Regression Analysis and Inversion Methods for Ground Stress Field Modeling in Underground Engineering

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Abstract. Accurate ground stress field modeling is essential for the design and construction of underground engineering projects. This paper presents a regression-based method for ground stress field inversion, employing least squares fitting to enhance precision and efficiency in stress field modeling. By selecting appropriate linear and nonlinear stress functions, this approach allows for adaptable modeling depending on geological complexity. A detailed case study is presented to demonstrate the application and effectiveness of this method, verifying its feasibility in underground engineering.

Keywords: ground stress field inversion, regression analysis, underground engineering, modeling

1 Introduction

Ground stress field modeling plays a crucial role in ensuring the safety and stability of underground engineering projects, such as tunnels and mines[1-3]. Given the geological complexity of underground formations, accurately modeling ground stress is a challenging but vital task[4-5]. Traditional methods rely heavily on extensive field measurements and empirical formulas, which often lack efficiency and precision[6].

This paper proposes a regression-based approach to model and invert the ground stress field by fitting stress functions using field measurements. This approach uses least squares fitting to enhance model precision while simplifying calculations. The methodology is implemented in MATLAB, making the process efficient and minimizing human errors during subroutine creation.

2 Mathematical Modeling

2.1 Choice of Stress Function

The selection of an appropriate stress function is crucial for modeling accuracy. The choice of stress functions can be either linear or nonlinear, depending on the geological conditions:

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1. Linear Stress Function: Suitable for relatively simple geological conditions with uniform stress distribution:

$$\sigma(x, y, z) = a \cdot x + b \cdot y + c \cdot z + d \tag{1}$$

where $\sigma(x, y, z)$ is the stress at point (x, y, z), and a, b, c, and d are coefficients determined using regression.

2. Nonlinear Stress Function: For complex geological formations where the stress field is highly variable:

$$\sigma(x, y, z) = a \cdot x^2 + b \cdot y^2 + c \cdot z^2 + d \cdot xy + e \cdot yz + f \cdot zx + g \tag{2}$$

Coefficients (a, b, c, d, e, f, g) are obtained using least squares regression, allowing for the flexibility needed to handle complex geological conditions.

2.2 Least Squares Fitting

Least squares fitting minimizes the sum of squared residuals between measured stress data and the stress function. Here is an example of the least squares fitting process using MATLAB[7]:

```matlab
% Define measured coordinates and stress values
coordinates = [x1, y1, z1; x2, y2, z2; ...];
measured_stress = [sigma1; sigma2; ...];
% Set up design matrix for linear regression
A = [coordinates, ones(size(coordinates, 1), 1)];
% Compute coefficients using least squares
coefficients = (A' * A) \ (A' * measured_stress);
```

For nonlinear functions, iterative methods such as the Levenberg-Marquardt algorithm can be used to improve fitting accuracy. Residual analysis is then performed to evaluate the precision of the model.

3 Application and Case Study

3.1 Determination of Buried Depth of Measurement Point *H*

To accurately model the ground stress field, determining the buried depth H of each measurement point is essential. Based on the node coordinates of the surface x, y, z, the following fitting equation is obtained:

$$z = b_1x^3 + b_2y^3 + b_3x^2y + b_4xy^2 + b_5x^2 + b_6y^2 + b_7xy + b_8x + b_9y + b_{10} \tag{3}$$

where $(b_1, b_2, \ldots, b_{10})$ are the coefficients to be fitted.

The fitted surface equation is given by:

$$z = -1.71 \times 10^{-9}x^3 + 1.98 \times 10^{-7}y^3 + 6.35 \times 10^{-8}x^2y + 2.19 \times 10^{-7}xy^2 - 7.58 \times 10^{-5}x^2 + 1.12 \times 10^{-4}y^2 + 2.39 \times 10^{-4}xy - 0.09x - 0.45y + 216.68(\text{m}) \tag{4}$$

By substituting the x and y coordinates of each node into Equation (4), the corresponding z coordinate value can be obtained. By subtracting this z value from the original z, the buried depth H of each node can be determined.

3.2 Fitting the Stress Function σ_x

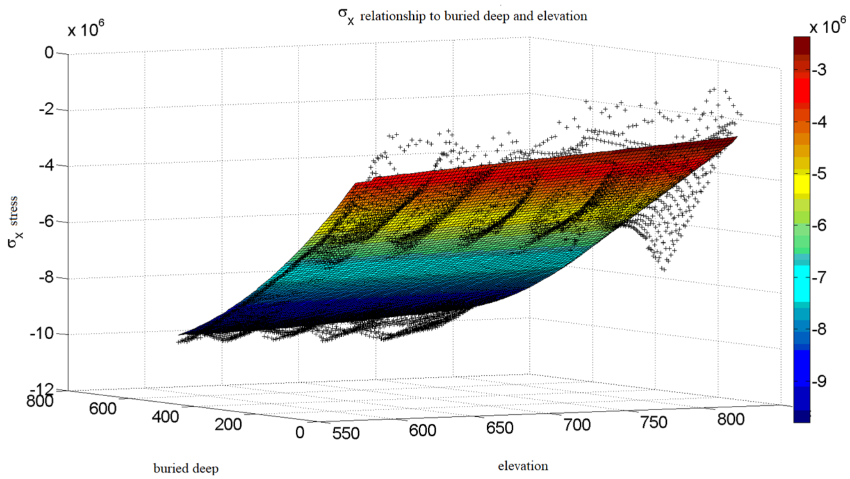
Using MATLAB, the stress component surface σ_x shown in Figure 1 was fitted, resulting in a correlation factor of 0.904. The fitted function is given as:

$$\sigma_x = -5.8179 - 0.0167H + 9.7960 \times 10^{-6}H^2 + 4.1956 \times 10^{-3}Z$$

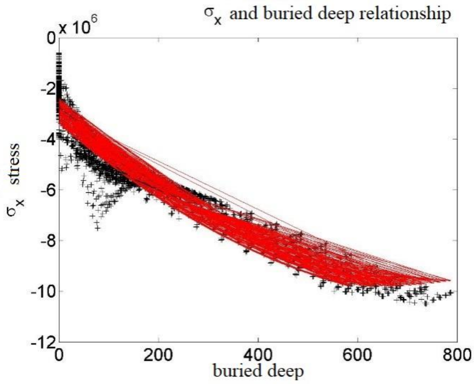
where:

- H is the buried depth of the measurement point in meters.
- Z represents the surface elevation corresponding to the calculated point in meters.
- The unit of stress is MPa.

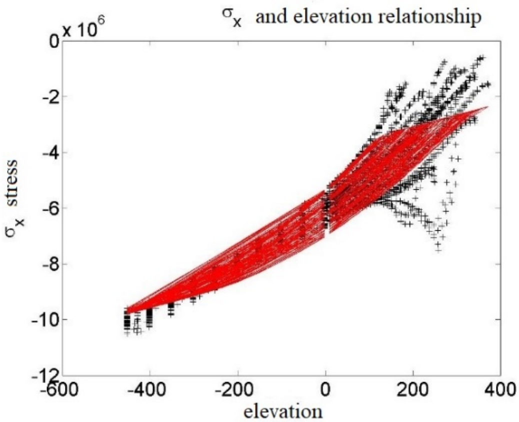
In Figure 1, the black points represent the original node values, while the curved surface represents the fitted equation. The proximity between the fitted curve and the black points indicates a successful fit.



- (a) Three-dimensional fitting surface.



- (b) Projection of the three-dimensional surface along the elevation axis.



- (c) Projection of the three-dimensional curved surface along the buried depth axis.

Fig. 1. Illustrates the fitted surface for σ_x:

4 Results and Discussion

4.1 Advantages of the Regression-Based Approach

The proposed regression-based method for ground stress field modeling offers several advantages:

- Improved Accuracy The regression-based fitting accurately represents the measured data, minimizing residual errors.

- Dynamic Monitoring Capability: This method is more effective than empirical models at capturing real-time stress variations.

- Computational Efficiency: Using MATLAB for data processing significantly enhances computational speed and reduces manual effort.

4.2 Limitations

- Dependence on Data Quality: The accuracy of the regression analysis depends on the quality and range of field measurements.

- Model Selection Complexity: In complex geological conditions, selecting an appropriate stress function can be challenging.

5 Conclusions

This paper presents a regression-based approach for ground stress field modeling and inversion in underground engineering. The proposed method was validated through a detailed case study, demonstrating its feasibility and advantages in terms of accuracy and efficiency. The approach allows for dynamic monitoring and provides reliable data for design and construction processes. Future work will focus on integrating this approach with machine learning techniques to improve model performance under complex geological conditions.

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