Theoretical Study on the Calculation Method of Ultimate Torque of Prestressed UHPC-RC I-shaped Composite Beam

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Abstract. The prestressed UHPC-RC I-shaped composite beam is a common new type of beam structure, but there is relatively little research on its torsional calculation method This article derives the theoretical formula for ultimate torque based on the theory of variable angle spatial trusses; Subsequently, the accuracy of the theoretical formula was verified by finite element calculation results, and the difference between the theoretical value and the finite element calculation value was only 2.3% The calculation results of the variable parameter analysis in this article indicate that increasing the initial prestress of the section appropriately or improving the tensile (post crack) strength of UHPC can enhance the ultimate torque of the section; Considering the initial pre stress and post cracking strength of UHPC, the ultimate torque of the section can be increased by 272.1 kN·m (57.9%) and 359.6 kN·m (94.1%), respectively.

Keywords: Structural engineering; prestressed UHPC-RC composite beam; I-shaped cross-section; Ultimate torque.

1 Introduction

The prestressed UHPC-RC I-shaped composite beam is a new type of beam structure with multiple advantages [1-2]. Currently, there is more research on the bending and shear mechanical properties of this type of structure, but less research on its torsional performance.However, the torsion resistance of the I-shaped section is poor, and in practical engineering, it may be affected by the bending torsion coupling effect or eccentric load of the curved beam, resulting in a large section torque. It is necessary to accurately analyze the section torsion resistance performance in the design stage.

At present, the design specifications (standards) for concrete (UHPC) structures mainly propose crack torque and ultimate torque calculation formulas for rectangular or box shaped sections [3-6]. The existing standard formulas are not applicable for calculating the ultimate torque of prestressed UHPC-RC I-beams. In recent years, scholars at home and abroad have made certain achievements in the research of ultimate torque calculation methods for UHPC beams [7-10]. Studies by some scholars have shown

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that the torsional bearing capacity of reinforced UHPC beams is mainly determined by the amount and type of steel fibers, reinforcement ratio, and section form. Steel fibers can significantly improve the ultimate torque of the section. Some scholars have proposed formulas for calculating the torsional bearing capacity of reinforced UHPC rectangular beams and reinforced UHPC box beams.

In summary, the current research mainly focuses on reinforced UHPC rectangular beams, with a small amount of literature involving reinforced UHPC box beams and failing to address prestressed UHPC-RC I-shaped composite beams The existing formulas for calculating ultimate torque are mainly based on fitting formulas proposed from model test data, and most of them cannot take into account the effects of prestress and post cracking strength of UHPC on the ultimate torque of UHPC-RC composite beams Therefore, it is necessary to conduct research on the torsional calculation method of prestressed UHPC-RC I-shaped composite beams based on their mechanical characteristics. This chapter focuses on the stress and failure characteristics of UHPC-RC I-beams under ultimate bearing capacity, derives the theoretical calculation formula for the ultimate torque of the section, verifies the theoretical formula through finite element calculation results, and finally conducts variable parameter analysis of engineering examples.

2 Theoretical Analysis of Ultimate Torque

Ultimate torque refers to the external torque corresponding to the ultimate failure state of a section under torsion The following first proposes a partition method for the anti torsion section of I-shaped composite beams, and then derives the theoretical calculation formula for the ultimate torque based on the theoretical model of variable angle spatial trusses.

2.1 Anti Torsion Section Partition

According to the distribution characteristics of shear flow in the cross-section of an I-beam under torsional action, the standard I-beam composite section is divided into three block areas (ABC) (as shown in Figure 1). The prestressed UHPC-RC composite beam is first prefabricated with UHPC beams (block B + block C), and then the RC bridge deck slab (block A) is poured and subjected to a second phase load. The sectional stress of each block area should be accumulated and calculated according to the specific construction process.

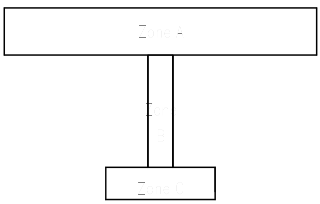

Fig. 1. Positioning principle based on location fingerprints

2.2 Ultimate Torque

The ultimate torque is analyzed based on the theory of variable angle spatial truss model (as shown in Figure 2), and the theoretical formula for the ultimate torque of the I-shaped section is derived below.

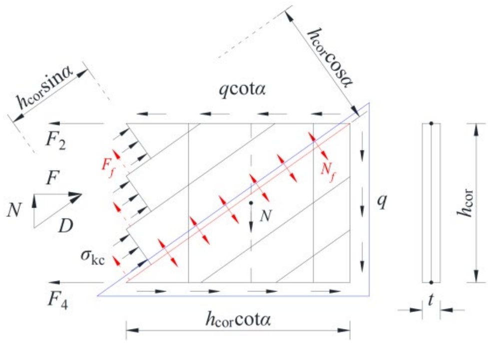

Fig. 2. Variable angle spatial truss model

The ultimate torque analysis of UHPC beams (divided into zones B and C) should consider the post cracking strength of UHPC between the diagonal strut gaps. Taking the lower half of the spatial truss model (within the blue box in Figure 2) as the isolation body, the vertical force balance relationship can be obtained as follows:

$$N = N_s + N_f \tag{1}$$

Among them:

$$N = q \cdot U_{cor} = T \cdot \frac{U_{cor}}{2A_{cor}} \tag{2}$$

$$N_s = \frac{A_{sv1} f_{sv}}{s_v} U_{cor} \cot\alpha \tag{3}$$

$$N_f = k f_t t_d U_{cor} \cot\alpha \tag{4}$$

In the formula: q is the shear flow acting on the sidewall; ft is the post cracking strength q of UHPC; fsv is the tensile strength of the hoop reinforcement; sv is the spacing between stirrups; Ns is the vertical tensile force provided by a single leg hoop reinforcement; Nf is the vertical tension provided between the gaps of UHPC diagonal compression bars; N is the vertical force formed by the shear flow in the section (the sum of the vertical tension provided by the single leg hoop reinforcement and UHPC); $A_{i,cor}$ is the core area of section i in the partitioned area, $A_{i,cor} = (a_i - 2t_d)(b_i - 2t_d)$; $U_{i,cor}$ is the circumference of the core area of section i in the partitioned area, $U_{i,cor} = 2(a_i - 2t_d) + 2(b_i - 2t_d)$; kis the fiber orientation coefficient; td is the equivalent wall thickness of the thin-walled section, $t_d = A/u$, Ais the total area enclosed by the outer perimeter of the solid cross-section; u is the outer circumference of the solid section. The equivalent wall thickness td shall not be less than twice the distance between the outer edge of the section and the center of the longitudinal steel bar. For hollow sections, the actual wall thickness is the upper limit value [4].

Substituting equations (2) to (4) into equation (1) yields:

$$\left(\frac{A_{sv1}f_{sv}}{s_v} + kf_t t_d\right) \cdot U_{cor} \cot\alpha = T \cdot \frac{U_{cor}}{2A_{cor}} \tag{5}$$

Taking the overall spatial truss model as the isolation body, the horizontal equilibrium relationship can be obtained as follows:

$$F_s + F_N + F_f \sin\alpha = D\cos\alpha \tag{6}$$

From the vertical balance relationship, it can be concluded that:

$$D\sin\alpha + F_f \cos\alpha = qU_{cor} \tag{7}$$

Among them:

$$F_f = kf_t t_d U_{cor} \sin\alpha \tag{8}$$

$$F_s = A_{st} f_s \tag{9}$$

$$q \cdot U_{cor} = T \cdot \frac{U_{cor}}{2A_{cor}} \tag{10}$$

In the formula: Fs is the sum of the tensile forces of longitudinal ordinary steel bars; Ff is the tension provided between the gaps of UHPC diagonal compression bars; FN is the cross-sectional prestress generated by the prestress beam; D is the sum of axial pressure of UHPC diagonal compression rods; Ast is the sum of the areas of longitudinal ordinary steel bars; Fs is the tensile strength of longitudinal ordinary steel bars.

Substituting equations (7) to (10) into equations (6) to (7) yields:

$$A_{st} f_s + F_N + kf_t t_d U_{cor} \sin^2\alpha = D\cos\alpha \tag{11}$$

$$D\sin\alpha + 0.5kf_t t_d U_{cor} \sin 2\alpha = T \cdot \frac{U_{cor}}{2A_{cor}} \tag{12}$$

The mechanical equilibrium relationship of the variable angle spatial truss model can be obtained as follows.

When the reinforcement yields:

$$T_1 = 2A_{cor}\left(\frac{A_{sv1}f_{sv}}{s_v} + kf_t t_d\right) \cdot \cot\alpha \tag{13}$$

When the longitudinal bars yield:

$$T_2 = 2A_{cor}\left[\left(A_{st}f_s + F_N + kf_t t_d U_{cor}\sin^2\alpha\right)\frac{\tan\alpha}{U_{cor}} + 0.5kf_t t_d \sin 2\alpha\right] \tag{14}$$

When the hoop reinforcement and longitudinal reinforcement yield simultaneously, the inclination angle α can be calculated based on $T_1 = T_2$:

$$\alpha = \arctan\left(\sqrt{\frac{A_{sv1}f_{sv}/s_v + kf_t t_d}{A_{st}f_s + F_N + kf_t t_d U_{cor}}U_{cor}}\right) \tag{15}$$

The ultimate torque of the overall cross-section is the sum of the ultimate torques of each segmented area:

$$T_{\max} = \sum_{i=1}^{3} T_{i,\max} = T_{A,\max} + T_{B,\max} + T_{C,\max} \tag{16}$$

The ultimate torque of block A is still considered according to the spatial truss theory, but the concrete fails after cracking. Let the f_t in equations (13) to (14) be zero.

3 Numerical Verification of Torsional Calculation

3.1 Example Parameters

This example is a pre-stressed pre tensioned UHPC-RC I-shaped composite beam with a span of 25m. The calculated span is 24m, and the cross-sectional dimensions of the beam are shown in Figure 3 Five and 36 steel strands are arranged on the top and bottom plates respectively, with a controlled tension stress of 1395MPa under the anchor There are no ordinary steel bars inside the UHPC beam, and two layers of ordinary steel mesh are arranged inside the RC bridge deck, with a spacing of 100mm between the steel bars The RC bridge deck is made of C50 concrete, the I-beams are made of UC150 grade UHPC, the ordinary steel bars are HRB400 steel bars, and the prestressed steel bars are Φ_s15.2mm prestressed steel strands. The material mechanical properties are shown in Table 1.

Table 1. Material Mechanical Properties Table

material	compressive strength (MPa)	tensile strength (MPa)	Elastic modulus (MPa)
C50	32.4	2.65	34500
UC150	105	7.2	43500
HRB400	400	400	200000
steel strand	/	1860	195000

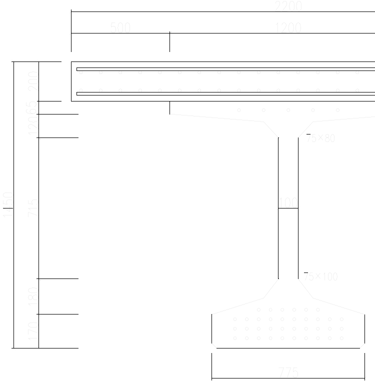

Fig. 3. Sectional View of Example Beam

The finite element model (as shown in Figure 4) of the simply supported UHPC-RC I-shaped composite beam was established in professional analysis software, and the entire bridge was divided into 1842540 elements and 255909 nodes.

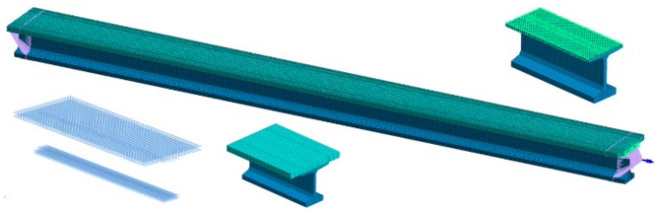

Fig. 4. Overall diagram of finite element model

3.2 Finite Element Model Calculation Results

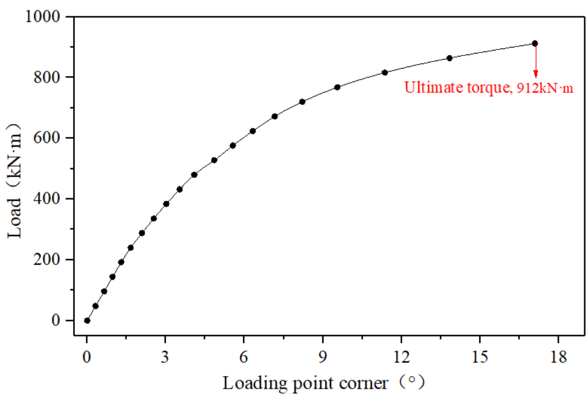

Fig. 5. Numerical calculation results of beam load angle curve for example

The load angle curve of the example beam calculated by the finite element model is shown in Figure 5. It can be seen from the figure that the theoretical value of the initial cracking torque of the example beam is 133KN·m, the theoretical value of the cracking torque is 420KN·m, and the theoretical value of the ultimate torque is 741KN·m.

3.3 Numerical Verification of Ultimate Torque

The results of the calculation based on the theoretical formula of ultimate torque derived in this article are shown in Table 2. The ultimate torque is considered whether to include the tensile strength of C50 concrete in the bridge deck.

Table 2. Theoretical and Finite Element Calculation Values of Sectional Ultimate Torque

Ultimate torque			
Theoretical value (Maximum torque considering C50 tensile strength)	Initial cracking torque	cracking torque	ultimate torque
	133.1	420.5	890.4
Finite element calculation value	Initial cracking torque	cracking torque	ultimate torque
	144	480	912

It can be seen that the theoretical values of the initial cracking torque and cracking torque of the beam section in the calculation example are relatively close to the finite element calculation values; After considering the tensile strength of C50 concrete in the bridge deck, the theoretical values of the initial cracking torque, cracking torque, and ultimate torque of the beam section in the calculation example are close to the finite element calculation values This indicates that the theoretical formula for the ultimate torque of prestressed UHPC-RC I-shaped composite beams proposed in this article can truly reflect the torsional process of the example beams For safety reasons, this article suggests not considering the contribution of concrete tensile strength in the calculation of ultimate torque of bridge decks, and ignoring the tensile strength of C50 concrete in subsequent analysis.

4 Analysis of Ultimate Torque Parameters

Based on the calculation parameters in Section 2.1, a parametric analysis of prestress and UHPC tensile (post crack) strength is conducted using the ultimate torque theory formula to investigate the effect of these parameters on the torsional performance of prestressed UHPC-RC I-shaped composite beams.

4.1 Initial Prestress

The influence of initial preloading stress on the ultimate torque value is shown in Figure 6. The overall sectional ultimate torque values are 779.7 kN·m, 741.7 kN·m, 697.7 kN·m, 644.1 kN·m, 566.7 kN·m, and 469.6 kN·m, respectively For the partitioned area

C, the compressive force in the spatial truss model is borne by the post crack stress of the crack gap between each compressive rod When the prestress value decreases to 25% of the original value, sectional tensile stress is generated in the segmented area C, but the crack gaps between the compression bars can still provide tensile strength, and the segmented area C has not yet reached the ultimate torque When the prestress value drops to 0%, the tensile stress of section C in the segmented area has exceeded the limit and cannot form a spatial truss model to resist external torque The calculation results indicate that increasing the prestress value can improve the ultimate torque of the overall cross-section.

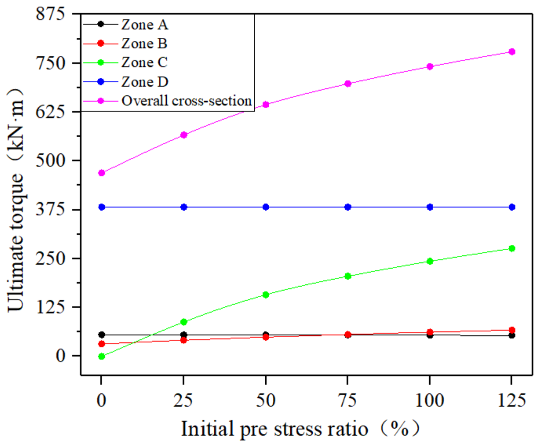

Fig. 6. Initial preloading stress ultimate torque

4.2 The Influence of UHPC Tensile Strength

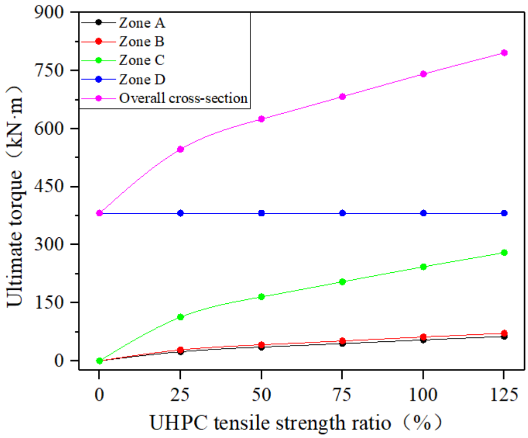

Fig. 7. UHPC tensile strength ultimate torque

The influence of UHPC tensile strength on the ultimate torque value is shown in Figure 7. The overall sectional ultimate torque values are 796.7 kN • m, 741.7 kN • m, 683.4 kN • m, 625.1 kN • m, 547.3 kN • m, and 382.1 kN • m, respectively The ultimate torques TuA, TuB, and TuC of the partitioned areas A, B, and C decrease with the decrease of UHPC tensile strength There is no reinforcement in the block areas A, B, and C, and the compressive force in the spatial truss model is borne by the post crack stress between the crack gaps of each compressive rod. When the material matrix has no tensile capacity, it cannot form a spatial truss model to resist external torque When the tensile strength of UHPC decreases from the original value to 0%, the ultimate torque of the overall cross-section can only be provided by the segmented zone D The post cracking strength of UHPC has a significant impact on the ultimate torque value of the overall cross-section, and the post cracking tensile strength of the material should be appropriately increased.

5 Conclusions

Based on the structural characteristics of prestressed UHPC-RC I-shaped composite beams, this article systematically studies the calculation method of their ultimate torque through theoretical analysis and numerical simulation. The main conclusions drawn are as follows:

(1) The ultimate torque calculation formula proposed in this article can consider the effects of prestress and UHPC tensile strength, and can more accurately reflect the torsional state of the structure than traditional formulas.

(2) Comparing the theoretical analysis and numerical calculation results of the ultimate torque of a 25m span example beam, the theoretical value of the ultimate torque differs only 2.3% from the finite element calculation value, verifying the accuracy of the anti torsion calculation formula derived in this paper.

(3) The initial pre stress has a significant impact on the ultimate torque of the section. In this study, applying pre stress to the beam can increase the ultimate torque value by 57.9%, and considering the tensile strength of UHPC, it can increase the ultimate torque value by 94.1%.

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