Prospective Mathematics Teachers' Learning Obstacles: A Case Study of Calculus II

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Abstract. The purpose of this paper is to describe the learning obstacles of prospective mathematics teachers in the calculus II course. This is a compulsory course in the mathematics education program at Universitas Negeri Gorontalo. This study is qualitative research; tests, interviews, and observations were used as methods of data collection. Six second semester students enrolled in the Mathematics Education program at Universitas Negeri Gorontalo during the 2022-2023 academic year were the subjects of this research. They were selected based on their academic ability in this course. The research results found several learning barriers, namely: 1) difficulty in answering questions that require concepts that have been previously studied (prior knowledge); 2) difficulty in applying formulas/definitions in solving problems; 3) difficulty answering problem-solving exercises or questions that require a number of steps before an answer can be obtained, particularly those involving graphs. The findings of this research can provide valuable insights into how the learning process can be improved. Additionally, the findings can help educators identify areas where students may need additional support or resources to better meet learning objectives especially in a calculus course.

Keywords: prospective mathematics teachers, learning obstacles, calculus II

1 Introduction

Teacher education is a priority program to improve the quality of human resources. The pre-service teaching program encourages students to explore their pedagogical, professional, personal, and social abilities since the teacher plays an important role in the learning process. As qualified teachers produce quality students, teacher education becomes a crucial component of providing competent teachers [1].

There is no doubt that mathematics is an essential subject for 21st century learners who must possess sufficient skills to pursue their higher education and become competent for their career path in the future. Mathematics is an accumulative discipline dealing with complex concepts that are built cumulatively on more simple concepts. Without a solid mathematical foundation, a student will have difficulty learning advanced mathematics [2]. Being a mathematics teacher is challenging for students in a mathematics education study program. Preservice mathematics teachers are prospective

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teachers who will teach mathematics after graduation [3]. As it pertains to Mathematics Education, teachers must have a thorough understanding of mathematics, pedagogy, as well as the methods for teaching the subject [4], because most teachers lack the proper mathematical knowledge for teaching [5]. A teacher's content knowledge relates to his or her proficiency in a particular subject or content area. Some researchers have debated the impact of teachers' content knowledge on students' achievement for many years [6–8].

It is mandatory for students to study calculus in school, but many students struggle to succeed in calculus courses in college. A calculus course can provide students with important foundations for a variety of further studies, particularly in mathematically intensive fields [9]. University students encounter many calculus concepts during their college education [10]. Calculus is one of very few areas of study in mathematics which very few pupil teachers have learned in depth before [11]. One of the types of this subject is Calculus II that is taught in the second semester in mathematics education study program at Universitas Negeri Gorontalo. It is one of the materials that students have obstacle in some previous years.

As students transition from Calculus 1 to Calculus 2, they often face difficulties grasping new concepts and applying previous knowledge. This case study analyzes the challenges students encounter in a Calculus 2 course. In this article, we will explore an analysis of the specific learning obstacles faced by students in a Calculus II course. By understanding the nature of these difficulties, students and instructors alike can work to develop strategies to overcome them. Therefore, teachers need to have a good understanding of mathematical problem solving and its teaching [12]. Calculus educators shall have an attitude that is positive, towards their learners' errors, and shall conceive learners' errors as a tool to adapt and correct their didactic methodologies [11].

2 Method

This study used a descriptive exploratory method with qualitative design. Subjects were six second semester students enrolled in the mathematics education program at Universitas Negeri Gorontalo during the 2022-2023 academic year that which selected by their academic ability in this course Data collection techniques used test, observation, and interviews. Observations were made to observe the learning activities of students in lectures, discussions, exercises and evaluations. Interviews were conducted to explore information about the difficulties experienced by students in learning Calculus II material, and test was used to obtain data on student scores on exam.

The tests were obtained from selected mid and final exams questions, the following are the questions used:

1. Find the area of the circumscribed polygon (Figure 1).
2. Find the value of $\int_{-2}^{1} (x^2)$ dx using definition:

   $\int_a^b f(x)dx = \lim_{|P|\to 0} \sum_{i=1}^{n} f(\bar{x}_i)\Delta x_i$
3. Use the three-step procedure (slice, approximate, integrate) to set up and evaluate an integral for the area of the indicated region shown in Figure 2.

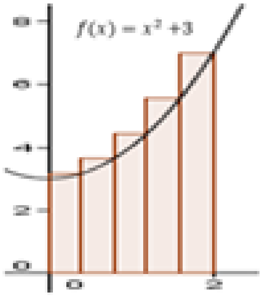

Fig. 1. Approximation of area under $f(x) = x^2 + 3$

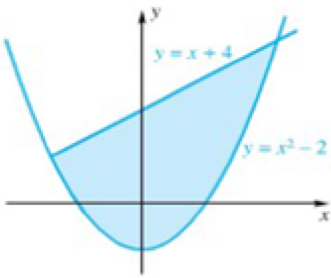

Fig. 2. Region bounded by $y = x + 4$ and $y = x^2 - 2$

3 Result and Discussion

3.1 Analyzing the error in question number 1

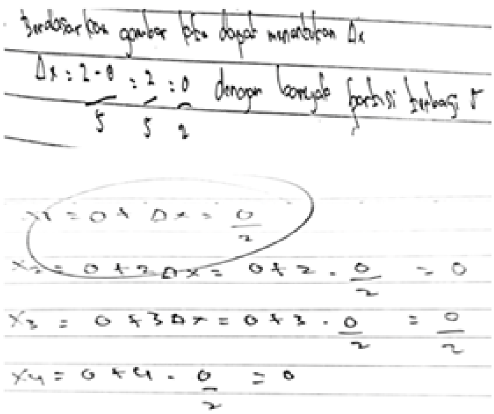

Fig. 3. First type of error – low and middle criteria student in the first question

Based on Figure 3, students in low and middle criteria clearly understand the question, which asks them to determine the area of five rectangles. There was, however, some difficulty in determining the final result since the area of the rectangle should have a meaningful *length x width*, which means Δx is the width of each rectangle and $f(xi)$ represents the height/length of the shape. The answer sheet shows that $\Delta x = 0$ and $f(xi) = 0$. Information obtained from interviews with respondent A6 is as follows. "I understand what the question asks for, but when it is entered into the formula/definition I am confused about continuing the process, then I do not know how to calculate the area of each rectangle"

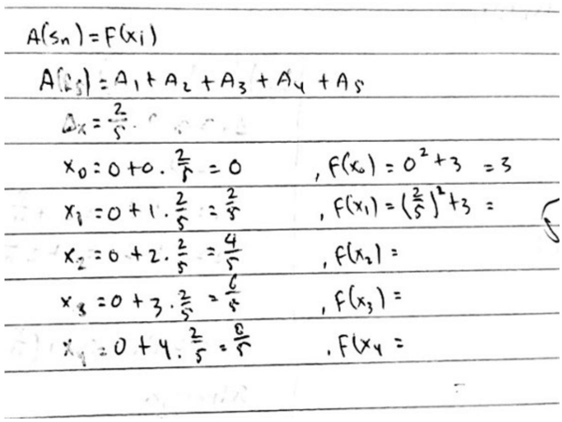

Fig. 4. Second type of error – low and middle criteria student in the first question

Additionally, respondent A4 stated that he had difficulty continuing his work because he did not know what steps to take next (Figure 4).
"After I was able to determine Δxs, then I was confused about what to do so I did not continue to solve this problem"

Furthermore, Figure 5 shows that high-performing students have difficulty understanding the formula and interpreting the question graph. There are five partitions in the question whose area needs to be calculated, while the students calculate the area for the nth partition, so that the formula is applied without checking whether it refers to the information provided. This was confirmed during the interview with respondent A1
"I tried to work on the problem by using the general formula for calculating circumscribed polygons and then substituting the known sigma value"

Based on this explanation, the error analysis in question number 1 shows that students have difficulty using the formula to calculate the area of the circumscribed polygon, and difficult to answer the question when it needs several steps to solve, namely you have to determine Δx and $f(xi)$ which then multiply and add up the results for each polygon. The results of this study are in accordance with previous studies that indicated when students memorize procedures, rules, and routines without much understanding, the concept is forgotten and they become accustomed to doing things in a procedural

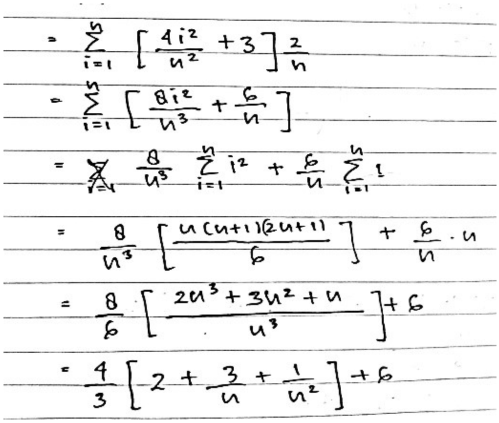

Fig. 5. Student high criteria error in first question

manner. Therefore, they were unaware of the real problem [2][13]. The ability to connect conceptual and procedural knowledge is one of the most fundamental tenets of understanding [10].

3.2 Analyzing the error in question number 2

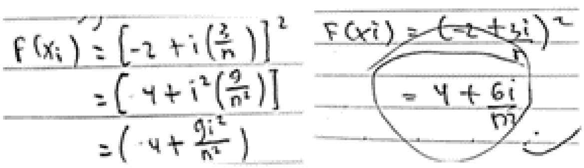

Fig. 6. Student low and middle criteria error in the second question

In question number 2 in Figure 6 depicts that students in the medium and low criteria are experiencing problems with their prior knowledge regarding the concept of exponents where it should be $(a+b)^2 = a^2 + 2ab + b^2$ while they assign a power of 2 to each value or multiply the number with its exponent. Additionally, based on the results of the interview, it can be concluded that they still encounter difficulties when answering questions requiring previously learned concepts.

Moreover in Figure 7, an error is made in the solution when x_0, which is the starting point or lower limit, which should be -2, is written by the student as 0 because it is assumed that all starting points are zero. It indicates students' obstacles to implementing the definition for solving the problem. Based on the interview to respondent A3, it shows that students have difficulty applying definitions.

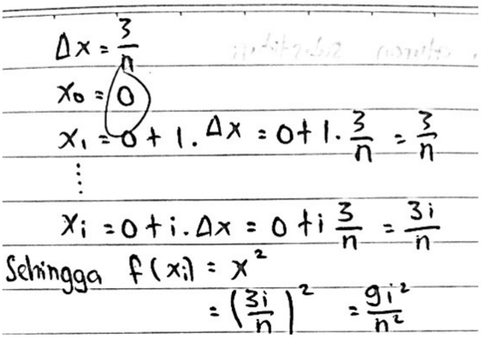

Fig. 7. Student high criteria error in the second question

"I have difficulty answering questions using definitions because the process is longer than if I just enter the values using the integral formula"

These finding is shown that students' orientation toward answering questions is directed toward the final result, rather than why the answer is correct. The difficulty the students experienced in understanding the steps for answering questions in accordance with the definition can be seen from their inability to answer them. In addition, they do not have a clue as to how to begin constructing the evidence, and cannot use the definition (concept) and principle already known, so they will likely begin by demonstrating what must be proved first [14]. Even though the problem would occur, they would not be able to identify and solve it. It is crucial for the teacher to develop students' logical reasoning skills for optimizing a learning process [15].

Furthermore, there are students who experience difficulties with the prerequisite material, and naturally they will encounter difficulties in solving the questions. Therefore, educators must know about students' existing knowledge and understanding before attempting to teach something new [16]

3.3 Analyzing the error in question number 3

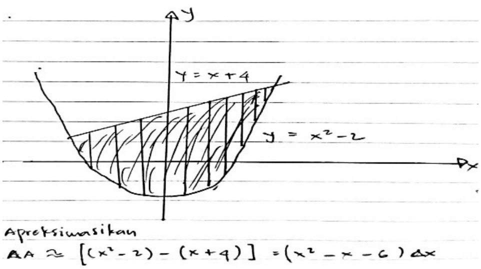

Fig. 8. First type of error – low and middle criteria student in the third question

For students with low and medium ability, they made several mistakes in solving this problem. Based on the Figure 8, it shows that students have difficulty determining which function has a greater value than the other so that the results can be accurate. Another mistake students make is in determining integration, in this case due to the fact that the picture is sliced parallel to the y-axis, thus the width of each partition is Δx and not Δy as shown in the Figure 9 . from interviews with respondent A3, information was

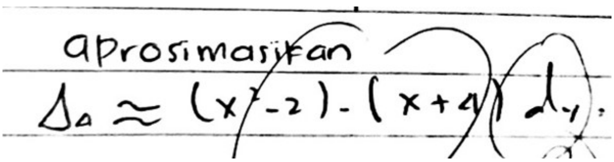

Fig. 9. Second type of error – low and middle criteria student in the third question

obtained, "I had difficulty solving problems with pictures, namely I was confused about choosing a function with a larger value, so I chose $y = x^2 - 2$, because it has a power of 2".

In addition, students with this type of ability do not take steps to find the intersection point of two curves to determine the upper and lower limits, which means that they only guess at the upper and lower limits of the shaded area based on the graph appearing (Figure 10).

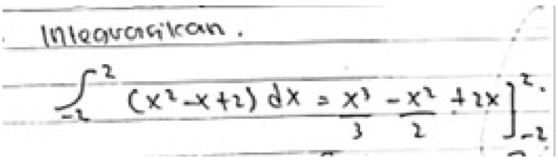

Fig. 10. Third type of error – low and middle criteria student in the third question

The interviews with respondent A5 obtained information,
"I do not know how to determine the upper and lower limits of the graph, so I just guessed that the upper limit is 2 and the lower limit is -2".

This fact shows that students have problems with prior knowledge and applying definitions. Based on what we have stated previously, prior knowledge refers to a mathematical skill or competence owned by a student that can be used to understand and solve problems relating to subsequent mathematical concepts. These skills and competencies must have been acquired by students at their previous educational levels or during discussions of previous mathematics topics. A student with high prior knowledge (adequate) will find it easier to solve problems on higher mathematical concepts (advanced), which will ultimately allow them to learn the next material more easily [17].

Furthermore, students encounter difficulty in solving problems requiring the interpretation of images with several steps involved. For a student to understand a graph, they must be able to make sense of the information presented in it. The ability of students to interpret graphs has been identified as a weakness by many researchers, previous research documents students' inability to understand more than simple graphs. Students often make the mistake of interpreting graphs as pictures, failing to note differences in scale, presuming that scales will start at zero, and connecting dots rather than fitting curves to noisy data [18] [19] [20].

4 Conclusion

Firstly, students have difficulty answering questions that require concepts that have been previously studied (prior knowledge). In this case, the prerequisite material was not sufficiently mastered. Thus, it is necessary for students to review the basic concepts of Calculus before studying Calculus 2.

Secondly, difficulty applying formulas/definitions to solving problems. Students tend to just memorize formulas without understanding their meaning in depth. As a result, students are confused about choosing the right formula or steps to solve certain problems. To overcome this, students need to get a lot of practice working on applicable questions that require an understanding of the concept of Calculus 2.

Lastly, difficulty answering problem-solving exercises or questions that require a number of steps before an answer can be obtained, particularly those involving graphs. Students are lacking in problem-solving skills in calculus, which requires creative and critical thinking. Consequently, students need regular training in problem-solving skills and interpreting graphs as a means of developing their problem-solving abilities.

On the other hand, educators must provide diagnostic assessments at the beginning of the material to ensure the students have mastered the prerequisite material. In the event that they still have difficulties, it is best to resolve them before moving on to the core material in order to avoid problems later on. As well as provide problems that can facilitate the development of thinking skills, the interpretation of graphs, and the application of definitions to the calculus learning process while considering and facilitating the students' learning needs.

References

1. M. Kurniasih, S. Waluya, Rochmad, J. Phys. Conf. Ser **1511**(1) (2020). DOI 10.1088/1742-6596/1511/1/012106.
2. R. Das, K. Das, J. Alam, Int. J. Sci. Res **7**(8), 1444–1449, (2018). DOI 10.21275/ART2019781.
3. P. Jiang, Y. Jiang, Int. J. Emerg. Technol. Learn **17**(11), 73–82, (2022). DOI 10.3991/ijet.v17i11.28611.
4. Y. Morales-López, Y. Chacón-Camacho, W. Vargas-Delgado, Mathematics **9**(15), 1–18, (2021). DOI 10.3390/math9151741.
5. F. Guiñez, S. Bustos, S. Martinez, P. Dartnell. A b-learning program for developing mathematical knowledge for teaching in in-service primary school teachers (2017)

6. M. Özden, Kuram ve Uygulamada Egit. Bilim **8**(2), 633–645, (2008)
7. U. Kultsum. The concept of pedagogical content knowledge (pck): Recognizing the english teachers' competences in indonesia (2017). DOI 10.2991/icirad-17.2017.11.
8. E. Washburn, E. Binks-Cantrell, R. Joshi, S. Martin-Chang, A. Arrow, Ann. Dyslexia **66**(1), 7–26, (2016). DOI 10.1007/s11881-015-0115-x.
9. J. Hagman, E. Johnson, B. Fosdick, Int. J. STEM Educ **4**(1) (2017). DOI 10.1186/s40594-017-0070-7.
10. D. Serhan, Int. J. Res. Educ. Sci **1**(1), 84–88, (2015). DOI 10.21890/ijres.00515.
11. A. Alam, J. Educ. Gift. Young Sci **8**(March), 407–433, (2020). URL https://dergipark.org.tr/en/pub/jegys/issue/52150/660201%0Ahttps://dergipark.org.tr/en/download/article-file/991238. Available:
12. P. Jiang, Y. Zhang, Y. Jiang, B. Xiong, Front. Psychol **13**(November), 1–16, (2022). DOI 10.3389/fpsyg.2022.998586.
13. A. As'ari, A. Mahmudi, E. Nuerlaelah, J. Math. Educ **8**(2), 145–156, (2017). DOI 10.22342/jme.8.2.3961.145-156.
14. M. Noto, N. Priatna, J. Dahlan, J. Math. Educ **10**(1), 117–125, (2019). DOI 10.22342/jme.10.1.5379.117-126.
15. A. Adinda, I. Purwanto, T. Chandra, Eurasian J. Educ. Res **2021**(95), 17–35, (2021). DOI 10.14689/EJER.2021.95.2.
16. J. Young-Loveridge, Teach. Curric **8**(1) (2005). DOI 10.15663/tandc.v8i1.83.
17. M. Muntazhimah, T. Turmudi, S. Prabawanto, J. Phys. Conf. Ser **1731**(1) (2021). DOI 10.1088/1742-6596/1731/1/012043.
18. H. Chang, M. Linn, J. Res. Sci. Teach **50**(7), 858–886, (2013). DOI 10.1002/tea.21089.
19. J. Clement, in *Proceeding of ninth conference of the International Group for the Psychology of Mathematics Education* (1985), p. 1–7
20. K. Lai, J. Cabrera, J. Vitale, J. Madhok, R. Tinker, M. Linn, J. Sci. Educ. Technol **25**(4), 665–681, (2016). DOI 10.1007/s10956-016-9621-9.