




Portfolio Optimization in Practice: A Comparative Analysis of the Markowitz and Index Models

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Abstract. Portfolio optimization is the foundation of modern investing theory, enabling investors to balance risk and return in the dynamic stock market environment. This paper examines two seminal models of portfolio optimization—the Markowitz model and the Index model—exploring their theoretical foundations, methodologies, and practical applications. The Markowitz model, with its rigorous mean-variance optimization framework, emphasizes diversification by accounting for asset correlations, offering unmatched precision in portfolio construction. Conversely, the Index model simplifies optimization by linking asset returns to a single market index, reducing computational demands and focusing on systematic risks. Using Goldman Sachs’ \$621 billion stock portfolio as a case study, the paper evaluates the applicability and performance of these models in managing large-scale, diversified investments. Findings highlight the Index model’s practicality and alignment with Goldman Sachs’ strategic focus on market indices and technology sector investments, underscoring its suitability for large institutional portfolios. This comparative analysis illuminates how theoretical frameworks can be effectively integrated into real-world financial strategies to optimize performance and manage risk in a complex, ever-changing market landscape.

Keywords: portfolio optimization; Markowitz model; Index model; financial strategy.

1. Introduction

The stock market – an ecosystem driven by supply, demand and investor sentiment – has long been the focus of financial invention and strategy. It’s where businesses issue shares and investors swap securities, in an exchange that brings risk and reward to life. The stock market volatility – its price action and market movement – are indicative of general economic activity, business performance, and investor [1]. Portfolio optimization has taken on the role of a central principle of investment theory in this rapidly changing world and is a methodology to take one step forward to deal with the volatility of the markets.

Portfolio optimization is fundamental in the stock market because it directly tackles one of the biggest concerns of an investor — achieving a balance between risk and return. It is about creating a portfolio where returns are maximized for a minimized level of risk [2]. This process is especially important in a market where volatility and unpredictability will destroy value if investments are not proactively addressed. Through implementing optimization models, investors can optimize their asset portfolio, spread risk and take decisions in a way that fits their financial objectives [3].

The Markowitz model and the index model are two of the most common models used in this space and they provide very different ways of thinking about how to construct portfolios. They each tackle the fundamental issue of risk and return into account. Markowitz revolutionized investment theory by revealing the benefits of diversification, introducing a systematic way to build portfolios that maximized risk-reward balance [1]. Its highlight on asset-relationship theory has shaped the foundation for modern finance and investment strategies.

In contrast, the index model has streamlined portfolio management and is especially useful in large-scale or complex investment environments. As the model connects portfolio performance with a market index, it renders simplified decision-making process, allowing investors to allocate more weights on managing systematic risks tied to overall market fluctuation [4]. Together, these models give investors robust tools to maneuver through the intricacies of the financial markets, bridging theory with practice to help make portfolios perform as effectively as possible [5].

The stock market's intricate nature underscores the importance of such optimization models. In an environment where sentiment, speculation, and out of the box factors can drive seismic shifts, having a strategic framework for portfolio construction is invaluable. Both the Markowitz and index models provide tools for understanding and managing market dynamics, helping investors make rational decisions in the face of uncertainty [6]. Together, these models represent how modern finance brings theoretical concepts with practical applications, empowering investors to overcome the complexities of the stock market effectively.

Goldman Sachs, a prominent worldwide investment bank, has always been a leader in stock investment portfolios, leveraging its deep market knowledge and advanced analytical tools. Its asset and wealth management division oversees trillions of dollars, underscoring its strong standing among institutional and individual investors [7].

This essay attempts to compare these two models by comprehensively examining their methodologies, foundational assumptions, and practical effectiveness in optimizing investment portfolios. To demonstrate their application, the analysis will evaluate each model in the context of Goldman Sachs' stock portfolio, boasting a detailed assessment of their performance and relevance within a real-world, high-stakes financial environment. This essay intends to examine the pros and cons of each strategy to illuminate how modern finance integrates theoretical frameworks with practical strategies to empower investors in achieving their financial goals.

2. The Markowitz Model and the Index Model: A Comparative Analysis

The realm of portfolio optimization has been significantly shaped by two seminal approaches: the Markowitz model and the Index model. Each of these models provides unique methodologies for balancing risk and return in investment portfolios, reflecting distinct theoretical underpinnings and practical applications. The evolution, conceptual framework, and comparative advantages of these models highlight their critical roles in modern finance [1].

2.1 The Markowitz Model: History and Evolution

The Markowitz model, developed by Harry Markowitz in 1952, marked the birth of modern portfolio theory. Before this discovery, investment strategies solely relied on selecting individual securities without systematically considering the risk-return trade-off. Markowitz's groundbreaking paper, "Portfolio Selection," laid out a mathematical framework for diversification that centered on the correlation between assets and their variances. This methodology aimed to construct an "efficient portfolio"—one that either maximizes returns for a specified amount of risk or minimizes risk for a maximized return [1].

Markowitz's work was revolutionary for its time, going against the grain of wisdom and introducing a systematic, quantitative approach to investment. His model demonstrated that risk could be managed through diversification if assets within a portfolio were not perfectly correlated. This insight was particularly influential during the mid-20th century when financial markets were expanding, and the demand for structured investment strategies was becoming significant [8].

Over the decades, the Markowitz model has been refined and expanded, becoming a basis of financial theory. Its reliance on a covariance matrix to measure the asset-to-asset correlations revolutionized investment management. However, as computational tools were limited in the mid-20th century, practical applications were at first limited by the model's computational complexity. With progression in technology, particularly in computing power and data availability, the model became more influential, driving the development of financial products and risk management measures. The Markowitz model also laid the groundwork for the development of other major financial theories, for instance, the Capital Asset Pricing Model (CAPM).

2.2 Markowitz Model: Concept and Methodology

The Markowitz model utilizes the mean-variance optimization framework, which demands investors to evaluate the expected return, variance, and covariance of each asset in the portfolio. Taking these into account, the model determines a sequence of efficient portfolios, graphically labeled as the "efficient frontier." This frontier illustrates the ideal trade-offs between risk and reward, assisting investors in making informed decisions according to their risk tolerance levels.

A key feature of the Markowitz model is its extensive use of the covariance matrix. This matrix details the correlation between asset returns, allowing investors to minimize portfolio risk through diversification. By considering how assets move relative to one

another, the model ensures that adding assets with low or negative correlations can minimize overall risk. This is different from simpler investment practices, where diversification is often employed without accounting for asset correlations.

The mathematical foundation of the Markowitz model is both its strength and its limitations. While it provides a theoretically sound framework for optimization, the requirement to estimate many inputs—specifically, the expected returns, variances, and covariances for all assets—can be a significant obstacle. Errors in these estimates can result in suboptimal portfolios, highlighting the model's dependence on reliable and timely data.

2.3 Application of the Markowitz Model

The Markowitz model is widely applied in environments where precision and theoretical rigor are paramount. It is especially suited for institutional investors, like pension funds and endowments, that manage complex portfolios with diverse assets. These organizations often have access to the resources needed to implement the model effectively, including sophisticated software and data analysis tools.

The model is also a staple in academic research, serving as a benchmark for evaluating portfolio performance and testing new financial theories. Its principles have been incorporated into a variety of investment products, from actively managed funds to structured financial instruments. Additionally, the Markowitz model has inspired numerous extensions, including multi-period optimization models that account for changes in investment conditions over time and models incorporating constraints such as transaction costs or regulatory requirements.

Despite its strengths, the Markowitz model's reliance on a large covariance matrix poses challenges in large-scale portfolio management. Estimating these matrices becomes computationally intensive and prone to estimation errors, especially as the number of assets increases. This computational complexity has limited its application in some practical settings and spurred the development of alternative approaches, including the Index model [9].

2.4 The Index Model: History and Evolution

The Index model emerged as a practical solution to the challenges posed by the computational intensity of the Markowitz model. Developed in the mid-20th century, this model leverages the insight that individual asset returns are often influenced by broader market movements. By focusing on a single market index as a proxy for these movements, the Index model simplifies portfolio optimization while retaining much of the functionality of its predecessor.

This innovation coincided with the rise of efficient market hypotheses and the increasing availability of market index data. Researchers and practitioners recognized that the performance of many individual securities could be explained primarily by their relationship with a broader market index, for instance, the S&P 500. This realization formed the basis for the Index model, which prioritized efficiency over theoretical completeness.

Over time, the Index model gained traction for its computational efficiency and its ability to approximate results like those of the Markowitz model. Its simplicity made it particularly attractive for large-scale portfolio management, where computational

resources and data accuracy were critical constraints. The model also aligned well with the growing popularity of passive investment strategies, which aim to match the performance of market indices rather than outperform them.

2.5 The Index Model: Concept and Methodology

The Index model's fundamental premise is that any asset's returns are determined by how sensitive it is to a particular market index, or "beta." The degree to which an asset's returns react to changes in the overall market is measured by its beta. This simplification reduces the need to estimate a full covariance matrix, as correlations between assets are implicitly captured through their relationships with the index.

The Index model employs regression analysis to estimate each asset's beta and its residual variance—risk that is not explained by the market. By combining these components, the model constructs a portfolio that balances market-related risk with asset-specific risk. This approach streamlines the optimization process, making it more accessible for practical applications.

While the Index model significantly reduces computational complexity, it is not without limitations. By focusing solely on a single market factor, the model overlooks the potential influence of other factors, such as sector-specific trends or macroeconomic variables. This reliance on a single index can lead to less robust diversification, particularly in markets characterized by high volatility or segmentation.

2.6 Applications of the Index Model

The Index model is extensively utilized in large-scale portfolio management, particularly in scenarios where computational resources are limited or the portfolio contains a vast number of assets. Mutual funds, exchange-traded funds (ETFs), and other investment vehicles often rely on the Index model to create portfolios that track or outperform market benchmarks. The model is also well-suited for passive investment strategies, where the goal is to emulate the performance of a market index.

In addition to its use in passive strategies, the Index model is employed by active managers seeking to construct portfolios that capitalize on specific market inefficiencies. By adjusting the weights of assets based on their betas, active managers can attempt to enhance returns while still adhering to the constraints of the model. This versatility makes the Index model a valuable tool for both passive and active investment strategies.

However, the simplicity of the Index model comes with trade-offs. By focusing on a single market factor, it overlooks the potential influence of other factors, such as industry trends or macroeconomic variables. This limitation may result in suboptimal diversification compared to the Markowitz model, particularly in volatile or segmented markets. Additionally, the Index model assumes a stable relationship between individual asset returns and the market index, which may not hold in periods of economic or financial disruption.

2.7 Comparing the Markowitz Model and the Index Model

The Markowitz and Index models exemplify different methodologies for portfolio optimization, each possessing unique advantages and drawbacks. The Markowitz model is renowned for its theoretical rigor and ability to account for intricate

relationships between assets. Its reliance on a full covariance matrix allows for precise diversification, making it ideal for investors with access to accurate data and computational resources.

In contrast, the Index model prioritizes simplicity and efficiency. By reducing the computational burden, it enables investors to construct portfolios more quickly and cost-effectively. While it may sacrifice some of the precision of the Markowitz model, the Index model often produces similar results in practice, particularly in diversified markets where a single index captures much of the relevant information.

One key distinction lies in their respective applications. The Markowitz model is better suited for institutional investors and scenarios requiring customized, high-precision portfolios. The Index model, on the other hand, is favored for its practicality in large-scale and passive investment strategies, making it accessible to a broader range of investors.

Both models also differ in their underlying assumptions. The Markowitz model assumes that investors can accurately estimate returns, variances, and covariances for all assets, which may not be realistic in practice. The Index model, by contrast, simplifies these assumptions by focusing on a single market factor, but this simplification can lead to oversights in certain market conditions.

3. Goldman Sack Stock Portfolio Optimization

Goldman Sachs, one of the top global investment banking and securities companies, oversees a diverse portfolio of stock holdings. As of the third quarter of 2024, the firm's 13F filing reported approximately \$621 billion in managed 13F securities, with a top 10 holdings concentration of 19.46%. The largest holding was Apple Inc., comprising 86,514,737 shares.

The top five stock holdings included SPDR S&P 500 ETF Trust (SPY), NVIDIA Corporation (NVDA), Microsoft Corporation (MSFT), Apple Inc. (AAPL), and Amazon.com Inc. (AMZN), collectively representing 19.32% of Goldman Sachs' stock portfolio. This concentration indicates a strategic focus on major technology firms and broad market indices.

In terms of portfolio adjustments, Goldman Sachs added significant positions in NVIDIA (+\$25B), Apple (+\$3.0B), and Microsoft (+\$2.1B) during the same period. Conversely, the firm reduced holdings in SPDR S&P 500 ETF Trust (-\$5.9B), iShares Russell 2000 ETF (-\$4.8B), and iShares Core S&P 500 ETF (-\$3.7B). These movements reflect the firm's dynamic approach to portfolio management, adjusting exposures in reaction to market conditions and investment strategies.

The firm's size and investment goals must be considered when determining whether the Markowitz model or the Index model would be more appropriate for Goldman Sachs' portfolio management. The Markowitz model, or mean-variance optimization, constructs a portfolio that maximizes anticipated return for a given level of risk by analyzing the covariance of asset returns. Extensive computational resources and accurate estimation of predicted returns, variances, and covariances for every asset are

necessary for this method. The Markowitz approach would be difficult and resource-intensive to adopt given Goldman Sachs' extensive and varied portfolio.

The Index approach, on the other hand, makes portfolio optimization easier by presuming that the returns of individual assets are mostly determined by a single market index. This model is more useful for large-scale portfolios since it simplifies computation and concentrates on the connection between each asset and the market index. The Index model fits in nicely with Goldman Sachs' investment approach given their significant holdings in broad market indices such as the SPDR S&P 500 ETF Trust.

Therefore, for a firm of Goldman Sachs' magnitude, the Index model would be more suitable. Its efficiency in handling large portfolios and focus on market index relationships aligns with the firm's existing investment approach, facilitating effective portfolio management without the intensive computational demands of the Markowitz model [10].

4. Conclusion

In the intricate landscape of the stock market, portfolio optimization serves as a vital tool for balancing the optimal between risk and return, allowing investors to navigate uncertainty with structured strategies. Both the Markowitz model and the Index model represent transformative approaches to portfolio construction, each offering distinct methodologies tailored to different investment contexts. The Markowitz model, with its rigorous mean-variance optimization framework, provides unparalleled precision in diversification by accounting for asset correlations. However, its computational intensity and reliance on accurate data make it best suited for institutions with significant resources and a need for highly customized portfolios.

Conversely, the Index model simplifies portfolio management by linking asset performance to a single market index, streamlining the decision-making process and reducing computational demands. This model aligns well with large-scale investment operations, where efficiency and practicality are paramount. Its focus on systematic risk management makes it particularly valuable for institutions like Goldman Sachs, which manage extensive portfolios heavily concentrated in broad market indices and leading technology firms.

In the context of Goldman Sachs' portfolio, the Index model emerges as the more fitting choice. With significant holdings in market index funds such as the SPDR S&P 500 ETF Trust and a focus on tech-heavy investments like Apple and NVIDIA, the firm's strategy is naturally aligned with the Index model's assumptions and strengths. The model's capacity to manage systematic risk effectively while maintaining operational efficiency suits Goldman Sachs' need for scalable portfolio management tools in a fast-paced and data-driven environment.

Ultimately, both models underscore the importance of bridging theoretical insights with practical applications. While the Markowitz model remains a benchmark in investment theory, the Index model's adaptability to real-world complexities highlights its enduring relevance in modern finance. Together, these frameworks exemplify how

financial innovation continues to empower investors to optimize performance, mitigate risk, and achieve their financial goals within the ever-evolving dynamics of the stock market.

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