Modeling System Dynamics Reverse Logistics of Polyethylene Terephthalate (PET): Equilibrium Approach

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Abstract. The incessant beverage industry, which uses a lot of PET plastic as single-use packaging, dramatically contributes to the problem of plastic waste in the world and Indonesia. On the other hand, PET is a material with high economic value because it is easily recycled many times. So, post-consumption PET recycling has the potential to create a circular economy. In a circular economy, reverse logistics is crucial because it enables waste reuse and reduces resource wastage and environmental pollution. In reverse logistics (RL), the quantity of PET waste collected can change linearly or non-linearly over time. These changes affect material flow in remanufacturing recycling systems, affecting rPET material flow to manufacturers in closed supply chain systems (CLSCs). This study proposes a dynamic approach to analyzing reverse logistics equilibrium in CLSC that considers parameter dynamics and product flow. In the PET supply chain equilibrium model, the supply chain is viewed as a system of interconnected entities, namely the manufacturer (M), Distributor (D), the consumer (C), collector (C_o), remanufacture (R), and other stakeholders (regulators, R) who interact with each other through the exchange of goods, services, and resources. To evaluate the performance of the reverse logistics PET system, three scenarios were given: uncontrolled scenario, scenario on government policy control to Manufacturing (u_1), and scenario on government policy control to consumers (u_2). The simulation results show that system performance becomes better for the sustainability of the circular economy when given control over the consumer because consumers are the largest entities in this system. This developed model contributes to helping understand the performance of CLSC systems in various situations and make informed decisions to maintain system balance.

Keywords: system dynamics, reverse logistics, polyethylene terephthalate, equilibrium approach

1 Introduction

The use of plastic that is not environmentally friendly is a problem for the environment, both in Indonesia and around the world, having a severe negative impact on the environment [1]. Plastic waste is not only a problem in cities but also in the ocean. Indonesia is the second largest contributor of plastic waste flowing into the sea after China [2]. If not

H. S. Panigoro et al. (eds.), *Proceedings of the 2nd International Conference on Sciences, Mathematics, and Education 2023 (ICOSMED 2023)*, Advances in Social Science, Education and Humanities Research 927,
https://doi.org/10.2991/978-2-38476-410-5_13

appropriately managed, plastic waste pollution can endanger human health, threaten the existence of various protected animals, and damage the environment as a whole [3]. The incessant beverage industry, which uses a lot of PET plastic as single-use packaging, dramatically contributes to the problem of plastic waste in the world and Indonesia. On the other hand, PET is a material with high economic value because it is easily recycled many times. So, post-consumption PET recycling can create a circular economy [[4–6]].

In a circular economy, reverse logistics is crucial because it enables waste reuse and reduces resource wastage and environmental pollution. Reverse logistics focuses on returning products from customers and restoring added value by reusing the entire product or multiple modules, components, and parts. By implementing RL, companies can reduce production costs, environmental pollution, and resource wastage, thus helping mitigate global ecological pressures [7–9].

In reverse logistics, the quantity of PET waste collected can change linearly or non-linearly over time. These changes affect the flow of material in the remanufacturing recycling system, which can affect the flow of rPET material to manufacturers in the Closed Loop Supply Chain (CLSC) chain system. Such changes may also affect the performance of the CLSC system, such as the price of rPET materials and the total profit obtained. Therefore, a CLSC equilibrium model is needed to understand and manage dynamic changes in product quantity in reverse logistics systems [9]. The CLSC network is a system that combines reverse logistics processes with conventional advanced logistics to facilitate the recovery, reuse, and recycling of materials and products [10].

The supply chain equilibrium model is a modeling framework that combines the concept of general equilibrium to analyze interactions and dynamics in a supply chain [11]. In the supply chain equilibrium model, the supply chain is viewed as a system of interconnected agents or entities, such as producers, consumers, recyclers, and other stakeholders, that interact by exchanging goods, services, and resources. The model captures dependencies and feedback between these agents and their decisions. It enables a comprehensive analysis of supply chain dynamics and the effects of various factors, such as pricing, policy, and technological change, on overall equilibrium [12]. The model provides a quantitative framework for evaluating the economic, social, and environmental implications of various strategies and interventions in the context of a circular economy [11].

Some previous studies related to equilibrium in CLSC include research by Fu and Meng (2020) [9], developing dynamic models for CLSCs that consider parameter dynamics and product flow. The result is a dynamic model that allows managers to make better decisions and understand the performance of CLSC systems in various situations. Zhou, Chan, and Wong (2018) [13] built a multi-period supply chain equilibrium model with uncertain demand to study retailer loss-aversion behavior's impact on the CLSC network. The results show how retailer loss-averse behavior will affect decisions, manufacturers' profits, and retailers' utilities on balance. Chan, Zhou, and Wong (2019) [10] analyze CLSC's network balance model that considers multiattribute behaviors, namely retailer risk-averse behaviors, loss-averse behaviors, and regret-averse behaviors, in making decisions from a behavioral finance perspective. The results showed that each of these three behaviors had a significant influence on balance decisions.

Hamdouch and Ghoudi (2020) [12] Developed a balanced supply chain network model consisting of several raw material suppliers, manufacturers, and retailers. The result is that this model sets the basis for supply chain equilibrium problems under common price-dependent demand.

The previous studies above found that implementing the CLSC balance model has been recognized as a proficient, powerful, and economical approach for CLSC actors. However, nevertheless, CLSC network design that balances economic and environmental considerations is highly complex and requires a multidisciplinary approach [14]. In the context of CLSC PET, the model is dynamic because it involves complex interactions between various factors, namely economic, environmental, social, and policy, that influence each other in the process of product recovery at the final stage of life. These factors can change over time and influence strategic and regulatory decisions within CLSC. So, this study proposes a System Dynamic model to model the circularity of PET in CLSC. System dynamic models are mathematical approaches and continuous simulations representing the world as stocks and flows, where supply is accumulated (e.g., goods, people, money). In contrast, flow adjusts stock levels (inflows increase stock and outflows decrease it). The amount of stock that changes continuously is a response to the balance of inflows and outflows continually (constantly) [15].

This study proposes an equilibrium model approach to System Dynamics CLSC to understand the interaction between various elements in forward logistics PET (with manufacturing entities, distributors, and consumers) and reverse logistics PET post-consumption (with entities of consumers, collectors, remanufactures to manufacturers as users of recycling PET materials). This equilibrium model allows government policy control as a strategy to improve PET waste collection, rPET reuse, resource efficiency, and reduce the environmental impact of PET waste discarded into the environment.

2 Analytical Results

2.1 System Model Dynamic of PET Deployment

The Dynamic System Model can be used to analyze the circularity dynamics of PET plastics spread across Indonesia with the concept of a compartment model. The compartment model in this system is divided into five different compartments. The System Dynamics model can be formulated by describing the relationship between rooms using a set of stocks and the flow of material between rooms to another on a time series time scale [16].

This study's PET plastic circular dynamics system model has five main entities. These entities are Manufacturing (M), an entity that produces drinking water in PET bottles; Distributor (D), an entity that distributes drinking water in PET bottles to consumers; Consumer (C), an entity that consumes drinking water in PET bottles, Collector as an entity that collects post-consumption PET bottles (C_o), and Remanufacturing (R) as an entity that recycles post-consumption PET bottles into rPET, which will be redistributed to Manufacturing. In the form of a dynamical system, the model can be formed as follows:

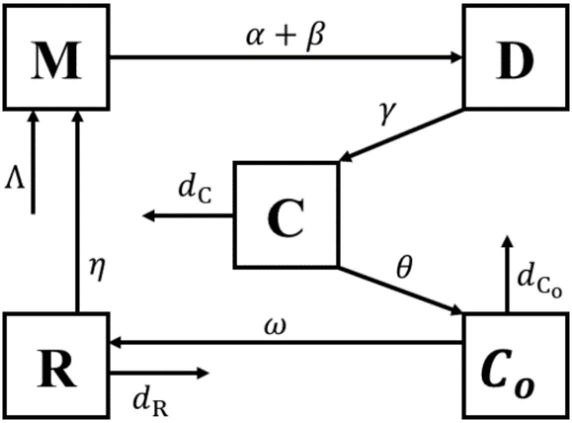

Fig. 1. System dynamic model

If the dynamic system in the compartment diagram is represented in its mathematical form, then the condition of the mathematical model is:

$$\dot{M} = \Lambda - \alpha M + (\eta - \beta)R \quad (1)$$

$$\dot{D} = \alpha M + \beta R - \gamma D \quad (2)$$

$$\dot{C} = \gamma D - (d_C + \theta)C \quad (3)$$

$$\dot{C}_o = \theta C - (d_{C_o} + \omega)C_o + \rho \quad (4)$$

$$\dot{R} = \omega G - (d_R + \eta)R \quad (5)$$

The parameters used in the mathematical model, as defined in Equations (1) to (5), are presented in Table 1.

2.2 Stability Analysis

Stability is a slight change in the initial condition that causes only a small effect on a solution [17]. Asymptotic stability is the effect of a subtle shift that tends to disappear altogether. Instability, on the other hand, is a slight change in an initial condition that significantly affects a solution [18]. Here is an example of the stability of a system of differential equations as follows [19]:

$$\frac{dy_1}{dt} = b_{11}y_1 + b_{12}y_2 \quad (6)$$

$$\frac{dy_2}{dt} = b_{21}y_1 + b_{22}y_2 \quad (7)$$

The system in equations (6) and (7) can be expressed in matrix form as follows.

$$\begin{bmatrix} \frac{dy_1}{dt} \\ \frac{dy_2}{dt} \end{bmatrix} = \begin{bmatrix} b_{11} & b_{12} \\ b_{21} & b_{22} \end{bmatrix} \begin{bmatrix} y_1 \\ y_2 \end{bmatrix}$$

Table 1. Parameter Model

Parameter	Information	Value
Λ	Amount of Raw Materials	70,000
α	Percentage of Raw Material Usage	0.7
β	Percentage of Recycle Material Usage	0.3
γ	Rate of PET plastic used	0.625
θ	Collected PET plastic speed	0.3246
ω	Rate of sorted PET plastics	0.9
η	Rate of recycled PET plastic	0.3
d_C	The rate of PET plastic discarded by the Customer	0.675
d_{C_o}	Rate of PET plastic that failed to be sorted by the Collector	0.1
d_R	The rate of non-recyclable PET plastic	0.167
ρ	PET Trash Import Variable	8864

The matrix above can be briefly written as $\frac{dy}{dt} = By$. Suppose the equation systems (6) and (7) have similar characteristics, namely: λ_1 and λ_2 which is called the Eigen value, where the eigenvalue is obtained from the determinant of the matrix resulting from equations (6) and (7). The eigenvalue serves as a determination of the type of stability of a system equilibrium point, where there are three (3) categories of stability in a system equilibrium [17]:

- Stable Asymptotic, when the eigenvalue of the matrix is worth a negative real.
- Stable Center, when the eigenvalue of the matrix is worth zero.
- Unstable when the eigenvalue of the matrix is worth a positive real.

2.3 Routh-Hurwitz Stability Criteria

Characteristic equations in a system of differential equations can be used to show that the system is stable, asymptotic, or unstable. Characteristic equations are obtained by finding the roots of a system of differential equations. Still, the characteristic equations of a system of ordinary n-dimensional differential equations are n-degree polynomial equations that may be difficult to find all roots explicitly. So, a method is needed to determine the stability of a system, namely the routh-Hurwitz stability criterion. Routh Hurwitz's stability criterion is a criterion that shows the stability of the system by taking into account the coefficients of the characteristic equation, where all the roots of the characteristic equation have a negative fundamental part without calculating the distinct sources directly. Here is an example of a condition on the coefficients of a characteristic equation [20]:

$$b_0\lambda^n + b_1\lambda^{n-1} + \cdots + b_{n-1}\lambda + b_n = 0 \tag{8}$$

Next, the coefficients of the characteristic equation are compiled into the Routh Hourwitz table as follows: [20]

Table 2. Routh-Hurwitz

λ^n	a_0	a_2	a_4
λ^{n-1}	a_1	a_3	a_5
λ^{n-2}	b_1	b_2	b_3
.	.	.	.
.	.	.	.
λ^0	q		

The Routh-Hurwitz criterion has several conditions. Here are certain conditions on the Routh-Hurwitz stability criterion [21]:

- Condition $n = 2$, has a Routh-Hourwitz conditions are: $a_1 > 0, a_2 > 0$
- Condition $n = 3$, has a Routh-Hourwitz condition $a_3 > 0, a_1a_2 > a_3$
- Condition $n = 4$, has a Routh-Hourwitz condition $a_4 > 0, a_2 > 0, a_1 > 0, a_3 > 0 and a_3(a_1a_2 - a_3) > a_1^2 a_4$
- Polynomial conditions of degree n, then there are n Routh-Hourwitz conditions.

2.4 Runge Kutta order 5

Runge Kutta's method of order 5 is one of the numerical methods used to solve differential equations with initial conditions, and this method is a development of Euler's plan so that this method has better accuracy than Heun's method and Euler's method [22]. Runge Kutta's form of order 5 has the following equation [23]:

$$y_{i+1} = y_i + \frac{h}{6}(k_1 + 4k_4 + k_5) = i \in \mathbb{N} \tag{9}$$

with:

$$k_1 = f(t_i, y_i) \tag{10}$$

$$k_2 = f\left(t_i + \frac{h}{3}, y_i + \frac{h}{3}k_1\right) \tag{11}$$

$$k_3 = f\left(t_i + \frac{h}{3}, y_i + h\left(\frac{1}{6}k_1 + \frac{1}{6}k_2\right)\right) \tag{12}$$

$$k_4 = f\left(t_i + \frac{h}{2}, y_i + h\left(\frac{1}{8}k_1 + \frac{3}{8}k_3\right)\right) \tag{13}$$

$$k_5 = f\left(t_i + \frac{h}{2}, y_i + h\left(\frac{1}{2}k_1 - \frac{3}{2}k_3 + 2k_4\right)\right) \tag{14}$$

Each of the k partitions in equations (10) through (14) can be described as follows:

- k_1 is the first step of the partition
- k_2 is the midpoint partition k_1
- k_3 is the midpoint partition k_2
- k_4 is the midpoint partition k_3
- k_5 is the midpoint partition k_4 and the last partition.

3 Optimal Control

The problem of controlling PET waste can be solved by providing optimal control. In this case, two different types of controls and four different scenarios will be given. In this case study, control will be given to the sub-compartments of Manufacturer (M) and Customer (C) so that the scenario consists of four types: a scenario without control and a scenario with control. u_1, scenarios with controls u_2 and scenario with controls u_1 and u_2. The following is a model of the distribution of PET waste after being given control, thus causing changes in the PET distribution model in equations (1), (2), (3), and (4) to:

$$\dot{M} = \Lambda - \alpha M + (\eta - \beta) - u_1 M \tag{6}$$

$$\dot{D} = \alpha M + \beta R - \gamma D + u_1 M \tag{7}$$

$$\dot{C} = \gamma D - (d_c + \theta)C - u_2 C \tag{8}$$

$$\dot{C_o} = \theta C - (d_g + \omega)C_o + u_2 C \tag{9}$$

After being given a PET deployment model with control, the next goal is to find the control equation (u) of each. Next, search for the control. u_1 then the Hamiltonian function must be obtained. The Hamiltonian procedure consists of a goal function and a model that has been given optimal control and variables λ as a substitute variable. Here is the purpose function used and the Hamilton equation generated in the equation below [24]:

$$J_{\text{Min}} = \int A_1 M + A_2 C + C_1 u_1^2 + C_2 u_2^2 dt \tag{10}$$

$$H = (A_1M + A_2C + C_1u_1^2 + C_2u_2^2) + \lambda_1(\Lambda - \alpha M) + (\eta - \beta) - u_1M$$
$$+ \lambda_2(\alpha M + \beta R) - \gamma D + u_1M + \lambda_3(\gamma D) - (d_c + \theta)C - u_2C + \lambda_4(\theta C)$$
$$- (d_g + \omega)C_o + u_2C + \lambda_5(\omega G) - (d_R + \eta)R \tag{11}$$

After obtaining the equation corresponding to equation (11), the control u_1 searchable by: $\frac{\partial H}{\partial u} = 0$ It can also be used in search controls u_2. Based on this, it can be determined the general form of optimal control is as follows:

$$u_1 = \frac{\lambda_1 - \lambda_2}{2C_1}M \tag{12}$$

$$u_2 = \frac{\lambda_3 - \lambda_4}{2C_2}C \tag{13}$$

Based on equations (12) and (13), two general forms of control are obtained u_1 and control u_2, the exact value is 0.9.

4 Stability Analysis Over Equilibrium Point

In finding the equilibrium point, the first step is to make equations (1)-(5) equal to zero. It aims to obtain optimal conditions, as it is known in derivative applications in calculus that, to accept optimal conditions, the first derivative of an equation must be equal to zero. Since equations (1)-(5) are first-order differential equations or can be said to be the first derivative, they can be explicitly equalized to zero. An equilibrium point is obtained from the model system with a scenario in the absence of control so that the equilibrium point will be displayed in the following equation.

$$K^* = (M^*, A^*, C^*, C_o^*, R^*) \tag{14}$$

where:

$$M^* = \frac{G_1}{L_1} \tag{15}$$

$$A^* = \frac{G_2}{L_2} \tag{16}$$

$$C^* = \frac{G_3}{L_3} \tag{17}$$

$$C_0^* = \frac{G_4}{L_4} \tag{18}$$

$$R^* = \frac{G_5}{L_5} \tag{19}$$

Where in the equation (13), (14), (15), (16) and (17) elements $G = G_1, G_2, ...G_5$ and $L = L_1, L_2, ..., L_5$. This is a simplification of the equilibrium point result that can be seen in Appendix A. Based on equation (12), the parameters that have been given will be substituted based on Table 1. If the value of each parameter is replaced, the value of each equilibrium point in the entity is: $M^* = 150.000$, $D^* = 210.110$, $C^* = 140.239$, $C_o^* = 45.521$, and $R^* = 87.729$.

The equilibrium point obtained next is to represent Equations (1) to (5) that have been linear into matrix form. The matrix is obtained as follows:

$$\begin{bmatrix} \dot{M} \\ \dot{D} \\ \dot{C} \\ \dot{C}_o \\ \dot{R} \end{bmatrix} = \begin{bmatrix} -\alpha & 0 & 0 & 0 & \eta-\beta \\ \alpha & -\gamma & 0 & 0 & \beta \\ 0 & \gamma & -(d_c+\theta) & 0 & 0 \\ 0 & 0 & \theta & -(d_{C_o}+\omega) & 0 \\ 0 & 0 & 0 & \omega & -(d_R+\eta) \end{bmatrix} \tag{20}$$

Then substitute the equilibrium point into the matrix, such that the Eigenvalue is obtained by: $\lambda_1 = -0.2314+0.0000i, \lambda_2 = -0.7759+0.4252i, \lambda_3 = -0.7759-0.4252i, \lambda_4 = -1.3084+0.0000i, and \lambda_5 = -0.7000+0.0000i$. The model is stable since all eigenvalues obtained are negative in the real number part.

5 Numerical simulation

Simulation is used to analyze the dynamics of the numerical representation of the System Dynamics model. There are three simulation scenarios carried out. In the first scenario, there is no control in the model system; the model describes existing conditions without any regulatory control over each entity. The second control scenario is exerted on the Manufacturing entity (M), while in the third scenario, control is given to the consumer entity (C). In the first scenario, simulation results are obtained as shown in Fig. 2:

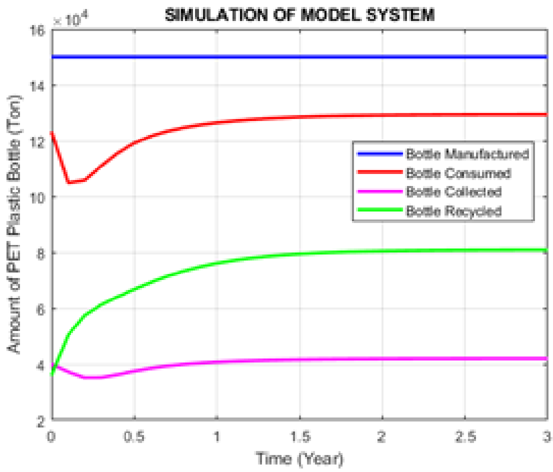

Fig. 2. System simulation model scenario 1

Based on the simulation results in Fig. 2, the model system takes 2.5 years to achieve stability at the equilibrium point. In this condition, regulations have not been applied in a disciplined manner to each entity. So, the collection rate of used PET bottles is still low compared to the level of production of new PET bottles by manufacturers. Due to the intense collection rate, imports used PET from used PET bottles outside those produced by Manufacturing to meet the needs of remanufacturing production. In the second scenario, regulatory control is given to the Manufacturing entity (M) so that simulation results are obtained as shown in Fig. 3.

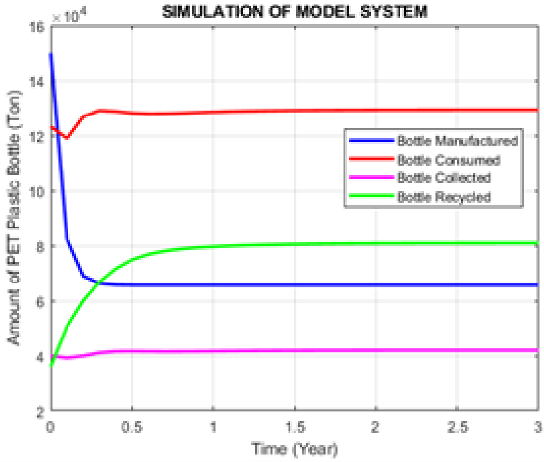

Fig. 3. System simulation model scenario 2

Fig. 3 shows a change in the equilibrium point in the model system, and this is due to regulatory controls applied to Manufacturing so that Manufacturing is obliged to recall post-consumption products produced through Extended Responsibility Producer (EPR). As a result of the implementation of EPR, the collection rate of used PET bottles has increased, but the number of increases is still less significant than the existing conditions. So, the recycling rate of Remanufacturing has also only increased slightly. A new equilibrium point is obtained, namely.$M^* = 65.625, D^* = 206.822, C^* = 129.313, C_o^* = 41.974, and R^* = 80.886$. Model systems can take up to 2 years to achieve stability at equilibrium. Furthermore, in the third scenario, regulatory control is given to the Consumer entity (C). The simulation results are obtained as follows:

Fig. 4 shows that the regulatory controls applied to consumers significantly impact the collection rate of used PET bottles. Regulations make consumers more disciplined and wise in plastic—increased public concern for the environment due to the government's educational efforts to the community. With the increasing collection rate of used PET bottles, the recycling rate of Remanufacturing has increased dramatically. In this scenario, the equilibrium point occurs with the value. $M^* = 150.000, D^* = 310.485, C^* = 59.045, C_o^* = 154.152, and R^* = 296.974$. So, the model takes up to 1.7 years to reach equilibrium.

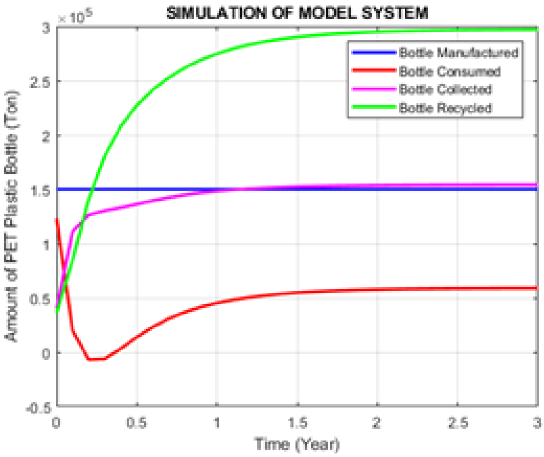

Fig. 4. System simulation model scenario 3

The simulation results of the three scenarios above show that the existence of regulatory controls contributes positively to the increase in the collection of used PET bottles so that the recycling rate also increases. This is in line with the research of Zand et al. [25], Fander and Yaghoubi [26], Ghassemi et al. [27], Zheng et al. [28]. Increased collection and recycling of used PET ultimately contribute positively to the income of the entities and a better environment.

6 Conclusion

Green supply chain, a more environmentally conscious industry, is in demand, which has contributed to an increase in emphasis on the concept of supply chain balance in supply chain management literature. Our study contributes to research in supply chain balance by providing insight into how control affects supply chain performance. We develop a supply chain equilibrium model in a network of Manufacturing, Distributors, Consumers, Collectors, and Remanufacturers. Equilibrium analysis is more focused on reverse logistics on post-consumption of PET with control in the form of regulations applied to manufacturing entities (u_1) and government policy control on consumers (u_2). The simulation results demonstrate that, since consumers comprise most of the system's entities, giving them regulation control will improve system performance for the circular economy's sustainability. The system requires a shorter period to achieve equilibrium by providing regulatory control to consumers.

This model is limited to the equilibrium of quantities on each entity. This model does not consider prices due to changes in demand for each entity. Future research could expand models to overcome these limitations and examine different types of incentives that could lead to the coordination of supply chains within networks with general demand functions

7 Appendix A. Proof of Equilibrium Stability

In finding the equilibrium point, the first step is to make equations (1)-(5) equal to zero. It aims to obtain optimal conditions, as it is known in derivative applications in calculus that, to accept optimal conditions, the first derivative of an equation must be equal to zero. Since equations (1)-(5) are first-order differential equations or can be said to be the first derivative, they can be explicitly equated to zero. Generally, the notation for equilibrium points on variables uses stars, such as (x^*, y^*, z^*). Here are the first steps in determining the equilibrium point:

$$\begin{aligned}\frac{dM}{dt} &= \Lambda - \alpha M + (\eta - \beta)R \\ 0 &= \Lambda - \alpha M^* + (\eta - \beta)R^* \\ \alpha M^* &= \Lambda + (\eta - \beta)R^* \\ M^* &= \frac{\Lambda + (\eta - \beta)R^*}{\alpha}\end{aligned} \tag{A.1}$$

$$\begin{aligned}\frac{dA}{dt} &= \alpha M + \beta R - \gamma A \\ 0 &= \alpha M^* + \beta R^* - \gamma A^* \\ \gamma A^* &= \alpha M^* + \beta R^* \\ A^* &= \frac{\alpha M^* + \beta R^*}{\gamma}\end{aligned} \tag{A.2}$$

$$\begin{aligned}\frac{dC}{dt} &= \gamma A - (d_C + \theta)C \\ 0 &= \gamma A^* - (d_C + \theta)C^* \\ (d_C + \theta)C^* &= \gamma A^* \\ C^* &= \frac{\gamma A^*}{d_C + \theta}\end{aligned} \tag{A.3}$$

$$\begin{aligned}\frac{dC_o}{dt} &= \theta C + \rho - (d_{C_o} + \omega)C_o \\ 0 &= \theta C^* + \rho - (d_{C_o} + \omega)C_o^* \\ (d_{C_o} + \omega)C_o^* &= \theta C^* + \rho \\ C_o^* &= \frac{\theta C^* + \rho}{d_{C_o} + \omega}\end{aligned} \tag{A.4}$$

$$\begin{aligned}\frac{dR}{dt} &= \omega C_o - (d_R + \eta)R \\ 0 &= \omega C_o^* - (d_R + \eta)R^* \\ (d_R + \eta)R^* &= \omega C_o^* \\ R^* &= \frac{\omega C_o^*}{d_R + \eta}\end{aligned} \tag{A.5}$$

So that the equilibrium point will be obtained through the substitution of the equation as follows.

$$K^* = (M^*, A^*, C^*, C_o^*, R^*) \tag{A.6}$$

Obtained $M^*, A^*, C^*, C_o^*, R^*$ They are as follows.

$$M^* = \frac{L_1}{G_1}$$
$$A^* = \frac{L_2}{G_2}$$
$$C^* = \frac{L_3}{G_3}$$
$$G^* = \frac{L_4}{G_4}$$
$$R^* = \frac{L_5}{G_5}$$

Where for each variable L_1, L_2, L_3, L_4, L_5 and variable G_1, G_2, G_3, G_4, G_5 is an example that contains results as written below.

- $L_1 = \Lambda\beta\omega\theta - \Lambda d_C d_{C_o} d_R - \Lambda d_C d_{C_o}\eta - \Lambda d_C d_R\omega - \Lambda d_C\eta\omega - \Lambda d_{C_o} d_R\theta - \Lambda d_{C_o}\eta\theta - \Lambda d_R\omega\theta - \Lambda\eta\omega\theta + \beta\omega\rho\theta - \eta\omega\rho\theta$
- $L_2 = \Lambda d_C d_{C_o} d_R + \Lambda d_C d_{C_o}\eta + \Lambda d_C d_R\omega + \Lambda d_C\eta\omega + \Lambda d_{C_o} d_R\theta + \Lambda d_{C_o}\eta\theta + \Lambda d_R\omega\theta + \Lambda\eta\omega\theta + \eta\omega\rho\theta$
- $L_3 = (d_{C_o} + \omega)(d_R + \eta)(\Lambda + \rho)$
- $L_4 = (d_R + \eta)\theta(\Lambda + \rho)$
- $L_5 = \theta\omega(\Lambda + \rho)$
- $G_1 = \alpha(d_C d_{C_o} d_R + d_C d_{C_o}\eta + d_C d_R\omega + d_C\eta\omega + d_{C_o} d_R\theta + d_G\eta\theta + d_R\omega\theta)$
- $G_2 = \gamma(d_C d_{C_o} d_R + d_C d_{C_o}\eta + d_C d_R\omega + d_C\eta\omega + d_{C_o} d_R\theta + d_{C_o}\eta\theta + d_R\omega\theta)$
- $G_3 = d_C d_{C_o} d_R + d_C d_{C_o}\eta + d_C d_R\omega + d_C\eta\omega + d_{C_o} d_R\theta + d_{C_o}\eta\theta + d_R\omega\theta$
- $G_4 = d_C d_{C_o} d_R + d_C d_{C_o}\eta + d_C d_R\omega + d_C\eta\omega + d_{C_o} d_R\theta + d_{C_o}\eta\theta + d_R\omega\theta$
- $G_5 = d_C d_{C_o} d_R + d_C d_{C_o}\eta + d_C d_R\omega + d_C\eta\omega + d_{C_o} d_R\theta + d_{C_o}\eta\theta + d_R\omega\theta$

Appendix B. Proof of Stability Analysis of Mathematic Model After the equilibrium point in the model has been obtained, the next step is to change the model's shape, which was initially a differential equation, into a matrix. Observed in Equations (1)-(5), the equation is a 1st-order differential equation, so the equation needs to be linearized in the following way:

$$J = \begin{bmatrix} \frac{\partial M}{\partial M} & \frac{\partial M}{\partial A} & \frac{\partial M}{\partial C} & \frac{\partial M}{\partial C_o} & \frac{\partial M}{\partial R} \\ \frac{\partial A}{\partial M} & \frac{\partial A}{\partial A} & \frac{\partial A}{\partial C} & \frac{\partial A}{\partial C_o} & \frac{\partial A}{\partial R} \\ \frac{\partial C}{\partial M} & \frac{\partial C}{\partial A} & \frac{\partial C}{\partial C} & \frac{\partial C}{\partial C_o} & \frac{\partial C}{\partial R} \\ \frac{\partial C_o}{\partial M} & \frac{\partial C_o}{\partial A} & \frac{\partial C_o}{\partial C} & \frac{\partial C_o}{\partial C_o} & \frac{\partial C_o}{\partial R} \\ \frac{\partial R}{\partial M} & \frac{\partial R}{\partial A} & \frac{\partial R}{\partial C} & \frac{\partial R}{\partial C_o} & \frac{\partial R}{\partial R} \end{bmatrix} |M^*, A^*, C^*, G^*, R^* \tag{B.1}$$

Such that a linear result matrix is obtained as follows:

$$\begin{bmatrix} \dot{M} \\ \dot{A} \\ \dot{C} \\ \dot{C}_o \\ \dot{R} \end{bmatrix} = \begin{bmatrix} -\alpha & 0 & 0 & 0 & \eta-\beta \\ \alpha & -\gamma & 0 & 0 & \beta \\ 0 & \gamma & -(d_C+\theta) & 0 & 0 \\ 0 & 0 & \theta & -(d_G+\omega) & 0 \\ 0 & 0 & 0 & \omega & -(d_R+\eta) \end{bmatrix} \tag{B.2}$$

Based on the matrix, the eigenvalues are obtained as follows: $\lambda_1 = -0.2314+0.0000i, \lambda_2 = -0.7759+0.4252i, \lambda_3 = -0.7759-0.4252i, \lambda_4 = -1.3084+0.0000i, and \lambda_5 = -0.7000+0.0000i$. The model is stable since all eigenvalues obtained are negative in the real number part.

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