



A Method for the Calculation of Discharge through Check Gate Based on Biharmonic Spline Interpolation and Dimensional Analysis

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Abstract. The calculation for the discharge through check gate is a crucial reference for scheduling operations, and it also constitutes a significant internal boundary condition in hydraulic models. In the actual operation of open channel water transfer projects, although a large amount of real-time water situation data is accumulated during dispatching operation, due to the influence of control mode and water demand, the monitoring data values are mostly concentrated in a small local range in practice, and it is difficult to obtain representative data covering a wide range, resulting in insufficient generalization ability of the calculation model of discharge through check gate. To solve the above problems, this paper proposes a method for the calculation of the discharge through check gate based on the coupling of biharmonic spline interpolation and dimensional analysis. The method of biharmonic spline is used to increase the representativeness of the monitoring data, and the method of dimensional analysis is used to calculate discharge through check gate. The paper takes the Gang-tou check gate and Xi Heishan check gate of the Middle Route of South-to-North Water Transfer Project as the research object, and comparative analysis was conducted on the determination coefficients and calculation accuracy of measured samples and fitted curves before and after the coupling of biharmonic spline interpolation and dimensional analysis. The results show that compared to the single dimensional analysis method, the calculation method for the calculation of discharge through check gate based on the coupling of biharmonic spline interpolation and dimensional analysis effectively improves the calculation accuracy in the absence of representative data. The fitting coefficient of the relationship curve increases by about 8%, the absolute and relative errors of the calculation results of the gate flow have been reduced.

Keywords: Biharmonic Spline Interpolation; Dimension Analysis; Open Channel Water Diversion Project; Discharge through check gate.

1 Introduction

Check gate is the critical control structure in open channel water transfer project, where adjusting the gate opening to achieve targeted water levels before check gate and discharge through check gate. The calculation of discharge through check gate is a fundamental function of hydrodynamic simulation models, and is also an essential part of hydraulic calculation for water transfer structures and control structures^[1-2]. These calculations serve as the basis for operational staff to control water level and discharge through check gate, plays a crucial role in ensuring the safety and scientific management of open channel water projects^[3]. With the comprehensive advancement of digital twin watershed and engineering construction, the calculation of discharge through check gate has become an integral hydraulic model in digital twin modeling^[4], where its accuracy significantly impacts the application of digital twin construction in water transfer project.

Currently, for different flow forms through check gate such as orifice flow, weir flow, free outflow, and submerged outflow in open channel water transfer projects, the common methods for calculating discharge through check gate based on the energy equation^[5-7], the formulas from Nanjing Hydraulic Research Institute^[8], the Henry formula^[9], and the empirical formula of Wuhan university^[10], and the formula for discharge through check gate based on dimensional conservation^[10], and Cui^[11] conducted a comparative analysis for the calculation of discharge through check gate using energy equation and dimensional analysis method in the Middle Route of South-to-North Water Transfer Project. Furthermore, black-box models basing data driving^[12], such as artificial neural networks, random forests, and genetic programming, have been developed to establish nonlinear relationship between water level, gate opening of check gate, discharge through check gate, and apply the established nonlinear relationship to calculate the discharge through check gate.

While empirical formulas for calculating discharge through check gate are widely used, they have limitation in application condition and parameter calibration. Hydraulic parameters calibrating such as discharge coefficient and submersion coefficient in empirical formulas is often complex, and these methods can vary significantly in accuracy under different operational conditions. The calculation of discharge through check gate basing on dimensional analysis generally offer higher accuracy than empirical methods, with simpler processes of parameter calibration. However, they still require a large amount of representative data for parameter calibration, and the fitting results can be significantly affected under the conditions of insufficient representative samples. With the advancement of modern intelligent technologies, numerous black-box models basing data driving have achieved good fitting accuracy under certain conditions. However, purely mathematical models abandon physical mechanisms and often have weak generalization abilities, the prediction effect can be poor, or even the prediction value is theoretically incorrect if the simulation data significantly exceed the training sample range. In conclusion, combining mechanistic models and black-

box models effectively, leveraging their respective strengths, can achieve better results. However, in practical operational scheduling, although a large amount of hydrological data can be accumulated, the water level and discharge through check gate are often controlled within a narrow range due to the control mode and actual user demands of water transfer project. This can lead to non-representative hydrological data, directly impacting the effectiveness of parameter calibration.

The Middle Route of South-to-North Water Transfer Project serves as a pilot project for digital twin engineering. The paper focuses on the typical check gate of the main canal in the Middle Route of South-to-North Water Transfer Project, employing both mechanistic and data driven approaches to systematically study the issue of calculation of discharge through check gate in actual operational scheduling. Since its full operation in December 2014, the project has accumulated extensive data on upstream and downstream water level, discharge through check gate, and check gate opening. The design plan maintains constant upstream water level before check gate (constant downstream water level after canal), meaning that most collected data represent conditions near the target water level, with significant deviations being rare. Furthermore, due to water demand in water receiving areas along the route, monitored data often concentrate within a limited local range, making it challenging to obtain broadly representative data of discharge through check gate.

To address these issues, particularly the lack of representative water level data and discharge data under the control model of the constant water level before check gate in open channel water transfer projects, the paper proposes a calculation model of discharge through check gate based on biharmonic spline interpolation and dimensional analysis. This model combines the strengths of both approaches: first, using biharmonic spline interpolation to fully utilize the positional information of existing data for interpolating hydrological data, thus enhancing the representativeness of the sample data; and then employing dimensional analysis, which allows for high accuracy and easy parameter calibration, to establish a control gate flow calculation model. By comparing the accuracy of the biharmonic spline interpolation and dimensional analysis with that of the dimensional analysis alone, the effectiveness of the model is validated, and the paper provides a scientific basis for the operational scheduling of Middle Route mainline project.

2 Method

2.1 Biharmonic Spline Interpolation Method

The biharmonic spline interpolation method, introduced by American scholar Sandwell in 1987, utilizes Green's functions as its base functions to interpolate minimal curvature surfaces based on known points^[13]. The interpolation surface is formed by a linear combination of Green's functions centered around each data constraint, with the relative intensities of these points determined by solving a system of quadratic linear equations. This method boasts high interpolation accuracy, simple calculations, and flexible application.

Step 1: Calculate the distances between known points and the desired output point.

Assuming a surface has n known points x_i , and the desired output point is p_0 , interpolated using the k closest known points x_i to p_0 , the formula for calculating the distance from the desired output point p_0 to the known point x_i is:

$$r_{oi} = |x_i, x_0|, \quad i = 1, 2, \dots, n. \quad (1)$$

Where r_{oi} is the distance from the desired output point p_0 to the i_{th} known point; x_0 is the position vector of the desired output point p_0 ; x_i is the position vector of the i_{th} known point.

Step 2: Calculate the weight matrix.

Set up the coordinate matrix for known points as $\mathbf{X}=[x_1, x_2, \dots, x_k]$, $\mathbf{Y}=[y_1, y_2, \dots, y_k]$, and the attribute matrix $\mathbf{Z}=[z_1, z_2, \dots, z_k]$. The $k \times k$ Green's matrix $\mathbf{G}$ can be described as:

$$\mathbf{G} = \begin{bmatrix} d_{11} & d_{12} & \cdots & d_{1k} \\ d_{21} & d_{22} & \cdots & d_{2k} \\ \cdots & \cdots & \ddots & \vdots \\ d_{k1} & d_{k2} & \cdots & d_{kk} \end{bmatrix}. \quad (2)$$

Where d_{ij} is the distance between known points, calculated as:

$$d_{ij} = \begin{cases} 0, & r_{ij} = 0, \\ \left[\ln(r_{ij}) - 1.0 \right] r_{ij}^2, & r_{ij} \neq 0, \end{cases} \quad i = 1, \dots, k, j = 1, \dots, k. \quad (3)$$

The r_{ij} is the distance projected by any two known points on the xy plane, and its calculation formula is:

$$r_{ij} = \sqrt{(x_i - x_j)^2 - (y_i - y_j)^2}. \quad (4)$$

The weight matrix $\mathbf{W}$ is calculated using the Green's function matrix $\mathbf{G}$ and the attribute matrix $\mathbf{Z}$:

$$\mathbf{W} = \mathbf{G}^{-1} \mathbf{Z}. \quad (5)$$

Step 3: Calculate the attribute values of the desired output point. The $1 \times k$ Green's matrix $\mathbf{G}_p$ for the desired output point can be described as:

$$\mathbf{G}_p = [d_{01}, d_{02}, \dots, d_{0k}]. \quad (6)$$

Where $d_{01}, d_{02}, \dots, d_{0k}$ are obtained from the previous calculation (Step 2)

The attribute value z_p for the desired output point p_0 is calculated as:

$$z_p = \mathbf{G}_p \mathbf{W}. \quad (7)$$

Where W is the weight matrix obtained from Step 2.

Step 4: Repeat Steps 1-3 until attribute values for all desired output points are obtained (z_{pi}).

2.2 Dimensional Analysis method

Dimensional analysis is a technique where the correct application helps in formulating, analyzing, and solving mathematical problems^[14-15]. It is entirely based on the dimensions of variables and parameters, which are closely related to the complete statement of the problem. Dimensional analysis leads to a transformation of the original problem statement, typically using dimensionless variables and parameters, resulting in an equivalent statement with fewer dimensionless entities. These entities are a systematic assembly of the dimensioned variables and parameters, allowing for a simplified representation that more deeply reflects the inherent relationships between physical quantities. Therefore, the results of dimensional analysis are easier to express succinctly. Some scholars in the 1980s first derived formulas for calculating discharge through check gate using dimensional analysis, highlighting the method's clear physical concepts and practicality. Considering that the single-width discharge is a function of gate width, gravity acceleration, head difference, and absolute viscosity, its functional relationship can be described as follows:

$$q = f(e, g, H_e, \mu), \quad (8)$$

$$q = m(e^a, g^b, H_e^c, \mu^d). \quad (9)$$

Where q is the flow rate per unit width (m^2/s), e is the gate opening (m), μ is the absolute viscosity ($Pa \cdot s$), g is the acceleration due to gravity (m/s^2), and H_e is the head difference between upstream and downstream (m).

Using dimensional analysis:

$$(q^2/g)^{1/3} = m^{2/3} e (H_e/e)^{2c/3}. \quad (10)$$

Reduced formula by simplification:

$$K/e = i (H_e/e)^j. \quad (11)$$

Where $K = (q^2/g)^{1/3}$, m;

Following the typical format for calculating gate flow, the formula derived from dimensional analysis can be simplified to:

$$Q = B \sqrt{g \left[e i (H_e/e)^j \right]^3} \quad (12)$$

Following the typical format for calculating gate flow, the formula derived from dimensional analysis can be simplified to:

$$\lg(K/e) = j \lg(H_e/e) + \lg(i) \quad (13)$$

Where i, j is the discharge coefficient, respectively.

$$y = kx + l \quad (14)$$

Where $y = \lg(K/e), x = \lg(H_e/e), b = \lg(i)$; k and l are the slope and intercept, respectively.

For submerged outflow:

$$H_e = z_{up} - z_{down} \quad (15)$$

Where z_{up} and z_{down} are the water levels upstream and downstream of the gate, respectively.

For free orifice flow:

$$H_e = z_{up} - z_B \quad (16)$$

Where z_B is the crest height of the gate chamber.

2.3 Method Based on Biharmonic Spline Interpolation and Dimensional Analysis

As the operating conditions of open-channel water transfer projects continuously evolve, the accumulated hydrological monitoring data becomes more comprehensive. However, these projects often operate under a single control mode, such as constant upstream and downstream levels. In the case of the Middle Route of South-to-North Water Diversion Project, which consistently employs a constant upstream water level control mode to ensure smooth operation, the monitored data represent conditions near the target water levels, thus lacking significant deviations. Moreover, during the actual water supply process, user demand follows certain patterns, limiting the representativeness of the measured discharge through check gate. To address the lack of hydrological monitoring data at check gates, this paper first uses the formulas of the biharmonic spline interpolation method (1)-(7) to interpolate the existing measured data, calculating the attributes of all desired water points (z_{pi}). By utilizing the biharmonic spline interpolation, the positional information of the limited sample points is fully leveraged, enhancing the representativeness of the hydrological data samples, resulting in richer and more representative sample data. Then, using the formulas of dimensional analysis (8)-(16), the coefficients i and j are calibrated, ultimately providing a practical formula for calculating discharge through check gate.

This article assumes that the monitoring error of water situation data has no impact on the calculation results of the method proposed in this article.

3 Case Study

The South-to-North Water Diversion Project is a major national initiative, planned with eastern, middle, and western routes. Both the eastern and middle routes have been operational and have shown significant comprehensive benefits. The Middle Route of this project diverts water from the Danjiangkou Reservoir to provinces such as Henan, Hebei, Beijing, and Tianjin, covering a total length of 1,432 km. The project primarily uses open channel water conveyance with 64 control gates, 97 water diversion ports, and 54 backwater gates distributed along its length. The Middle Route functions through gravity flow without inline storage reservoirs, relying on the coordinated operation of over 500 gates to achieve its water supply objectives, posing substantial challenges in scheduling technology. As a vital means for hydraulic regulation, precise analysis and calculation of gate flow are crucial. The Middle Route also serves as a pilot for digital twin project construction, where gate flow calculation is a key hydraulic model for achieving non-steady flow simulations and precise pre-plays. This paper examines Gangtou check gate and Xi Heishan check gate in the Beijing-shijiazhuang section of the Middle Route as study sites, with a schematic diagram of the flow through these control gates shown in Fig. 1.

In Fig. 1, e represents the check gate opening; H is the water depth before the gate; H_0 is the head before the gate (composed of water depth and velocity head), v_0 is the water velocity before the gate; α is the velocity distribution coefficient, h_s is the water depth after the gate, g is the acceleration due to gravity, r is the radius of the arc gate.

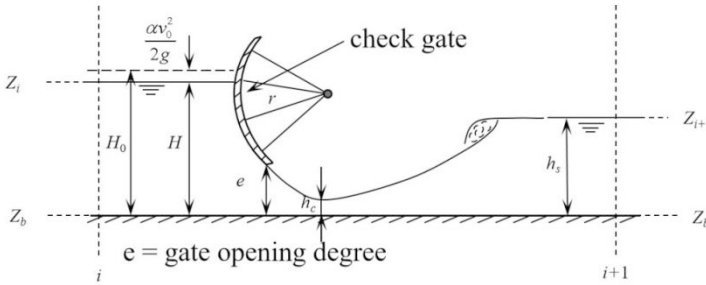


Fig. 1. Schematic diagram of the discharge form at check gate in the Middle Route of South-to-North Water Diversion Project

The onsite hydrological data collection equipment has a certain data filtering capability, performing a primary filter of raw hydrological data before transmitting it to the gate station monitoring system (automated scheduling system). The monitoring system then applies a second filtering based on set methods, ensuring the authenticity and reliability of the measured hydrological data. The hydrological data used in this paper have undergone two levels of filtering by onsite automated equipment and the monitoring system, making the data stable and reliable.

Gangtou check gate features two arc gates, each with a width of 7.8 m, a design water level of 65.99 m before the gate, a bottom elevation of 60.59 m, and a design water depth of 5.40 m, with a design flow rate of 125 m³/s. For this study, 37 repre-

sentative sets of actual hydrological data were selected, including upstream water level, downstream water level, check gate opening, and discharge through check gate. The range of measured gate openings was 0.6 to 3.3 m, the difference between upstream and downstream water levels was 0.7 to 2.1 m, and the discharge varied from 30 to 120 m³/s.

Xi Heishan check gate has three arc gates, each with a width of 5 m, a design water level of 65.28 m before the check gate, a bottom elevation of 60.78 m, and a design water depth of 4.50 m, with a design discharge of 100 m³/s. For this gate, 42 representative sets of actual hydrological data were chosen, including similar measurements to those at Gangtuo check gate. The range of check gate openings was 0.5 to 3.5 m, the difference in water levels was 0.1 to 1.4 m, and the discharge ranged from 27 to 85 m³/s.

First, to minimize the potential negative impacts of anomalous sample data, the 37 sets of actual hydrological data were normalized (preprocessed), confining the normalized data within the range of [0,1]. Next, these 37 sets of normalized data were used as input for the biharmonic spline interpolation model, detailed in formulas (1) to (7). The interpolation results were then denormalized, providing an additional 36 sets of interpolated data.

Fig.2 shows the interpolation results, where it is evident that the actual hydrological data points at Gangtuo control gate are evenly distributed on the three-dimensional smooth surface generated by the biharmonic spline interpolation. The relationship between gate flow, gate opening, and the difference in water levels before and after the gate is notably correlated. Specifically, if the water head difference remains constant, the flow through the gate increases as the gate opening widens. Conversely, if the gate opening remains constant, the flow increases with an increase in the water head difference. There is not a 'one-to-one' correspondence between discharge and check gate opening or water level difference; rather, there is a "one-to-many" relationship where the same discharge can result from combinations of 'smaller gate openings' with 'larger head differences' or 'larger gate openings' with 'smaller head differences'.

Subsequently, the 37 sets of original data and the 36 sets of interpolated data, totaling 73 sets, were input into the dimensional analysis model, detailed in formulas (8) to (16). This yielded $\lg(K/e)$ and $\lg(\Delta h/e)$, plotted on a scatter diagram with $\lg(\Delta h/e)$ on the x-axis and $\lg(K/e)$ on the y-axis, leading to the regression results seen in Fig.2.

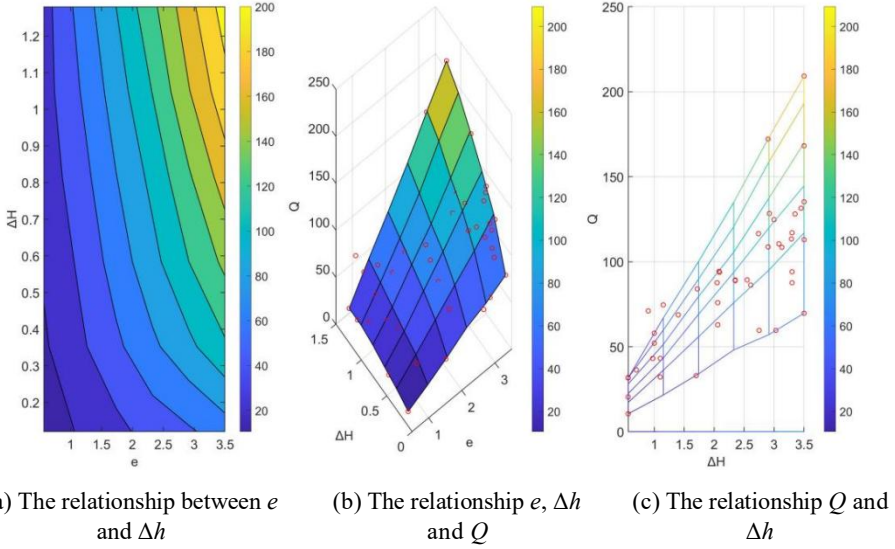


Fig. 2. Interpolation results of measured data based on biharmonic spline interpolation

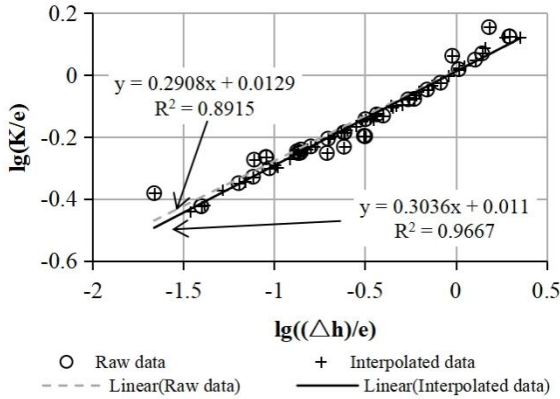


Fig. 3. Fitting Curve of the check gate relationship based on biharmonic spline interpolation and dimensional analysis

From Fig.3, it is apparent that the fitting formula for the flow relationship curve based on single-dimensional analysis is $y=0.2908x+0.0129$, with flow coefficients $i=1.0300, j=0.2908$, and a determination coefficient R^2 of 0.89. The fitting formula for the flow relationship curve based on both biharmonic spline interpolation and dimensional analysis is $y=0.3036x+0.0129$, with flow coefficients $i=1.0257, j=0.3036$, and a determination coefficient R^2 of 0.97. The 36 sets of interpolated data are uniformly distributed near the fitting curve. Compared to the fitting curve based on single-dimensional analysis, the proposed method combining biharmonic spline interpolation and dimensional analysis enhances the determination coefficient from 0.89 to 0.97, an

increase of about 8%. This indicates that considering the distribution of actual samples, the combined application of biharmonic spline interpolation and dimensional analysis can improve the fitting accuracy of the dimensional analysis-based model to a certain extent.

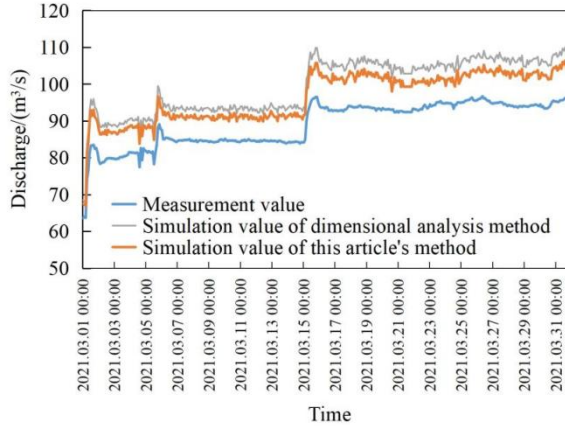


Fig. 4. Calculation results of discharge through check gate based on biharmonic spline interpolation and dimensional analysis

Lastly, to test the practical application effect of the discharge relationship calculation method based on biharmonic spline interpolation and dimensional analysis, measured data from March 2021, showing relatively significant variations in gate flow at Gangtou check gate (ranging from 65 to 97 m^3/s) and Xi Heishan check gate (ranging from 31 to 51 m^3/s), were selected as verification samples. These were used to compare the calculation accuracy of the proposed method against the traditional dimensional analysis method. The calculation results for Gangtou check gate are shown in Fig.4, with the accuracy detailed in Table 1.

Table 1. Calculation accuracy of discharge through check gate based on biharmonic spline interpolation and dimensional analysis

Station	Method	Validation sample size	absolute error (m^3/s)		relative error/%	
			mean	maximum	mean	maximum
Gangtou check gate	Dimensional analysis	366	10.4	14.0	11.7	15.0
	The paper method		7.5	10.4	8.4	11.5
Xi Heishan check gate	Dimensional analysis	366	11.3	15.4	12.7	15.6
	The paper method		6.7	12.4	7.8	11.0%

From Table 1, it can be seen that compared to the single dimensional analysis method, the gate flow calculation method proposed in this paper, which utilizes both

biharmonic spline interpolation and dimensional analysis, achieves higher calculation accuracy. For Gangtou check gate, the average absolute error in flow calculation results decreased from 10.4 m³/s to 7.5 m³/s, and the maximum absolute error reduced from 14.0 m³/s to 10.4 m³/s. The average relative error decreased from 11.7% to 8.4%, and the maximum relative error decreased from 15.0% to 11.5%. For the Xi Heishan check gate, the average absolute error reduced from 11.3 m³/s to 6.7 m³/s, the maximum absolute error dropped from 15.4 m³/s to 12.4 m³/s, the average relative error went from 12.7% to 7.8%, and the maximum relative error from 15.6% to 11.0%. This demonstrates that the method proposed in this paper can significantly enhance the accuracy of flow calculations compared to the single dimensional analysis method.

To study the impact of measurement errors on the accuracy of the calculation method, considering the measurement requirements of South-to-North Water Diversion Middle Route project, typically, the gate opening measurement error does not exceed 20 mm, and the water level measurement error does not exceed 2 cm. In this paper, with a gate opening e of 1 m and a water level difference Δh of 1 m as the benchmark, the gate opening measurement error is ± 0.02 m, and the water level difference error is ± 0.02 m. The results of calculations under different measurement errors are shown in Table 2.

Table 2. Impact of measurement errors on calculation results of discharge through check gate

e/m	$\Delta h/m$	$Q/(m^3/s)$
1.00	1.00	50.75
1.02	1.00	51.81
0.98	1.00	49.69
1.00	1.02	51.21
1.02	1.02	52.28
0.98	1.02	50.14
1.00	0.98	50.29
1.02	0.98	51.34
0.98	0.98	49.24

From Table 2, it can be observed that when the gate opening e is 1 m and the water head difference measurement error is ± 0.02 m, the flow calculation results range from 50.29 m³/s to 51.21 m³/s; when the water head difference Δh is 1 m and the gate opening measurement error is ± 0.02 m, the flow calculation results range from 49.69 m³/s to 51.81 m³/s. In summary, the maximum impact of the measured water head difference error on the flow calculation results is up to ± 0.92 m³/s, while the maximum impact of the measured gate opening error on the flow calculation results is up to ± 1.42 m³/s. Under the same measurement error conditions, the water head difference measurement error has a greater impact on the calculation results compared to the measurement error of check gate opening.

4 Conclusion

Check gate is extensively used in open channel water transfer project, and its precise calculation of discharge through check gate is crucial for ensuring the safe operation and effectiveness of these projects. In the construction of digital twin projects, the calculation model of discharge through check gate is one of the fundamental hydraulic models, playing a significant role. Currently, various the calculation models of discharge through check gate have been widely used in practice, but whether the mechanistic models or data driving models are all require a substantial amount of representative hydrological data for parameter calibration or model structuring. In actual operational scheduling, although a large amount of hydrological data can be accumulated, the representativeness of the data is often lacking due to factors such as water conveyance control methods and user water demand, leading to insufficient generalization capabilities of the models.

This paper focuses on Gangtuo check gate in the Middle Route of South-to-North Water Transfer Project and proposes a calculation method of discharge through check gate basing on the advantages of biharmonic spline interpolation and dimensional analysis. By analyzing and comparing actual data, the following main conclusions are drawn:

1) The control mode of constant upstream water level (downstream section level) is adopted in the Middle Route of South-to-North Water Transfer Project, the monitoring water levels are mostly concentrated in a small local range in practice.. Additionally, the water-using process of users along the route also shows strong regularity. Therefore, in actual water conveyance scheduling, the data on water levels before the gates are relatively concentrated, and the range of flow data is not extensive.

2) Regarding the lack of representativeness in measured water situation data, the paper proposed A method for the calculation of discharge through check gate based on biharmonic spline interpolation and dimensional analysis. By introducing the biharmonic spline interpolation method, the distribution characteristics of the original data can be fully utilized to obtain data samples that can more comprehensively reflect various operating conditions. Then, the dimensional analysis method can be used to obtain a more accurate formula for calculating relationship of the check gate.

The method proposed in the paper can enhance the accuracy of the calculation of discharge through the check gate when the availability of representative data is limited. The paper used 66 actual samples from Gangtuo check gate and Xi Heishan check gate, the proposed method was compared with the traditional dimensional analysis method. The results showed reductions in both the average and maximum values of absolute and relative errors, indicating improved calculation accuracy.

3) Through practical validation in the Middle Route of South-to-North Water Transfer Project, the calculation method of the discharge through check gate proposed in this paper has proven to be highly practical. It can support the construction of digital twin projects along the Middle Route and be applied to similar projects elsewhere.

4) The method proposed in this article can be extended to the calculation of gate flow in other open channel water diversion projects, and can be used to solve the

problem of insufficient accuracy in gate flow calculation caused by insufficient representativeness of water situation data samples.

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