



Method for Optimizing Sliding Distance of Track Quality Assessment

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Abstract. Objective assessment of railway track quality is vital to determine track maintenance operations to ensure train running safety and passenger riding comfort. A unit section is represented as a window on the chart of track quality parameters measurement data along the kilometrage x-axis. To obtain complete track quality values in the spatial dimension, running windows with some sliding distance are adopted. If the sliding distance equals the window size, two quality values at adjacent sections are independent, which has high risk of underestimating actual track quality. To address this issue, the sliding distance should be smaller than the window size. The smaller the sliding distance, the more the windows. More windows require a bigger storage disk and a larger computation for track quality indices calculation and analysis. Therefore, the sliding distance should be determined in an optimal manner so that the effect of all track irregularities is captured, and the amount of track quality indices is minimized. Based on the precision of the management specifications of track quality indices, the Jensen-Shannon divergence and Monto Carlo simulation method are employed to obtain the optimal sliding distance. The sliding distance of 5 meters is appropriate to tracks with the allowable speed no higher than 200km/h, while 10 meters to high-speed tracks (i.e., >200). Compared with traditional methods, the new method can reflect the most realistic quality status of the line and accurately determine all sections of track defects, which is conducive to railway maintenance plans.

Keywords: Track Quality Assessment, Sliding Unit, Information Entropy, Jensen-Shannon Divergence, Monte Carlo Method.

1 Introduction

Railways constitute a critical component of modern transportation infrastructure, with continuous advancements in speed and load capacity necessitating increasingly stringent standards for track quality [1]. The operational safety and ride comfort of railway vehicles are intrinsically dependent on track geometric integrity [2-4], which is systematically quantified through track geometry irregularities [5,6]. Defined as deviations from theoretical geometric parameters, track geometry irregularities include gauge, cross level, longitudinal level, lateral alignment, and twist [7]. Contemporary

assessment methodologies rely on track inspection data acquired through track recording vehicles [8,9], with global railway authorities employing diverse statistical models and empirical formulas for analytical evaluation. Internationally, track quality assessment method predominantly utilizes standard deviation-based formulations, exemplified by China's Track Quality Index (TQI), India's Track Geometry Index (TGI), and the Track Roughness Index (TRI) in the United States [10]. But in other countries, like France and Austria, different assessment methodologies are employed [11].

Recent scholarly efforts have significantly expanded methodological frontiers in track quality assessment. Chen et al. [12] developed a novel irregularity identification algorithm for high-speed railway bridges, utilizing vertical acceleration responses of vehicle bodies to estimate track irregularities. Sadeghi and Asadi [13] proposed an innovative defect density formula employing sleeper-based track segmentation for localized defect quantification. Through dimensionality reduction techniques, Lasisi and Attoh-Okine [14] effectively eliminated data redundancy in multidimensional track geometry datasets, subsequently employing machine learning and principal component analysis to establish a refined assessment framework. Li et al. [15] enhanced vertical geometry evaluation through second-order derivative analysis of longitudinal level data, thereby addressing track-vehicle dynamic interactions frequently overlooked in conventional methods. Yan and Corman's comparative analysis [16] emphasized the necessity of implementing complementary evaluation approaches for comprehensive track condition classification, a perspective further extended by Diao et al. [17] through their mechanistic analysis of complex irregularity patterns in high-speed rail systems.

The track quality assessment method divides railway tracks into fixed-length unit sections, represented as windows along the kilometer x-axis. However, the size of these windows varies across countries. For example, China use 200-meter-long windows, Japan uses 500 meters, and the United States uses 0.2 miles [18]. In China, the unit sections are adjacent and non-overlapping, meaning the sliding distance equals the window size. When the waveform of track geometry irregularities spans two-unit sections, the traditional method fails to accurately assess the severity of the condition in any single unit section, as depicted in Figure 1.

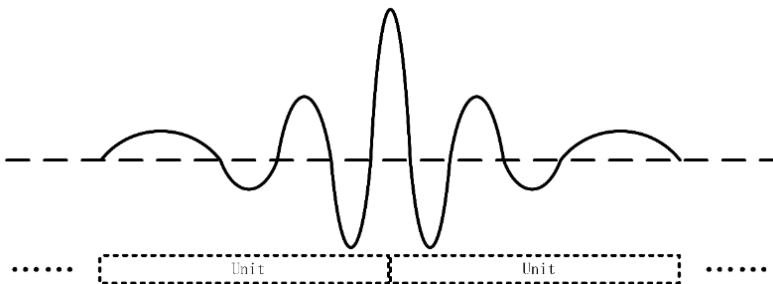


Fig. 1. Track geometry irregularities waveform.

In response to this challenge, many scholars have proposed reducing the window size or sliding distance to enhance the resolution of track quality assessment. Based on

the principle of maximum information entropy, Chen et al. [19] demonstrated that reducing the Chinese window size from 200 meters to 15 meters effectively identifies train-induced vibrations. However, the 200-meter window size is stipulated in China's Railway Maintenance Rules, making a reduction in unit length impractical in current practice. Therefore, this paper is grounded in the use of the 200-meter window size as its research basis. Additionally, Xiao et al. [20] evaluated and compared the geometrical condition of polyurethane-reinforced ballasted track (PRBT) using standard deviations with a sliding distance of 1 meter. Nevertheless, for a given railway length, the sliding distance is inversely proportional to the volume of data collected, which indicates that the smaller the sliding distance, the more the windows. Considering China's nearly 160,000 kilometers of railways, using a 1-meter sliding distance method results in about 160-million-unit sections for a single-track inspection. This immense data volume necessitates significant data processing capabilities from the railway department, making it crucial to determine an optimal sliding distance. The optimal sliding distance is defined as the maximum value that still captures all track irregularities. Therefore, this paper proposes an optimal sliding assessment method designed to meet these requirements. This method is tailored to China's specific conditions and can evaluate track quality more effectively and accurately than existing methods.

2 Track Quality Optimal Sliding Assessment Method

Railway track quality is evaluated by analyzing the track geometrical standard deviation of each unit section. In the traditional Chinese assessment method, the unit section is a 200-meter-long window, and the sliding distance is equal to the window size. This paper aims to shorten the sliding distance D to an appropriate value to capture all track irregularities. Balancing the relationship between data volume and assessment effectiveness is necessary. A better assessment effect requires capturing more track irregularities along the railway. Thus, the data should contain more information about track irregularities to reflect the actual track state, which can be expressed using information entropy. Consequently, this paper uses information entropy to compare the assessment effectiveness at different sliding distances.

Let the TQI values of the railway obtained at different sliding distances fall within the interval $[Inf, Sup]$. And the interval $[Inf, Sup]$ is divided into n equal parts at length of 0.01, where the i th part is denoted as $(d_i, d_{i+1}]$ and $d_1 = Inf, d_{n+1} = Sup$. Therefore, TQI information entropy at D -long sliding distance is defined as

$$H_D = -\sum_{i=1}^n p_D(x_i) \log_2 p_D(x_i), \quad (1)$$

where x_i denotes event that the TQI falls on the part $(d_i, d_{i+1}]$, D denotes D -long sliding distance, $p_D(x_i)$ denotes the probability of event x_i .

In this paper, data from 73 geometry inspections across 14 routes in the Chinese railway network were selected for analysis. Figure 2 shows the TQI information entropy curve obtained by Equation (1), where $D \in \{1, 2, 5, 10, 15, 20, 25, 50, 75, 100, 150, 200\}$. From (a) and (b) in Figure 2, the information entropy of each track geometrical standard deviation on the Hurong Line and

Jingjiu Line decreases gradually as the sliding distance increases. This indicates that the sliding distance is inversely proportional to the information entropy, and the gap between evaluated results and the actual state widens with increasing sliding distance. Additionally, the information entropy regression curve for high-speed levels is above that for low-speed levels, as shown in Figure 2(c). This suggests that information entropy is also related to speed class. This is due to the better track quality and reduced vibration on high-speed lines, resulting in lower information entropy for the same sliding distance. Judging the difference in information entropy between sliding distances of less than 10 meters in Figure 2 is difficult, making it challenging to accurately determine the optimal sliding distance for the track quality sliding assessment method for each speed class. Therefore, further calculations and analyses are required.

3 Determine the Optimal Sliding Distance

The assessment effect for different sliding distances corresponds with information entropy as discussed in the previous section. However, this relationship alone cannot directly determine an optimal sliding distance. An optimal sliding distance should consider both data volume and assessment effect. Since data volume is an inverse function of the sliding distance, it is essential to distinguish the best assessment effect for an optimal sliding distance. This can be achieved using the Jensen-Shannon divergence and the Monte Carlo simulation method, as detailed in Section 3.

3.1 JS Divergence Between Datasets at Each Sliding Distance

The concept of Kullback-Leibler Divergence (KL Divergence), also known as relative entropy, was first introduced by Kullback and Leibler in 1951[21]. It is primarily used to quantify the difference between two probability distributions based on information entropy. In 1991 Lin [22] proposed the concept of Jensen-Shannon Divergence (JS Divergence) based on information entropy and KL Divergence. Both KL Divergence and JS Divergence can be used to measure the difference between two probability distributions.

Let D_m, D_n be the dataset obtained by dividing the geometry inspection data with sliding distances m and n . The standard deviation probability distribution of track geometry g obtained at 0.1 value interval is defined as $P_m^g = \{p_{m,1}^g, p_{m,2}^g, \dots, p_{m,k}^g\}$, $Q_m^g = \{q_{n,1}^g, q_{n,2}^g, \dots, q_{n,k}^g\}$. And the JS Divergence is defined as

$$JS(P_m^g, Q_m^g) = \frac{1}{2} \sum_{i=1}^k p_{m,i}^g \log \left(\frac{2 \cdot p_{m,i}^g}{p_{m,i}^g + q_{n,i}^g} \right) + \frac{1}{2} \sum_{i=1}^k q_{n,i}^g \log \left(\frac{2 \cdot q_{n,i}^g}{p_{m,i}^g + q_{n,i}^g} \right). \quad (2)$$

Taking the left longitudinal level of the Jingjiu Line as an example, the heat map of JS Divergence obtained with different sliding distances using Equation (2) is shown in Figure 3. The JS Divergences between each sliding distance are symmetrically distributed, indicating that the larger the gap between sliding distances, the higher the JS Divergence between them. It can be inferred from Figure 2 and 3 that the dataset with a

1-meter sliding distance is the closest to the actual quality state of the line among all the integer sliding distances. Therefore, the optimal sliding distance should have a similar assessment effect to the 1-meter sliding distance within an acceptable error range.

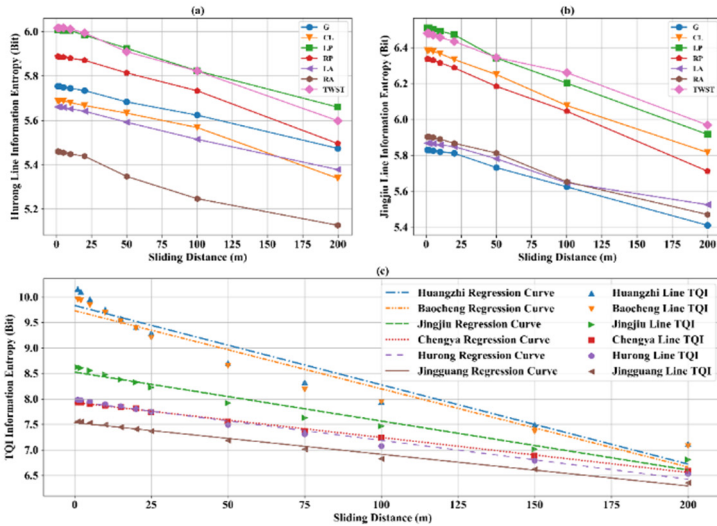


Fig. 2. Information entropy curves for different sliding distances.

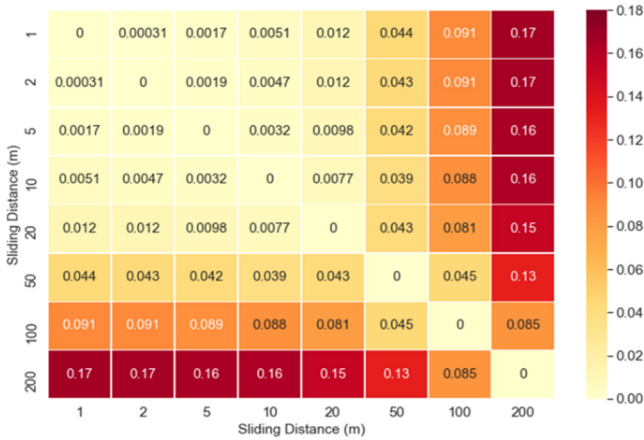


Fig. 3. Heat map of JS Divergence of the left longitudinal level on the Jingjiu Line.

Constructing a similarity range for JS Divergence(0, s], D_m and D_n can be considered similar in terms of evaluating quality state of the track geometry g when $JS(P_m^g, Q_m^g) \leq s$. Let the dataset with 1m-long sliding distance be $D_1 = \{d_{1,i}, i = 1, 2, \dots, n\}$. According to the China Railway Maintenance Rules, the maximum accuracy of the management value for each track geometrical standard deviation is 0.1.

Based on the 3σ principle and the maximum accuracy of the management value, a new dataset D_1^{new} with similar track quality assessment effect as D_1 can be constructed:

$$D_1^{new} = \{d_{1,i}^{new} | d_{1,i}^{new} = d_{1,i} + \varepsilon_i, i = 1, \dots, n\}, \varepsilon_i \sim N(0, 1/60). \quad (3)$$

The D_1^{new} is composed of D_1 and an error term that follows a normal distribution with a three standard deviation of 0.05 and a mean of 0, which is within the allowable error range of D_1 . Therefore, the JS Divergence between these two datasets can be used as a similarity threshold s to determine whether the probability distribution is similar to that of dataset D_1 . Furthermore, the similarity threshold s can be used to determine whether a new dataset has a similar track quality assessment effect to dataset D_1 .

3.2 Estimating Similarity Threshold Using Monte Carlo Method

The Monte Carlo method, a numerical calculation technique introduced in the 1940s and based on probability and statistics theory, is employed in this paper to estimate the similarity threshold s of each track geometrical standard deviation across different railway lines.

Using geometry inspection data from the Baocheng Line and Hurong Lines examples, this paper determines the similarity threshold using the Monte Carlo method under conditions of an allowable error of 0.01 and a confidence level of 99.7%, as shown in Table 1. To improve convergence speed, the JS divergences and similarity thresholds are transformed into corresponding negative natural logarithmic values. In Table 1, gauge, cross level, left longitudinal level, right longitudinal level, left lateral alignment, right lateral alignment, and twist are abbreviated as G, CL, LP, RP, LA, RA, and Twist, respectively. The bolded values in Table 1 indicate data values greater than the similarity threshold, signifying that the dataset has a similar probability distribution to the dataset with a 1m sliding distance. The optimal sliding distance for each railway line is the smallest of the sliding distances for each track's geometrical standard deviation.

3.3 Speed Level and Sliding Distance

Railway lines have varying requirements for track quality, making it essential to determine the relationship between speed level and sliding distance. By analyzing the JS Divergence of railway lines at different sliding distances, the relationship between speed level and sliding distance is depicted in Figure 4. The mean value curve and the minimum value curve of the sliding distance for each speed level exhibit an upward trend. Moreover, the sliding distance for all speed levels exceeds 5m, and for high-speed railways with a speed level of 200 km/h or above, the sliding distance exceeds 10m. Based on these findings, this paper concludes the following:

1. For railway lines with a speed level less than 200 km/h, a track quality assessment method with a sliding distance of 5m is recommended.
2. For high-speed railways with a speed level of 200 km/h or above, a track quality assessment method with a sliding distance of 10m is recommended.

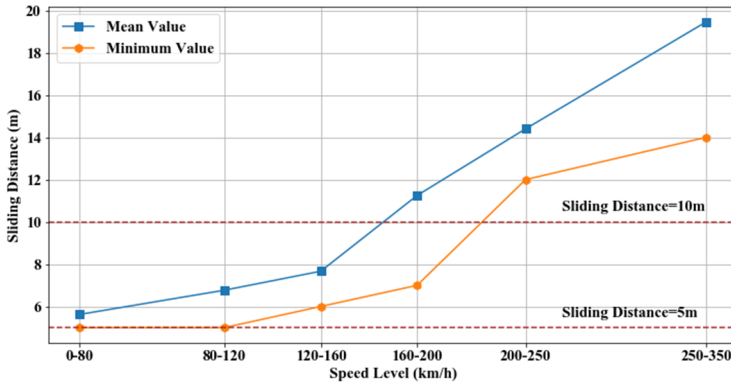


Fig. 4. Speed level and sliding distance.

Table 1. JS Divergences and similarity thresholds at each sliding distance.

Line name	Track geometry	Negative natural logarithmic value of JS Divergence between data with sliding distance of 1m and Lm					Similarity threshold
		2m	5m	10m	20m	50m	
Hurong	G	9.499	7.641	7.024	5.585	4.155	5.462
	CL	9.157	7.413	6.353	5.411	4.050	4.611
	LP	9.193	7.326	6.053	5.024	4.005	5.236
	RP	8.954	7.356	6.354	5.149	3.560	4.914
	LA	10.006	8.212	7.090	6.330	4.494	5.433
	RA	9.501	8.379	6.818	5.593	4.438	5.132
	TWST	9.227	7.328	5.893	4.940	4.031	5.361
Baocheng	G	7.864	5.804	4.749	3.906	2.711	5.239
	CL	7.178	5.900	4.957	3.891	2.662	4.424
	LP	8.247	6.532	5.255	4.289	2.960	5.039
	RP	8.163	6.478	5.110	4.221	3.048	4.978
	LA	7.056	5.490	4.674	3.806	2.781	3.612
	RA	7.319	5.636	4.656	3.829	2.768	3.743
	TWST	6.454	5.063	4.092	3.238	1.643	4.808

4 Case Study

This paper selects several geometry inspection data to study the effect of maintenance work using methods with and without the optimal sliding distance (i.e., the segment of K1259+400 to K1260+400 on the Jingjiu Line and K618+000 to K619+000 on the Hurong Line). The horizontal axis of Figure 5 indicates the start mileage points of the corresponding 200m unit section. In the legend of Figure 5, "200m" indicates the method with a 200m sliding distance (i.e., the traditional method), while "5m" and

"10m" indicate the optimal sliding distances (i.e., the new method). The red arrows indicate the track maintenance work on the gauge.

The work segment of the Jingjiu Line is K1259+800 to K1260+000, and for the Hurong Line, it is K618+200 to K618+400 and K618+400 to K618+600. According to China Railway Maintenance Rules, the gauge management value of the Jingjiu Line is 1.1-1.3. Under the traditional method, it is reasonable to select this segment for maintenance work. However, the post-maintenance data show that the gauge standard deviation in the K1259+600 to K1259+800 segment still has a large peak, proving that the traditional method cannot effectively capture all track irregularities. The curve obtained by the new assessment method before maintenance work shows two peaks, suggesting that the maintenance work segment should be extended to K1259+600 to K1260+000. Additionally, the standard deviation curve of the Hurong Line obtained using the traditional method before the work shows high values for these two segments, meeting the maintenance requirement. However, the curve obtained using the new method reveals additional segments with poor track quality that should also be included in the maintenance plan.

In conclusion, the new method more accurately reflects the actual quality state of the railway line and provides a more scientific and reasonable basis for the railway department to develop maintenance work plans.

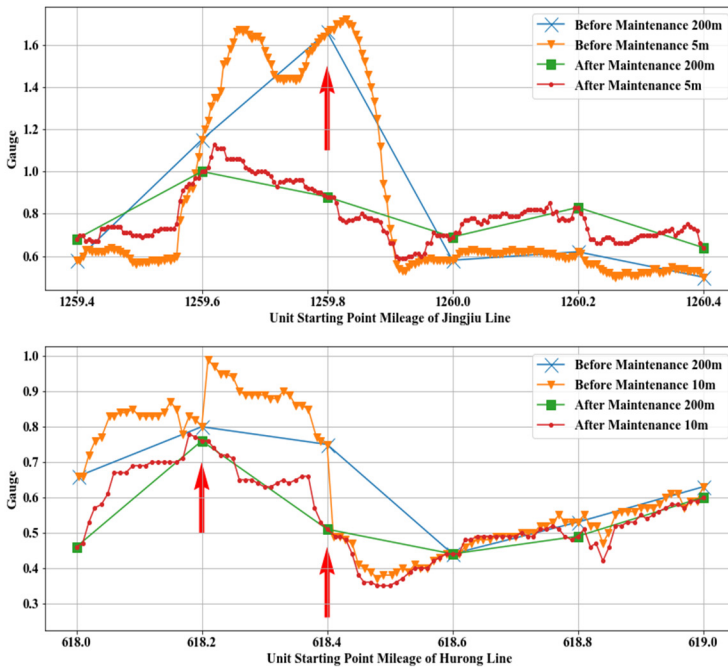


Fig. 5. Gauge standard deviation curve of Jingjiu Line and Hurong Line before and after the maintenance operation.

5 Conclusions

In most countries, the assessment of track quality is a prerequisite for track maintenance work, typically done by analyzing the standard deviation of each unit along the railway. The primary limitation of the traditional assessment method, which uses a 200m sliding distance, is its inability to capture all track irregularities. This underestimation of track quality can lead to running unsafe and passenger discomfort. To address this limitation, this paper proposes a new assessment method with an optimal sliding distance. This distance should not only align the assessment results with the actual track state but also minimize the data calculation requirements for railway departments.

In this study, information entropy is used to evaluate the impact of sliding distance on assessment results. The findings indicate that sliding distance is inversely proportional to information entropy, meaning that shorter sliding distances yield results closer to the actual track state. However, when the sliding distance is reduced beyond a certain point, its impact on information entropy diminishes. This reduction also leads to increased storage and computational demands, which is impractical for railway departments. Therefore, while the optimal sliding distance should be shorter than 200 meters, it should not be excessively short to avoid excessive data requirements.

To determine the optimal sliding distance, 73 geometry inspection datasets from 14 routes in the Chinese railway network are analyzed to obtain similarity thresholds using JS Divergence and the Monte Carlo method. These thresholds represent the limit at which the sliding distance yields similar assessment results. Based on these analyses, optimal sliding distances are determined for six classes of railway tracks in China.

The paper finally concludes that a new track quality assessment method with a 5m sliding distance is suitable for Chinese railways with speeds less than 200 km/h, while a 10m sliding distance is recommended for Chinese high-speed railways with speeds greater than 200 km/h. And the results of case study show that this new method can not only find the section with the worst track quality, but also find more sections that exceed the management limit.

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References

1. Choi I. I. Yoon, Um Ju-Hwan, Lee Jun S., and Choi Hyun-Ho: The Influence of Track Irregularities on the Running Behavior of High-speed Trains. *Proceedings of the Institution of Mechanical Engineers, Part F: Journal of Rail and Rapid Transit* 227(1), 94–102 (2012).
2. Haigermoser Andreas, Luber Bernd, Rauh Jochen, and Grafe Gunnar: Road and Track Irregularities: Measurement, Assessment and Simulation. *Vehicle System Dynamics* 53(7), 878-957 (2015).

3. Liu Chi, Thompson David, Griffin Michael J., and Entezami Mani: Effect of Train Speed and Track Geometry on the Ride Comfort in High-speed Railways Based on ISO 2631-1. *Proceedings of the Institution of Mechanical Engineers, Part F: Journal of Rail and Rapid Transit* 234(7), 765-778 (2019).
4. Hesser D.F., Altun K., and Markert B.: Monitoring and Tracking of a Suspension Railway Based on Data-driven Methods Applied to Inertial Measurements. *Mechanical Systems and Signal Processing* 164(1), 1-18 (2022).
5. Karttunen K., Kabo E., and Ekberg A.: The Influence of Track Geometry Irregularities on Rolling Contact Fatigue. *Wear* 314(1),78-86 (2014).
6. Soleimanmeigouni Iman, Ahmadi Alireza, and Kumar Uday: Track Geometry Degradation and Maintenance Modelling: A Review. *Proceedings of the Institution of Mechanical Engineers, Part F: Journal of Rail and Rapid Transit* 232(1) 73-102, (2016)
7. De Rosa A.: Monitoring of Lateral and Cross Level Track Geometry Irregularities Through Onboard Vehicle Dynamics Measurements Using Machine Learning Classification Algorithms. *Proceedings of the Institution of Mechanical Engineers, Part F: Journal of Rail and Rapid Transit* 235(1), 107-120 (2020).
8. Xu Weitong, Yu Ning, Wei Shibin, and Qu Jianjun: Application of Track Inspection Data Vehicle-ground Positioning Information Fusion Technology. *China Railway* 5, 125-130 (2021).
9. Fontul Simona, Fortunato Eduardo, De Chiara Francesca, Burrinha Rui, and Baldeiras Marco: Railways Track Characterization Using Ground Penetrating Radar. *Procedia Engineering* 143(1), 1193-1200 (2016).
10. Liu Rengkui, Xu Peng , Sun Peng , Zou Peng , and Sun Quanxin: Establishment of Track Quality Index Standard Recommendations for Beijing Metro. *Discrete Dynamics in Nature and Society*, 1-9 (2015).
11. Li Jiajie: Research on Data-Driven Detection of Track Longitudinal Level Irregularity. Xi'an University of Technology (2024).
12. Xiao Xiang, Shen Wenai, and He Xuhui: Track Irregularity Monitoring on High-Speed Railway Viaducts: A Novel Algorithm with Unknown Input Condensation. *Journal of Engineering Mechanics* 147(6), 1-13 (2021).
13. Sadeghi Javad and Askarinejad Hossein: An Investigation into the Effects of Track Structural Conditions on Railway Track Geometry Deviations. *Proceedings of the Institution of Mechanical Engineers, Part F: Journal of Rail and Rapid Transit* 223(4), 415-425 (2019).
14. Lasisi Ahmed and Attoh-Okine Nii: Principal Components Analysis and Track Quality Index: A Machine Learning Approach. *Transportation Research Part C: Emerging Technologies* 91(1), 230-248 (2018).
15. Li Martin, Persson Ingemar, Spannar Jan, and Berg Mats: On the Use of Second-order Derivatives of Track Irregularity for Assessing Vertical Track Geometry Quality. *Vehicle System Dynamics* 50(1),389-401 (2012).
16. Yan T.H. and Corman F.: Assessing and Extending Track Quality Index for Novel Measurement Techniques in Railway Systems. *Transportation Research Record* 2674(8), 24-36 (2020).
17. Diao Hongbao, Yang Fei, Gao Yansong, Sun Jialin, Zhao Gao, and Xie Wanru: Research on Mechanism and Evaluation Standard of Influence of Track Complex Irregularities on Highspeed Railway. *Journal of the China Railway Society* 46(11),102-111 (2024).
18. Gao Jianmin, Huo Wanming, Xu Yong, and Chen Dongsheng: Analysis on the Management Length of Geometry Irregularities Sections of Railway Lines. *Railway Construction* 5(1), 105-108 (2009).

19. Chen Rong, Li Shuai, Wang Yuan, Wang Ping, and Chen Junwen: Evaluation Method of Rail Regularity State Based on Track Local Fluctuation in High-speed Railway. *Journal of the China Railway Society* 39(2),105-111 (2017).
20. Xiao Jieling, Jing Pu, Yu Sixin, and Wang Ping: Analysis on the Track Quality Evolution Law of Polyurethane-Reinforced Ballasted Track in High-speed Railway. *Proceedings of the Institution of Mechanical Engineers, Part F: Journal of Rail and Rapid Transit* 235(8), 993-1005 (2020).
21. Kullback S. and Leibler R. A.: On Information and Sufficiency. *Institute of Mathematical Statistics* 22(1), 79-86 (1951).
22. Lin: Divergence Measures Based on the Shannon entropy. *IEEE Transactions on Information Theory* 37(1), 145-151 (1991).

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