Multi-agent Simulation of New Product Diffusion: Based on Gravity Theory and Directed Network

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Abstract. As the penetration rate of the mobile Internet approaches saturation, the diffusion of new products has shifted from pursuing new user growth to enhancing user activity and interaction, that is, the directed and contingent nature of online social networks. This study combines the gravitational field theory and the evolutionary game model to design a mechanism for the formation of directed and contingent networks, and uses multi - Agent simulation to study the evolutionary laws and characteristics of consumer decision - making under different network structures. The research findings are as follows: (1) The number of Hub nodes has a negative impact on the diffusion stability in the introduction period. (2) The initial degree of nodes has a positive impact on the final diffusion proportion and a significantly positive impact on the diffusion speed in the introduction period. (3) A larger proportion of out - edges will intensify the fluctuations in group decision - making during the introduction period of new product diffusion, delay the time for product diffusion to reach a stable state, and lead to a reduction in the final diffusion proportion. (4) The initial adoption proportion is positively correlated with the diffusion convergence speed, and this effect shows a marginal diminishing return. This paper introduces the gravitational field theory and directed and contingent networks into the new product diffusion model, providing a new perspective for product diffusion research.

Keywords: new product diffusion; gravity theory; directed network; multi-agent simulation

1 Introduction

As the mobile Internet spreads widely, online social networks are nearly saturated. This means new product diffusion now centers on user activity and interaction, not just new user growth. Thus, we must consider the directivity of these networks. These fields change network connections, greatly affecting new product diffusion. So, considering the impact of these fields on diffusion in directed networks from the consumers' dynamic behavior perspective is crucial.

Research on new product diffusion began with the Bass model from a macroscopic perspective[1-2], and then derivative models were developed. For example, Horsky[3] expanded it from the perspective of the potential market, and Dodson J A et al. ana-

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lyzed it from the diffusion stage[4]. The complex network theory provides a new perspective for the study of product diffusion[5]. For instance, Jiang Guoyin[6] analyzed the interaction behavior between service providers and consumer groups in a scale - free network. Multi - Agent simulation provides an entry point for the study of complex networks[7-8], extending the research on diffusion in terms of individual heterogeneity and decision - making behavior.

Establishing a social gravitational field by analogy with the gravitational field using the gravitational field theory is a new perspective for studying group behavior. For example, Meizi Li[9] constructed a group gravitational field model to analyze the evolution of group opinions. In reality, consumers' decisions are highly correlated with social relationships, and the degree of influence varies with social distance[10-11] , making the gravitational field theory applicable. The evolutionary game theory has strong analytical capabilities and compensates for the shortcomings of the gravitational field theory in describing interaction behavior[12-14].

The innovations of this paper are as follows:(1) Introduce gravitational field theory to new product diffusion, describe micro - level fields from individual interactions, and explore network's role in group decision - making evolution. (2) Based on the model, consider network directivity and propose the formation mechanism of the directed network.

In summary, based on the gravitational field theory, this paper considers the impact of the gravitational field from individual interactions on social networks. It constructs models and studies new product diffusion laws in different network topologies through simulation.

2 Model Design

2.1 Gravitational Field Theory in Social Networks

The law of universal gravitation states that an object exerts a gravitational force on other objects within its gravitational field. In this paper, we calculate the gravitational force between individuals by drawing an analogy with the universal gravitation. Specifically, we analogize the social distance between individuals to the distance between objects, and the decision - making utility of individuals and the utility difference to the mass of objects. Based on this, we propose the gravitational field theory in social networks.

In social networks, the edges between nodes represent the intimacy and interaction degree among individuals. The gravitational field changes with the variation of strategies, which in turn leads to an increase, decrease, or severance of social distance [15-16]. Social distance, in return, affects consumers' decision - making adoptions [17].

Social distance and gravitational force between individuals are negatively correlated. As social distance nears 0, gravity nears infinity; as it approaches infinity, gravity nears 0. Gravity is assumed inversely proportional to social distance squared. Based on utility maximization, individuals generate forces affecting social distance.

2.2 Evolutionary Game Process in Social Networks

Consumers in a directed network interact multiple times and update their strategies after each round of games. Thus, this paper uses evolutionary game theory to set the interaction rules for consumers.

There is a free - riding behavior during the spread of new products to non - adopters[18] . This paper stipulates that if an individual makes an adoption decision, they will obtain the utility of using the new product and share the diffusion cost. If an individual makes a rejection decision, they can also get utility from other purchasers, but there is a utility penalty. The single - game utility matrix for an individual is shown in Table 1.

Table 1. 2*2 Game Matrix for New Product Adoption Decision - making.

Consumer1	Consumer2	
	adopt	rejecter
adopt	$b-\frac{c}{nx}, b-\frac{c}{nx}$	$b-\frac{c}{nx}, b-\frac{d}{n(1-x)}$
reject	$b-\frac{d}{n(1-x)}, b-\frac{c}{nx}$	0,0

b is the individual's decision - making utility; c is the diffusion cost of the new product, shared by adopters; d is the utility penalty, shared by rejecters, n is the total number of individuals; x is the proportion of adopters. The replicator - dynamic equations are as follows:

The game equilibrium equation at the equilibrium point is:

$$b-\frac{c}{nx}=x\left(b-\frac{d}{n(1-x)}\right) \tag{1}$$

After each round of games, individuals decide whether to update their strategies. The higher the neighbor's utility, the larger the utility difference, and the smaller the social distance, the greater the probability that an individual will update their strategy.

After multiple periods of evolution and decision - making, the group game finally reaches a relatively stable state.

3 Model Construction

3.1 Construction of the Individual Adoption Decision - Making Model

To construct the individual adoption decision - making model, the following assumptions are made about new products in this paper:

1) The new products studied in this paper only consider general and abstract public attributes.

2) This paper only considers the independent diffusion of this new product and does not take into account the diffusion of other products.

3) The new products studied in this paper have no substitutes or complementary products.

According to the evolutionary game process, when an individual makes a single adoption decision, it will go through four stages: updating decision - making information, changing social distance, obtaining a psychological threshold, and making an individual decision.

(1) Updating Decision - Making Information

At this stage, under the limitation of bounded rationality, the individual weighs its own decision - making utility, calculates the utility difference with neighbors, and updates its own product preferences and neighbors' decision - making information. According to the game matrix, the individual's decision - making utility is $\mathrm{b} - \mathrm{c}/(\mathrm{nx})$ (adopt), and $\mathrm{b} - \mathrm{d}/(\mathrm{n} - \mathrm{nx})$ (reject).

(2) Changing Social Distance

The change in the utility difference between individuals leads to a change in the gravitational field, which in turn causes the social distance to increase, decrease, or be severed. The social distance between individual i and neighbor j at time t can be expressed as:

$$s_{i\to j}^{(t)} = s_{i\to j}^{(t-1)} - \frac{1}{2}\left(v_{i\to j}^{(t-1)} + G\frac{U_j - U_i}{\left[s_{i\to j}^{(t-1)} \right]^2}\Delta t \right)\Delta t \tag{2}$$

(3) Obtaining the Psychological Threshold

When decision - makers form their own strategies by referring to others' strategies, there is an egocentric bias, resulting in an advice discount effect[19].The degree of advice discount equal to the result of subtracting the minimum value of $S_{i\to j}$and then adding 1, divided by the result of subtracting the minimum value of S from the maximum value of S and then adding 2. An individual decides to learn and adopt a neighbor's strategy only when the psychological threshold is greater than zero. The psychological threshold Y_i is equal to the product of $1 - \mathrm{r}_{\mathrm{i}\to\mathrm{j}}$ and U_j, minus U_i.

(4) Making an Individual Decision

According to the replicator dynamics[20], players adjust their evolutionary strategies to improve their own utility.

3.2 Construction of the Directed Scale - Free Network Model

This paper designs a directed scale - free network model based on the BA model. The scale - free model depicts a large - scale network structure in which the degree of nodes follows a power - law distribution[21]. In real - world directed scale - free networks such as Weibo and YouTube, the network nodes are unevenly distributed, including Hub nodes with a large degree and numerous small nodes.

(1) Constructing the Initial Network: Select n_0 nodes from the n nodes contained in the network as Hub nodes, and randomly direct - connect them to other nodes to form m_0 directed edges.

(2) Node Growth Mechanism: Each period, a new node is added and randomly directed - connected to other nodes in the original network to construct m directed edges. Among them, there are $\left[um\right]$ out - edges and $m-\left[um\right]$ in - edges. u is the in - and - out - edge probability control parameter, representing the proportion of out - edges in the total edges.

(3) Preferential Attachment Mechanism:

1) Out - Edge Terminal Preference: Out - edges select nodes in the original network as the edge - connection terminals with probability u_1, and the probability of selecting node i is positively correlated with the in - degree d_i^{in} of the node.

In - Edge Starting - Point Preference: In - edges select nodes in the original network as the edge - connection starting points with probability u_2, and the probability of selecting node i is positively correlated with the out - degree d_i^{out} of the node.

4 Experiments and Result Analysis

4.1 Model Setting

In this section, an AnyLogic 8.8.3-based multi-Agent simulation experimental model is constructed. The number of nodes in the network is set, with the initial proportion of adopters being 50%. The utility parameters in the model are fixed: the initial value of product revenue is set to 10, the initial value of diffusion cost is 8, the initial value of loss revenue is 10, the initial value of the gravitational constant is 10, and the initial value of information noise is 50.

4.2 Simulation Research on New Product Diffusion in Directed Scale - Free Networks

The construction results of the directed scale - free network are shown in Figure 1. Purple and gray represent two - way and one - way edges respectively. Among them, the number of Hub nodes is negatively correlated with the connection aggregation degree of small nodes; the initial degree of nodes is positively correlated with the number of directed edges. The initial nodes are directed - connected to a large number of small nodes, which is in line with the power - law distribution characteristics of the directed scale - free network.

4.2.1 Impact of the Number of Hub Nodes on New Product Diffusion

Fix the initial node degree $m = 5$ and out - edge proportion $u = 0.5$, and adjust the number of Hub nodes. It's evident that the number of Hub nodes significantly impacts diffusion stability during the introduction phase, but has less effect after reaching a steady state (see **Fig. 1**). Firstly, it's negatively correlated with introduction - phase diffusion stability. For example, when $n_0 = 10$, the adoption - decision proportion fluctuates violently in the early stage, while it's more stable when the number is 6 or 8. Secondly, it's positively correlated with the diffusion speed in the introduction phase. For instance, when $t = 9$, the adoption - decision proportion is highest and the diffusion speed is fastest at $n_0 = 10$. This indicates that the number of Hub nodes positively affects the diffusion speed and negatively impacts the diffusion stability during the introduction phase. A large number of Hub nodes dilutes the influence of individual nodes. When consumers face Hub nodes with different attitudes, decision - making disagreements increase, leading to chaotic early - stage decisions. Thus, when formulating new product diffusion strategies, choose a network environment with a moderate number of initial nodes and a consistent overall strategy.

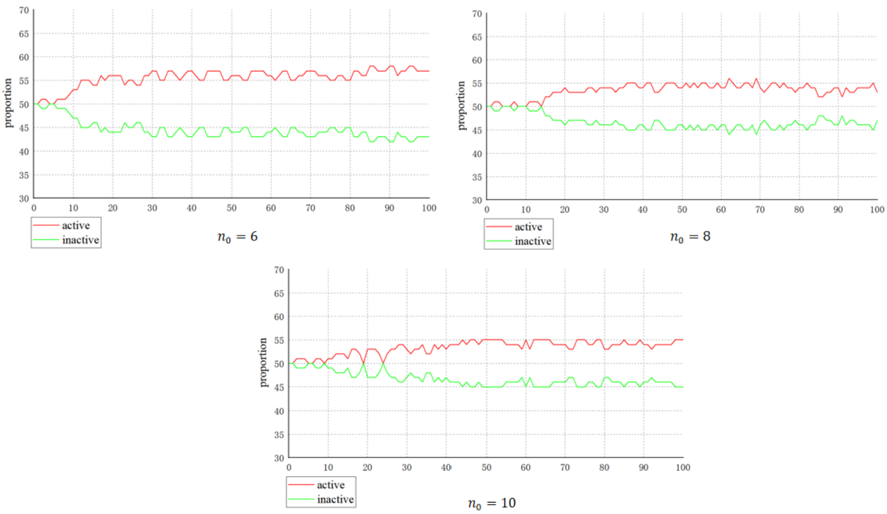

Fig. 1. The Impact of Changes in the Value of   on the Diffusion of New Products.

4.2.2 Impact of the Initial Degree of Nodes on New Product Diffusion

Fix the number of Hub nodes $n_0 = 10$ and the proportion of out - edges $u = 0.5$, and adjust the initial degree of nodes. As m_0 increases, the time required for the diffusion to reach a steady - state gradually shortens (see **Fig. 2**). For example, when m_0 takes values of 5 and 7 respectively, the time for the diffusion to reach a steady - state is 30 and 15 respectively. When $m_0 = 9$, the diffusion reaches a steady - state at

$t = 5$ .This shows that the initial degree of nodes has a negative impact on the time for the diffusion to reach a steady - state. The larger the initial degree of nodes, the more intimate relationships an individual has, the faster the decision - making speed, and the individual can end the information update stage more quickly and make an adoption decision. For example, when new users register on Weibo, they need to follow at least 4 recommended bloggers, which enables them to make adoption decisions more quickly.

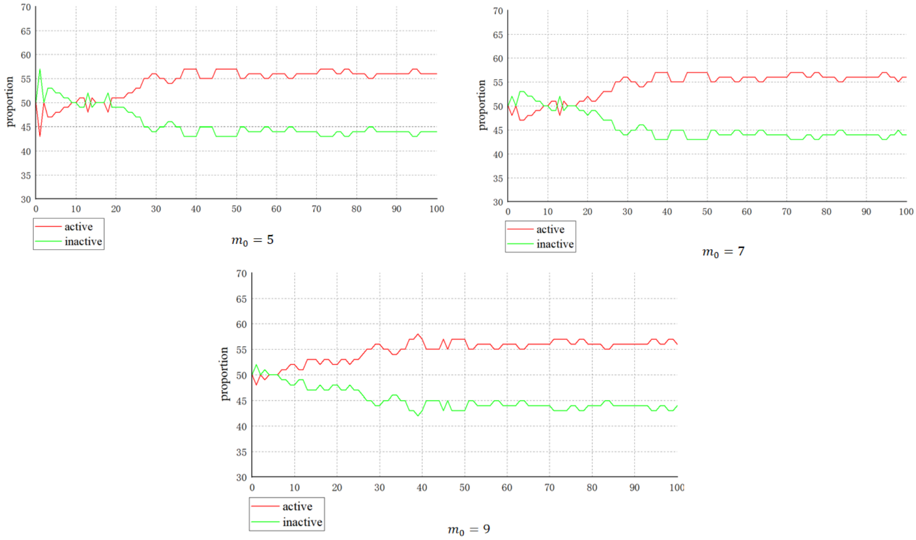

Fig. 2. The Impact of Changes in the Value of on the Diffusion of New Products.

4.2.3 Impact of the Proportion of Out - edges on New Product Diffusion

Set the number of Hub nodes $n_0 = 8$ and the initial degree of nodes $m = 5$, and adjust the proportion of out - edges. As u increases, the diffusion stability during the introduction period decreases, and the time required to reach a steady - state increases (see **Fig. 3**). For example, when u is 0.5, 0.7, and 0.9 respectively, the time to reach a steady - state is extended to 10, 30, and 55 in sequence, and the fluctuation range increases significantly. This indicates that increasing the proportion of out - edges will reduce the diffusion stability during the introduction period, extend the time to reach a steady - state, and reduce the final diffusion proportion. When the proportion of out - edges is large, the difference between Hub nodes and ordinary nodes is obvious. When individuals make adoption decisions, the decision - making information mostly depends on Hub nodes, which is likely to cause fluctuations in group decision - making. When formulating a new product diffusion strategy, multiple parties can be encouraged to evaluate the new product, increase the information available to potential customers, and reduce decision - making fluctuations.

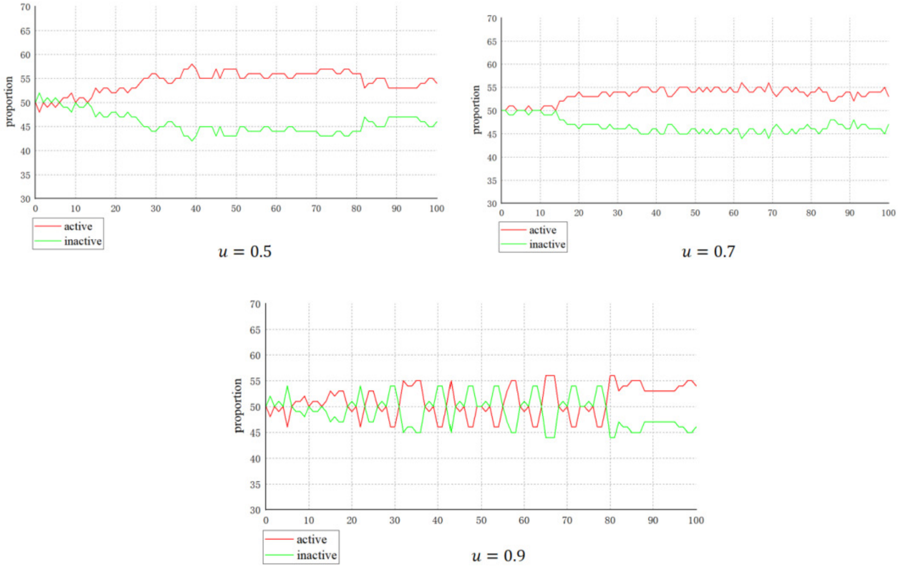

Fig. 3. The Impact of Changes in the Value of on the Diffusion of New Products.

4.2.4 Impact of the Initial Adoption Proportion on New Product Diffusion

Fix the number of Hub nodes $n_0 = 8$, the initial degree of nodes $m = 5$, and the proportion of out - edges $u = 0.5$, and adjust the initial adoption proportion. Randomly select $q \times n_0$ initial nodes to maintain an adoption attitude, fix $U_i = b$, and randomly generate initial strategies for the remaining nodes. As shown in **Fig. 4**, as q increases, the time required for the diffusion to reach a steady - state gradually shortens, but there is a marginal diminishing effect. For example, when q is 0, 0.3, 0.6, and 0.9 respectively, the diffusion reaches a steady - state at 30, 20, 10, and 10 respectively. When it increases from 0.6 to 0.9, the effect of shortening the convergence time is not obvious. This shows that the initial adoption proportion is positively correlated with the diffusion convergence speed, and this effect has a marginal diminishing property. In reality, selecting an appropriate number of seed users with high stickiness and loyalty can help promote the product. However, if there are too many such users, it may make consumers bored. During the product introduction period, enterprises should select an appropriate number of seed users.

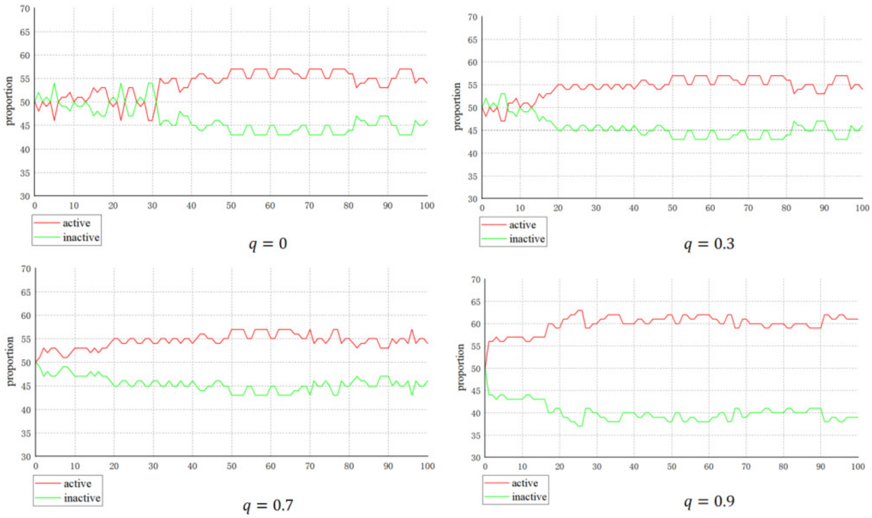

Fig. 4. The Impact of Changes in the Value of on the Diffusion of New Products.

5 Conclusions and Implications

In this paper, by combining the gravitational field theory and the evolutionary game theory, a formation mechanism of directed contingency networks is proposed, and a new product diffusion model based on multi - Agent simulation under different network structures is established.

Specifically, the gravitational field theory is introduced to describe the mechanism of changes in social distance among individuals. The evolutionary game theory is integrated to depict the interactions among individuals and construct an individual product adoption game model, and the gravitational field generated by individual interactions is studied as the formation mechanism of directed contingency networks. Multi - Agent simulation experiments are carried out under different network structures, and the effectiveness of the model is verified by comparison with real - world cases.

The analysis of simulation results shows that: (1) The number of Hub nodes has a negative impact on the diffusion stability in the introduction period. (2) The initial degree of nodes has a positive impact on the final diffusion proportion and a significantly positive impact on the diffusion speed in the introduction period. (3) A larger proportion of outgoing edges will intensify the fluctuations in group decision - making during the introduction period of new product diffusion, delay the time for product diffusion to reach a steady state, and lead to a reduction in the final diffusion proportion. (4) The initial adoption proportion is positively correlated with the diffusion convergence speed, and this effect shows a marginal diminishing effect.

Of course, there are also some limitations in this paper. In particular, this paper conducts research based on short - term dynamic networks, while in the long - term, significant changes may occur in network nodes and connections. In - depth research on new product diffusion in directed contingency networks from a long - term perspective can be carried out.

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