cosine similarity, and Pearson correlation. In summary, the TUM LMT Haptic Dataset 108 gives a robust platform for exploring how visual and aural aspects of surfaces interact.

4   Methodology

This part describes the approach for Multi-Resolution Haptic Rendering, followed by a contrast of various haptic rendering techniques. Every technique analyzes the input image to create tactile feedback signals, which are then assessed using various performance measures.

4.1   Approach to Haptic Rendering at Multiple Resolutions

The Multi-Resolution Haptic Rendering method aims to offer a rich and intricate sense of touch by analyzing the input image at various levels of detail. The multiresolution haptic rendering pipeline is consists of the following steps shown in Fig. 1:
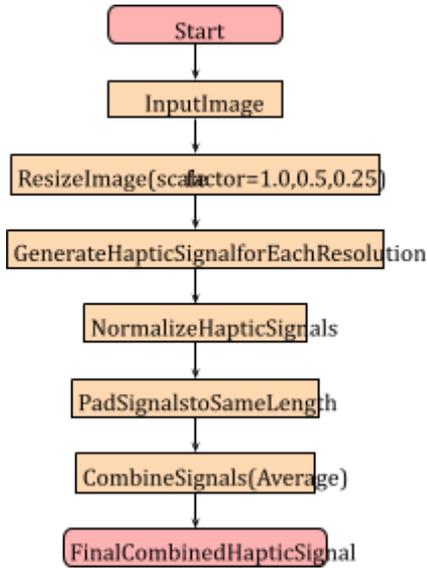
Fig.1. Process of Multi-Resolution Haptic Rendering

- **Step 1: Image Resizing at Multiple Scales**

  Denote the input image as *I*. The picture is adjusted to various magnification factors $s_1, s_2, ..., s_n$, with each s_i denoting a distinct quality of the image.
  The scaled image is denoted as s_i:

  $$I_{si} = \text{Resize}(I, s_i) \tag{1}$$

  Linear interpolation is used to perform this resizing. An instance of typical scale factors could be $s_1 = 1.0$, $s_2 = 0.5$, and $s_3 = 0.25$, which represent full, half, and quarter resolutions, respectively.

- **Step 2: Generation of Haptic Signals**

  After resizing the image, a one-dimensional haptic signal is generated for each scale by summing the pixel intensities along the vertical axis:

  $$H_{si}(y) = \sum^{W} I_{si}(x, y) \tag{2}$$

$$x=1$$

where W is the width of the resized image and y is the vertical position. After generating the haptic signal for each scale, it is normalized to a range between 0 and 1:

$$H_{s\text{norm}i} = \frac{H_{s_i} - \min(H_{s_i})}{\max(H_{s_i}) - \min(H_{s_i})} \tag{3}$$

This normalization ensures that all haptic signals are consistent across different resolutions.

• **Step 3: Padding Signals to Equal Length**

Since the signals generated from different scales vary in length, they are padded to ensure uniformity. The maximum signal length, $L_{\max}$, is given by:

$$L_{\max} = \max(L(H_{s1}), L(H_{s2}), ..., L(H_{sn})) \tag{4}$$

Each haptic signal is padded with zeros to match this maximum length:

$$H_{s\text{padded}i} = \text{Pad}(H_{s_i}^{\text{norm}}, L_{\max}) \tag{5}$$

• **Step 4: Combining Haptic Signals**

After padding, the haptic signals from all scales are combined by averaging them:

$$H_{\text{combined}} = \frac{1}{n} \sum_{i=1}^{n} H_{s_i\_\text{padded}} \tag{6}$$

This final combined signal represents the tactile properties of the image, capturing both fine and coarse textures.

4.2   Comparison of Haptic Rendering Techniques

To compare different haptic rendering methods, we evaluate five techniques (Fig. 2): Multi-Resolution Haptic Rendering, Image-Based Haptic Rendering, Edge Detection Haptic Rendering, Fourier Transform Haptic Rendering, and Height Map Haptic Rendering.
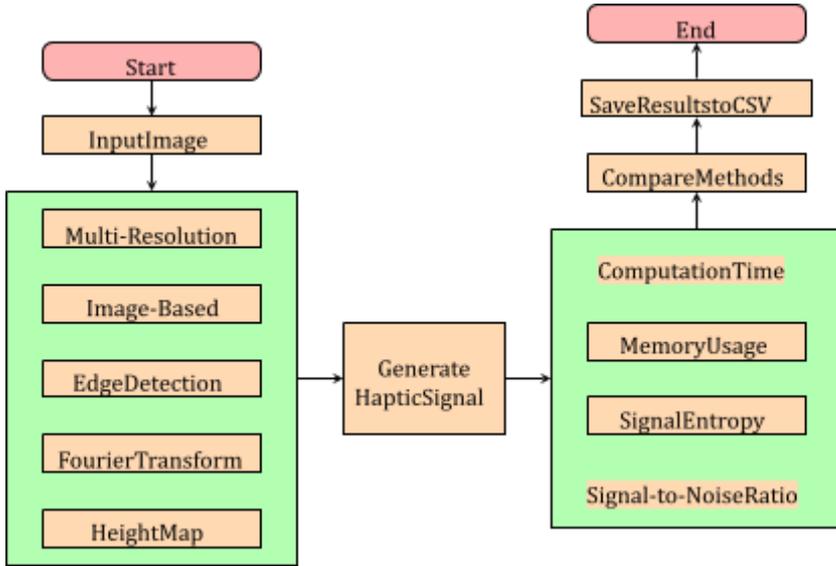
Fig.2. Flowchart of comparison of Methodology

4.2.1 Multi-Resolution Haptic Rendering This method has already been explained. The final haptic signal is formed by integrating data from several scales:

$$H_{multi} = \frac{1}{n} \sum_{i=1}^{n} H_{s_i \_ padded} \tag{7}$$

4.2.2 Image-Based Haptic Rendering In the image-based technique, the haptic signal H_{image} is generated by summing the pixel intensities of the original picture along the vertical axis:

$$H_{image}(y) = \sum_{x=1}^{W} I(x,y) \tag{8}$$

4.2.3 Edge Detection Haptic Rendering This approach employs a Canny edge detector to detect edges in the image. The haptic signal H_{edge} is created from the edge-detected image I_{edge} as:

$$H_{\text{edge}}(y) = \sum_{x=1}^{W} I_{\text{edge}}(x, y) \tag{9}$$

4.2.4 Fourier Transform Haptic Rendering The Fourier Transform method transfers a picture to the frequency domain. The haptic signal H_{fourier} is created from the magnitude spectrum M_{fourier} of the frequency-transformed image:

$$H_{\text{fourier}}(y) = X M_{\text{fourier}}(x,y) \tag{10}$$
$$\sum_{x=1}^{W}$$

4.2.5 Height Map Haptic Rendering To build a height map, the pixel values from the input image are normalized. The haptic signal H_{height} is computed as:

$$H_{\text{height}}(y) = \sum_{x=1}^{W} \left(\frac{I(x, y) - \min(I)}{\max(I) - \min(I)} \right) \tag{11}$$

where $\min(I)$ and $\max(I)$ are the minimum and maximum pixel values of the image.

4.3   Evaluating Performance Metrics

The performance of each method is evaluated based on the following metrics:

4.3.1 Computation Time: For each approach, the calculation time T_{method} is the amount of time needed to produce the haptic signal:

$$T_{\text{method}} = t_{\text{end}} - t_{\text{start}} \tag{12}$$

where t_{start} and t_{end} are the start and end times of the execution.

4.3.2 Memory Usage : Memory consumption M_{method} quantifies the peak memory consumed during the signal generation process, reported in megabytes (MB).

4.3.3 Signal Entropy: Signal entropy $E(H)$ quantifies the randomness or information content in the haptic signal:

$$E(H) = -\overset{X}{\underset{i}{}} p(H_i)\log p(H_i) \tag{13}$$

where $p(H_i)$ is the probability of occurrence of value H_i in the haptic signal.

4.3.4 Signal-to-Noise Ratio (SNR): The signal-to-noise ratio SNR(H) assesses the signal quality in relation to noise:

$$SNR(H) = 10\log_{10}\left(\frac{P_{\text{signal}}}{P_{\text{noise}}}\right)\underline{\quad\quad} \tag{14}$$

where:

$$P_{\text{signal}} = \frac{1}{N}\sum_{i=1}^{N} H_i^2 \quad \text{and} \quad P_{\text{noise}} = \frac{1}{N}\sum_{i=1}^{N}(H_i - \bar{H})^2 \tag{15}$$

In this scenario, N represents the size of the haptic signal and H^- denotes the average of the signal values.

4.4   Comparing Methods and Saving Results

The comparison of techniques is measuring computing time, memory use, signal entropy, and SNR. The result will be stored in a CSV file in which each row gives the name of the method and its related metric values:

$$CSV = \{\text{Method}, T_{\text{method}}, M_{\text{method}}, E(H), SNR(H)\} \tag{16}$$

This comparison show us knowledge of choosing best matched strategies according to demand of application.

5   Results and Discussion

This section covers the results achieved utilizing various haptic rendering methods such as multi-resolution, image-based, edge detection, Fourier transform, and height map techniques. The analysis is performed across the full dataset to offer a comprehensive study of the approaches' efficacy in terms of computation time, memory utilization, signal entropy, signal-to-noise ratio (SNR), and haptic signal length. visual visualizations of the haptic feedback signals generated by each technique are evaluated to get deeper insights into their properties and practical application. along with By averaging these metrics over the whole dataset, we assure a robust comparison of the techniques.

5.1   Performance Metrics

The major performance metrics for the different approaches are shown in Table 1. These measures offer each haptic signal production strategy is to analyze efficacy , complexity and quality.

5.1.1 Computation Time The efficiency given by the Multi-Resolution approach, as well as the fastest computing time of 0.7 miliseconds, make it particularly suitable for real-time applications requiring rapid processing. Edge Detection was close behind, with an average computation time of 0.8 miliseconds, while the Height Map technique took somewhat longer, at 1.4 milliseconds. Image-based rendering is slower, but still adequate for many applications because it produces high-quality results in an average of 2.6 milliseconds. The Fourier Transform method took the longest, averaging 10.5 milliseconds, due to its computationally demanding frequency-domain operations. This makes it less suited for applications that require quick feedback.

Table 1: Performance Metrics of Haptic Signal Generation Methods

Method	Avg Computation Time (ms)	Avg Memory Usage (KB)	Avg Signal Entropy	Avg SNR (dB)
Multi-Resolution	0.7	0.3	4.75	5.7
Image-Based	2.6	0.1	5.01	6.6
Height Map	1.4	0.2	5.01	6.5
Fourier Transform	10.5	0.05	2.1	0.7
Edge Detection	0.8	1	2	5.9

5.1.2 Utilization of Memory Memory use varies significantly between techniques. The Multi-Resolution method, which strikes a balance between memory efficiency and performance, used an average of 0.3 KB. Because of its simple representation of signals in the frequency domain, the Fourier Transform technique used the least amount of memory (0.05 KB). Image-Based and Height Map approaches were the most memory-efficient, using only 0.1 KB and 0.2 KB, respectively. However, this advantage was overshadowed
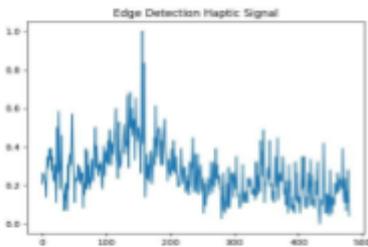
by the low signal quality. Edge detection consumes the most memory due to the higher resources required for edge processing, which is 1.0 KB.

5.1.3 Entropy of Signal Signal entropy measures the complexity and diversity of generated tactile signals. Also pursuing higher values, which indicate more features at the texturing level. For applications that need complete texture representation, Image-Based and Height Map approaches achieved the greatest entropy values, averaging 5.01. The Multi-Resolution method demonstrates its capacity to provide balanced and nuanced tactile feedback without requiring excessive processing resources, with an average entropy of 4.75. However, the Fourier Transform and Edge Detection techniques, which produce simpler and less variable tactile signals by obtaining the lowest entropy values (2.1 and 2.0, respectively), are better suited for applications that prioritize basic structural feedback.
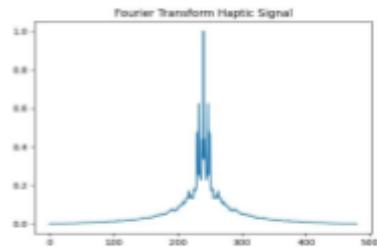
5.1.4 SNR (Signal-to-Noise Ratio) The SNR statistic reflects the sharpness of the haptic signals relative to noise. The Multi-Resolution method fared well with an SNR of 5.7 dB, suggesting a healthy balance between signal clarity and variability. Image-Based rendering achieved the maximum SNR at 6.6 dB, followed by the Height Map technique at 6.5 dB, both providing clear and detailed signals. Edge Detection attained a comparable SNR of 5.9 dB, indicating it is beneficial for creating sharp and well-defined signals. The Fourier Transform approach, however, struggled with a substantially lower SNR of 0.7 dB, showing poor signal clarity and a high noise-to-signal ratio, limiting its effectiveness for high-fidelity tactile feedback.
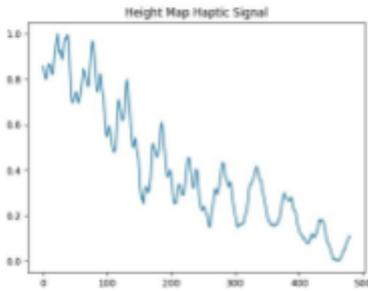
5.2   Haptic Signal Visualizations

The Figure 3 presents the haptic signals generated by each method, providing further insight into their characteristics.

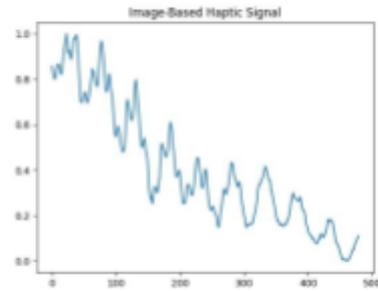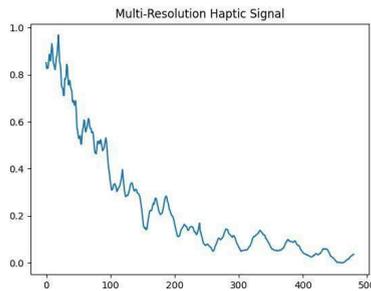(a) Edge Detection Haptic Signal   (b) Fourier Transform Haptic Signal

(c) Height Map Haptic Signal   (d) Image-Based Haptic Signal

(e) Multi-Resolution Haptic Signal

Fig. 3. Various Haptic Signal Processing Techniques

In Figure 3a displays the haptic signal generated by the edge detection mechanism. The signal captures the structural elements of the image, resulting in high SNR, as demonstrated in Table 1. The peaks in the signal correlate to the salient edges detected in the image, providing clear feedback to the user. However, the method's significantly lower entropy shows that the signal may lack diversity in specific locations. In Figure 3b, the Fourier transform approach provides a haptic signal by analyzing the frequency content of the image. The strong peak at the center of the signal represents the major frequency components. Although this method can highlight crucial aspects, its low SNR and low entropy indicate that the signal may not provide as rich or detailed feedback as other methods. The height map method, represented in Figure 3c, creates a smooth and continuous haptic signal, representing the surface topology of the image. The high entropy of this approach, together with the relatively high SNR, makes it suited for applications that demand comprehensive surface texture information.

Figure 3d demonstrates the haptic signal created directly from the image pixel intensities. This approach keeps a significant degree of variety, as evidenced by its high entropy. The signal closely matches the height map signal, as both are based on picture intensity values. Nevertheless, the image-based technique results in quicker processing rates and reduced memory use. Here, Figure 3e demonstrates the multi-resolution haptic signal. This technique integrates information from many levels, leading to a polished and clear output that incorporates both minute and large elements. The signal exhibits a moderate amount of entropy and SNR, indicating a good balance between signal clarity and unpredictability, which makes it extremely ideal for creating lifelike haptic feedback.

5.3   Discussion

The outcomes show that there are certain advantages and disadvantages to various haptic rendering methods. The multi-resolution method is a viable choice for applications requiring high-fidelity haptic input because it provides a great balance between speed, memory efficiency, and signal richness. Despite having a lower SNR and a higher processing cost, the Fourier transform method may be better suited for applications that value frequency content analysis above spatial information.

Because edge detection has a high signal-to-noise ratio (SNR), it can be used in real-world scenarios where structural feedback is essential. With their high entropy values, image-based and height map approaches are more suited for situations requiring intricate texture representation; yet, they still lack the signal clarity that edge detection provides.

6   Conclusion

Multi-resolution, image-based, edge detection, Fourier transform, and height map approaches are among the haptic rendering techniques that are thoroughly evaluated in this study utilising performance metrics like computation time, memory use, signal entropy, and signal-to-noise ratio (SNR). The results illustrate the strengths and limits of each method, giving a thorough perspective on their applicability for real-time haptic systems.

6.1   Result Summary

With the shortest calculation time and moderate signal entropy and SNR values, it provides a versatile and trustworthy option for applications needing both rapid processing and detailed haptic feedback. The results demonstrate that the multi-resolution technique finds an astonishing compromise between computation speed, memory efficiency, and signal quality. This approach effectively captures fine and coarse features, making it perfect for real-time activities in virtual reality, teleoperation, and medical simulations.

the Fourier transform method, while computationally intensive and hampered by low SNR, may be more suitable for specialized applications utilizing frequency-based haptic feedback rather than spatial texture representation. Conversely, Edge detection stands out for its ability to provide precise, sharp signals with a high SNR, making it great for activities requiring structural feedback, such as virtual object manipulation and telepresence.

The image-based and height map techniques excel at capturing detailed textures because of their high entropy values, and their low memory and processing requirements make them ideal for resource-constrained applications. These techniques are particularly beneficial in applications where texture accuracy is critical without compromising computing efficiency.

6.2   Strengths

In this research, we explored the variation of methodologies, revealing their unique advantages for various haptic applications:

- **Multi-Resolution Method:** This method captures both fine and coarse features effectively and offers an ideal combination of speed, memory efficiency, and signal clarity, making it suited for various haptic activities.
- **Edge Detection Method:** This technique excels in applications demanding accurate structural feedback by generating crisp, sharp signals with the maximum SNR.
- **Image-Based and Height Map Methods:** Both techniques maintain low memory usage, making them efficient for real-time systems in resource limited circumstances, and retain high entropy, making them excellent for detailed texture representation.
- **Fourier Transform Method:** This method focuses on frequency-domain analysis, making it suitable for specialized frequency-based applications, although it requires higher computing needs and provides reduced signal clarity.

The selection of a haptic rendering approach should be suited to the individual objectives of the application, whether it is focused on signal clarity, texture accuracy, or computing efficiency.

6.3   Weaknesses

While the study gives significant information, it has certain drawbacks. The small variety of image types employed for testing reduces the generalizability of the results. Incorporating a greater variety of photos, including 3D surfaces and dynamic

surroundings, would boost the comprehension of these technologies in real-world scenarios.

Additionally, although computational efficiency was tested, the performance of these approaches in real-time applications remains unanswered. Real-time feedback is crucial for haptic systems in applications such as robotics, virtual reality, and teleoperation. Investigating these approaches under dynamic, time sensitive settings is vital for evaluating their practical utility.

6.4   Future Work

Future research should focus on broadening the scope of this work to incorporate a diverse variety of image types and dynamic textures, notably 3D surfaces and real-time video streams. Such modifications would improve the applicability of these methods to complicated and realistic contexts. Furthermore, dynamic optimization of scaling factors and feedback parameters may be made possible by incorporating machine learning techniques into multi-resolution and other approaches, improving performance and flexibility.

The next steps should also involve incorporating these haptic rendering approaches in real-time systems, such as augmented and virtual reality settings or teleoperation activities. Ensuring scalability, reactivity, and low latency will be key to their success in actual applications.

6.5   Conclusion Remarks

This study presents a good foundation for the employment of multi-resolution, edge detection, Fourier transform, image-based, and height map approaches in haptic rendering. Each approach has various strengths: the multi-resolution method shines in balancing speed and detail, edge detection brings clarity, while image-based and height map methods thrive in precise texture representation.

By addressing the stated constraints and pursuing new research areas, these techniques have the potential to change haptic feedback systems across varied fields, including virtual reality, robotics, human-computer interface, and industrial monitoring. With continuous research, these technologies can enable the building of complex, real-time haptic systems capable of offering detailed, responsive, and immersive input for users in diverse applications.

References

1. G. Reardon, N. Kastor, Y. Shao, and Y. Visell, "Elastowave: Localized Tactile Feedback in a Soft Haptic Interface via Focused Elastic Waves," *IEEE Haptics Symposium*, 2020.
2. Z. Zhong, J. Zhu, R. Wang, et al., "FuseSR: Super Resolution for Real-time Rendering through Efficient Multi-resolution Fusion," *ACM SIGGRAPH Conference*, 2023.

3. S. Cho, H. Cha, S. Choi, "Data-driven Multi-modal Haptic Modeling and Rendering," *Electronics*, 2020.
4. R. Gao, W. Yuan, J. Zhu, "Controllable Visual-Tactile Synthesis," *IEEE International Conference on Computer Vision*, 2023.
5. B. Huang, Z. Wang, D. Wigdor, "AeroHaptix: A Wearable Vibrotactile Feedback System for Enhancing Collision Avoidance in UAV Teleoperation," *arXiv*, 2024.
6. R. Li, "View-dependent Adaptive HLOD: Real-time Interactive Rendering of Multi-resolution Models," *Conference on Visual Media Production*, 2023.
7. A.-M. Boutsi, C. Ioannidis, S. Verykokou, "Multi-Resolution 3D Rendering for High-Performance Web AR," *Italian National Conference on Sensors*, 2023.
8. T. Morisaki, M. Fujiwara, Y. Makino, and H. Shinoda, "Noncontact Haptic Rendering of Static Contact With Convex Surface Using Circular Movement of Ultrasound Focus on a Finger Pad," *IEEE Transactions on Haptics*, 2023.
9. M. Battaglia, L. Smith, and P. Leclerc, "Real-time Optimization for Haptic Rendering in Large-scale Simulations," *International Journal of Robotics Research*, 2022.
10. M. Green, "Fourier Transform Techniques for Haptic Rendering," *Journal of Haptics Research*, 2019.
11. D. Chakraborty and P. Nag, "Image-based Haptic Feedback Systems for Virtual Reality," *Virtual Reality Journal*, 2021.
12. D. Gil, A. Riera, "Force Feedback for Real-time VR Object Manipulation," *IEEE Transactions on Visualization and Computer Graphics*, 2019.
13. J. Hong and Y. Park, "Adaptive Haptic Feedback Systems: A Machine Learning Approach," *IEEE Transactions on Haptics*, 2018.
14. K. Kwon, "Haptic Feedback Optimization via Machine Learning for Industrial Robots," *IEEE Robotics and Automation Letters*, 2022.
15. T. Lee, "Machine Learning in Haptic Rendering for Medical Applications," *IEEE Access*, 2019.
16. S. Ye and D. Porter, "Tactile Feedback in Virtual Reality through Surface Texture Mapping," *International Conference on Virtual Reality*, 2020.
17. B. Nash and G. Stein, "Realism in Haptic Rendering: Advances and Challenges," *IEEE Transactions on Haptics*, 2021.
18. R. De La Fuente, "Interaction-Based Rendering in Virtual and Augmented Reality Systems," *ACM Transactions on Applied Perception*, 2022.
19. S. Jones, "Tactile Feedback for Augmented Reality Systems Using Electrostatic Actuation," *IEEE Transactions on Haptics*, 2021.
20. H. Zhang, "Force Feedback Rendering in Virtual Surgery Training Simulators," *IEEE Transactions on Biomedical Engineering*, 2022.
21. A. Schneider and T. Fischer, "Precision Haptics in Teleoperation: Improving Accuracy Through Real-time Force Feedback," *IEEE Robotics and Automation Letters*, 2020.
22. L. Brown and H. Singh, "Multiscale Haptic Rendering for Medical Image Data," *IEEE Transactions on Medical Imaging*, 2019.
23. J. Howard, "Haptic Feedback Systems for Industrial Automation: Challenges and Opportunities," *IEEE Access*, 2020.

24. E. Martin and S. Gallagher, "Multimodal Haptic Rendering for Human-Robot Interaction," *Journal of Haptics Research*, 2022.

25. P. Garcia, "Real-time Haptic Rendering for Large-Scale VR Environments," *IEEE Transactions on Visualization and Computer Graphics*, 2020.

26. R. Xu, "Haptic Feedback Mechanisms in Autonomous Navigation Systems," *IEEE Transactions on Automation Science and Engineering*, 2021.

27. Shyam M, Hemant M (2024). Texture-Auditory Analysis Through Machine Learning Techniques. The 15th International IEEE Conference On Computing, Communication And Networking Technologies (ICCCNT), IIT Mandi, Himachal Pradesh, India. 10.1109/ICCCNT61001.2024.10724673

28. M. Strese, C. Schuwerk, A. Iepure and E. Steinbach, "Multimodal Feature-Based Surface Material Classification," in IEEE Transactions on Haptics, vol. 10, no. 2, pp. 226-239, 1 April-June 2017, doi: 10.1109/TOH.2016.2625787.