Identifying Instability in the G7 Stock Market Using Two-Regime MS-GARCH Family Model as Risk Measurement Forecasting

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Abstract. This study aims to investigate the instability of the G7 stock market by forecasting value at risk (VaR) as a measure of risk with a two-regime MS-GARCH family model approach. Daily data of composite stock returns from January 2010 to September 2023 from each G7 country are studied. Of the seven stocks from the G7 financial markets, it is found that all returns have the properties of non-normally distributed, leptokurtic, stationary at the mean, time- dependency, volatility, clustering volatility, and fat-tailed. Based on backtesting, significance testing of parameter estimates, and stationarity of the regime-switching process, it is found that the Canadian (TSX Composite Index: ^GSPTSE) and UK (London Stock Exchange: LSEG.L) markets are not suitable to be modeled with the model used in this study. The other five c ountries fi t th e MS-EGARCH(1,1) (France - CAC 40: ^FCHI and Japan - Nikkei 225: ^N225) and MS-GJRGARCH(1,1) (United States - Nasdaq Composite: ^IXIC, Germany - Deutsche Boerse: DB1.DE, and Italy - FTSE MIB Index: FTSEMIB.MI) model approaches.

Keywords: two-regime MS-GARCH, G7 stock market, risk measurement forecasting

1 Introduction

It is well-documented that global events have triggerd rapid fluctuation in asset prices within financial markets. For instance, the September 11, 2001, Attacks on the Twin Towers which not only had a negative impact on the economic growth of the USA but also led to increased financial instabilitily and disrupted international trade which exacerbated the financial crisis in several countries [1–3]. Similarly, the Global Financial Crisis (GFC) in 2008 had an extreme impact including a severe decline in investment across various commodities, rising unemployment, declining financial transactions, asset losses in the US, Europe, and Japan, and the failure of risk measurement (VaR) led to critical invesment errors [4–9]. Then, the last horrific global event that may still have an impact is the Covid-19 pandemic which caused the cessation of economic activity from all sectors due to lockdown policies so that the stock market, GDP, export value, and the value of foreign direct investment (FDI) fell drastically [9–15].

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Hence, the objective of this study is to analyze the influence of events that trigger instability in stock prices within the G7 countries, which encompass Canada, France, Germany, Italy, Japan, the United Kingdom, and the United States. According to World Bank statistics, the G7 countries, while making up only about 10% of the world's population, collectively contribute to around 40% of the global Gross Domestic Product (GDP). Additionally, they are recognized as the largest stock market group globally, both in terms of trading volume and market capitalization [16–18].Despite the dominant position of the G7 countries in global financial markets, significant global events such as the Covid-19 pandemic can have severe adverse impacts on their economies and financial systems. The strong and historically bullish (the market is on an uptrend) US stock market since 2009 faced considerable high volatility in 2020. Similarly, stock markets in other G7 countries also experienced sizable declines, with some as high as 30-40% [19]. This underscores the interconnectedness of global financial markets and their vulnerability to unexpected shocks and crises.

Since many phenomena have a large impact on financial markets, it is necessary for issuers to measure the potential loss of assets or portfolios due to these phenomena. In many literatures, Value at Risk (VaR) is used as a reference to measure market risk which indicates the maximum possible loss of an asset or portfolio at a certain probability level (α) and time horizon. To estimate VaR, it is necessary to describe and analyze the behavior of the stock market over time. Many researchers have studied stock market behavior with various modeling approaches. Empirically, it is stated that stock indices or returns have non-normally distributed characteristics and time-varying volatility. The most well-known model approaches to capture the characteristics of returns are autoregressive conditional heteroscedasticity (ARCH) [20], generalized ARCH (GARCH) [21], and asymmetric GARCH namely Exponential GARCH (EGARCH) [22], Glosten-Jagannathan-Runkle (GJRGARCH) [23], and Threshold GARCH (TGARCH) [24].

However, based on many studies, it is known that the movement of returns is often influenced by phenomena that occur and have a systemic impact on the economy of a country as explained. Drastic changes in the movement of the index or stock returns between investment periods may last for some time or return to the conditions before the change or may change to another movement pattern. Therefore, within one time horizon there are several different structures/models of movement returns. Changes in movement between periods are often referred to as "regime-switching". Due to regime-switching, researchers have noted that empirical VaR estimation using the family of GARCH model approach cannot fully capture various patterns of return fluctuation shifts, resulting in inappropriate VaR prediction and forecasting [25–31].

To overcome the inability of GARCH, a hybrid approach between Markov-Switching (MS) and GARCH family models (MS-GARCH) is proposed, which allows the model parameters to change as the volatility level changes due to regime change [25], [29], [32]. In addition, MS-GARCH can also capture two volatility persistence, namely persistence within the regime and persistence between regimes [29, 30]. The MS-GARCH model has been widely used for VaR estimation in various stock markets. Kayalidere et al (2018) [33] identified the impact of Turkish Economic instability based on Credit Default Swap (CDS) using MS-GARCH(1,1). Torres et al (2020) (2021) [34], [35] modeled the coffee, chocolate, and sugar agricultural sector markets with MS-ARCH and MS-GARCH and used MS-AssymetricGARCH for US Stock Trading. Sema et al (2021) [14] used MS-GJRGARCH(1,1) on S&P 500 data which captured both pre and post Covid-19 regime changes. Chocholatá (2022) [15] estimated VaR on 3 stock markets also with MS-GJRGARCH(1,1). In this study, we will empirically examine the instability of the G7 stock market during 2010 to 2023 with the MS-GARCH family model approach with two conditions of high volatility and low volatility changes.

2 Methdology

Let P_t and P_{t-1} denote the closing price of a stock index at time t and $t-1$, respectively, then the return (r_t) is calculated as follows:

$$r_t = \ln\left(\frac{P_t}{P_{t-1}}\right) \tag{1}$$

Identification of return data characteristics can be determined based on the size of skewness and kurtosis [36], [37], and the Anderson-Darling and Liliefors test are used to determine normality [38], [39].

In this study, to capture time dependency, the Autoregressive Moving Average (ARMA) (p,q) model is used with the following equation [40–43]:

$$\begin{gathered} r_t = \sum_{i=1}^{p} \phi_i r_{i-1} + \varepsilon_1 - \sum_{i=1}^{p} \theta_i \varepsilon_{t-1} \\ \phi(B) r_t = \theta(B) \varepsilon_t \end{gathered} \tag{2}$$

with B as a backshift operator is the lag operator where $B^i r_t = r_{t-i}$ and ε_t is a white noise process i.e. series of ε_t are uncorrelated with mean 0 and variance $\sigma^2(\varepsilon_t\ WN(0,\sigma^2))$. The ARMA$(p,q)$ model is stationary if $\phi(B)r_t = 0$.

2.1 The Family of GARCH Model

The GARCH model was first proposed by Bollerslev (1986) [21] which is an extension of the ARCH model by Engle (1982) [20] that is able to capture volatility. An indication of volatility is seen in the error variance obtained from the ARMA(p,q) as a mean model containing heteroscedasticity which has an impact on the nature of parameter estimators that are no longer efficient. Identification of the presence of ARCH/GARCH influence on the model can be tested using the Lagrange Multiplier (LM test).

Let ε_t denote the error of the stochastic process for a discrete time period t and ψ_t denote the information set of the error variance over time period t, then the general form of the GARCH(P,Q) model is as follows

$$\varepsilon_t | \phi_{t-1}\ N(0,\sigma_t^2) \tag{3}$$

$$\sigma_t^2 = \omega + \sum_{i=1}^{Q} \alpha_i \varepsilon_{t-i}^2 + \sum_{j=1}^{P} \beta_j \sigma_{t-j}^2 \tag{4}$$

where σ_t^2 represents the residual variance at time t, ω is a constant, α_i is the $i-th$ ARCH parameter, with $i = 1,2,...,Q$, ε_{t-i}^2 represents the squared error at time $t-i$, β_j is the $j-th$ GARCH parameter, with $j = 1,2,...,P$, and h_{t-j} represents the residual variance at time $t-j$. In many studies, it is stated that the GARCH(1,1) model is sufficient to capture volatility clustering [44], [45].

The fact that stock return data is negatively correlated with volatility causes the GARCH model to have limitations and ignore the signs of returns. Ignoring the sign of the return causes changes in the residual variance value in period $t+1$ to be uncorrelated with the return data in period t so that the GARCH model becomes less relevant to apply.

The EGARCH Model

The EGARCH(P,Q) model as in equation (5) uses the logarithmic form of σ_t^2 so no restriction is needed in the model parameters [22]. The GARCH parameter is represented by β. The parameters α and γ capture the size and sign of the leverage effect.

$$\ln \sigma_t^2 = \omega + \sum_{i=1}^{Q} \alpha_i \left| \frac{\varepsilon_{t-i}}{\sigma_{t-i}} - E\frac{\varepsilon_{t-i}}{\sigma_{t-i}} \right| \tag{5}$$

$$+ \sum_{j=1}^{P} \beta_j \ln(\sigma_{t-j}^2)$$

$$+ \sum_{k=1}^{r} \gamma_k \frac{\varepsilon_{t-k}}{\sigma_{t-k}}$$

It is stated [46–48] that the indicator of asymmetric volatility occurs if and only if $\gamma \neq 0$. The value of $\gamma < 0$ means that negative phenomena affect volatility rather than positive policies. Conversely, if the value of $\gamma > 0$, then the return is positively correlated with volatility, which means that positive phenomena have more effect on volatility. An indicator of asymmetric volatility if and only if $\gamma \neq 0$. The effect of volatility from positive previous return data is expressed as $\alpha + \gamma$, while $\gamma - \alpha$ expresses the effect of volatility from negative previous return data [49].

The GJRGARCH Model

The GJRGARCH model is also a development of the GARCH model to capture asymmetric volatility with parameter γ in the GJRGARCH(P,Q) model and by adding $d_(t-1)$ as a virtual indicator worth 0 or 1 as in equation (6) [23],[49–51].

$$\sigma_t^2 = \omega + \sum_{i=1}^{Q} \alpha_i \varepsilon_{t-i}^2 + \sum_{j=1}^{P} \beta_j \sigma_{t-j}^2 \tag{6}$$

$$+\gamma \varepsilon_{t-1}^2 d_{t-1}$$

with

$$d_{t-1} = \begin{cases} 1, & \varepsilon_{t-1} < 0 \\ 0, & \varepsilon_{t-1} \geq 0 \end{cases}$$

At time t, if a positive phenomenon occurs, it will increase volatility by α_t and if a negative phenomenon occurs, it will increase volatility by $\alpha_t + \gamma_t$. The value of $\gamma > 0$ means that negative phenomena in the previous period have a greater influence on current volatility than positive phenomena, vice versa [49].

2.2 The MS-GARCH Model

Suppose s_t, with $t = 1,2,...,z$, is a regime variable that follows a first-order Markov chain with a non-periodic transition matrix $P_{z\times z}$, the MS-GARCH(P,Q) model is expressed by (7) [25], [26], [35], [52–54].

$$\begin{cases} \varepsilon_t = \sqrt{\sigma^2_{t(s_t)}}\eta_t \\ \sigma^2_{t(s_t)} = \omega_{(s_t)} + \sum_{i=1}^{q} \alpha_{i(s_t)}\varepsilon^2_{t-i(s_t-i)} \\ \quad + \sum_{j=1}^{p} \beta_{j(s_t)}\sigma^2_{t-j(s_t-j)}, \qquad \forall t \in \mathbb{Z} \end{cases} \tag{7}$$

The matrix $P_{z\times z}$ contains transition probabilities $p_{ij} = P\{s_t = j|s_{t-1} = i\}$ i.e. the probability of changing from the ith regime at time $t-1$ to the j-th regime at time t, with $0 < p_{ij} < 1, j \in \{1,2,...,z\}$, and $\sum_{j=1}^{k} p_{ij} = 1$. For example, for $s_t = 1,2$, the transition matrix formed is

$$P_{2\times 2} = \begin{bmatrix} p_{11} & p_{21} \\ p_{12} & p_{22} \end{bmatrix}$$

In this study, regime change will be investigated with 3 schemes: 1) MS-GARCH (Normal – t-Student) which means state 1 is defined as normal distributed GARCH and state 2 is t-student distributed, 2) MS-GARCH (Normal - EGARCH) which means state 1 is normal distributed GARCH and state 2 is EGARCH , and 3) MS-GARCH (Normal - GJRGARCH) which means state 1 is normal distributed GARCH and state 2 is GJRGARCH.

2.3 VaR and Backtesting

VaR is the expected loss of financial market risk of an asset or portfolio over a specified period at a set level of significance [55–57]. In general, VaR is measured at a confidence level of $(1-\alpha)100\%$ i.e. $P(Risk < -VaR_\alpha)$ with α being 1% to 5%. Let L denote the loss of an asset/stock/portfolio during period T, then VaR is a risk statistic that measures the risk of an asset/stock/portfolio during period T. Assuming that F_L is the distribution function of L, then according to [55], [58] at level $\alpha_\in (0,1)$ it is defined that

$$\begin{aligned} VaR^T_\alpha &= inf\{x \in \mathbb{R} : P(L > x) \leq 1-\alpha\} \\ &= inf\{x \in \mathbb{R} : F_L \geq \alpha\}. \end{aligned} \tag{8}$$

The accuracy of VaR estimation results and the effectiveness of the model used to estimate VaR can be evaluated by the backtesting method [59]. The VaR backtesting evaluation results provide specific information whether the VaR estimated from the modeling used is smaller (underestimate) or larger (overestimate) than the actual VaR. The simplest VaR backtesting method, which is widely used by VaR researchers, is the Kupiec-POF (proportion of failures) test.

If N is defined as the total observations and x is the number of VaR prediction errors, using the likelihood ratio (LR) statistic, the Kupiec-POF test is written as in equation (8) [60], [61].

$$LR_{POF} = 2\ln\left[\frac{(1-\frac{x}{N})^{N-x}(\frac{x}{N})^{x}}{(1-\alpha)^{N-x}\alpha^{x}}\right] \sim X^2_{\alpha,1}, \tag{9}$$

with α is the significance level under H_0. If $LR_{POF} < X^2_{\alpha,1}$, then the model is stated to provide correct VaR estimates, *vice versa*.

3 Empirical Results and Discussion

3.1 Data Characteristics and Descriptive Statistics

The data used are daily returns from G7 stock indices are the United States - Nasdaq Composite (^IXIC), Canada - TSX Composite Index (^GSPTSE), France - CAC 40 (^FCHI), Germany - Deutsche Boerse (DB1.DE), Italy - FTSE MIB Index (FTSEMIB.MI), United Kingdom - London Stock Exchange (LSEG.L), and Japan - Nikkei 225 (^N225) from January 2010 to September 2023 (N is around 5500 – 5950 obs.) with the returns movement pattern as shown in Figure 1.

Based on Figure 1, it can be seen that the behavior of each stock index is not stationary to the mean, which is indicated by the movement of the positive trend. According to Chocholatá (2022) [15], when the stock index behavior is not stationary, the return data is assumed to be stationary. This is also supported by the results of the stationarity test calculation using the Dickey-Fuller test statistics (Table. 1) which shows that the returns of the seven stocks are stationary.

It is known that the GFC that occurred in 2008 had a terrible impact on almost all financial markets around the world. However, since 2010, financial markets began to stabilize again until early 2011. However, the phenomenon of European Debt Crisis (EDC) since 2009, which began with the decline of interest rates in EU countries, continued to be volatile until 2012 due to serious economic crises in several countries such as Greece, Portugal, Ireland, Italy and the Republic of Cyprus [62], [63]. Financial markets returned to a more stable state and showed a positive trend until 2017 although between 2013 and 2017 there were high-risk phenomena such as the Russia - Ukraine war and the UK's planned exit from the European Union. This stable condition then experienced a strong shock due to the Covid-19 pandemic.

Based on the event analysis, Figure 1 also indicates that during the observation period there is fundamental volatility clustering so that conditional heteroscedasticity occurs because periods of high volatility mingle with periods of low volatility on very large positive and negative returns. Therefore, it seems that the use of the standard single regime GARCH model is inappropriate as it cannot capture the large persistence of individual shocks. Thus, the regime-switching model approach is more appropriate as it allows us to account for within-regime and between-regime persistence due to switching from a high volatility regime to a low volatility regime.

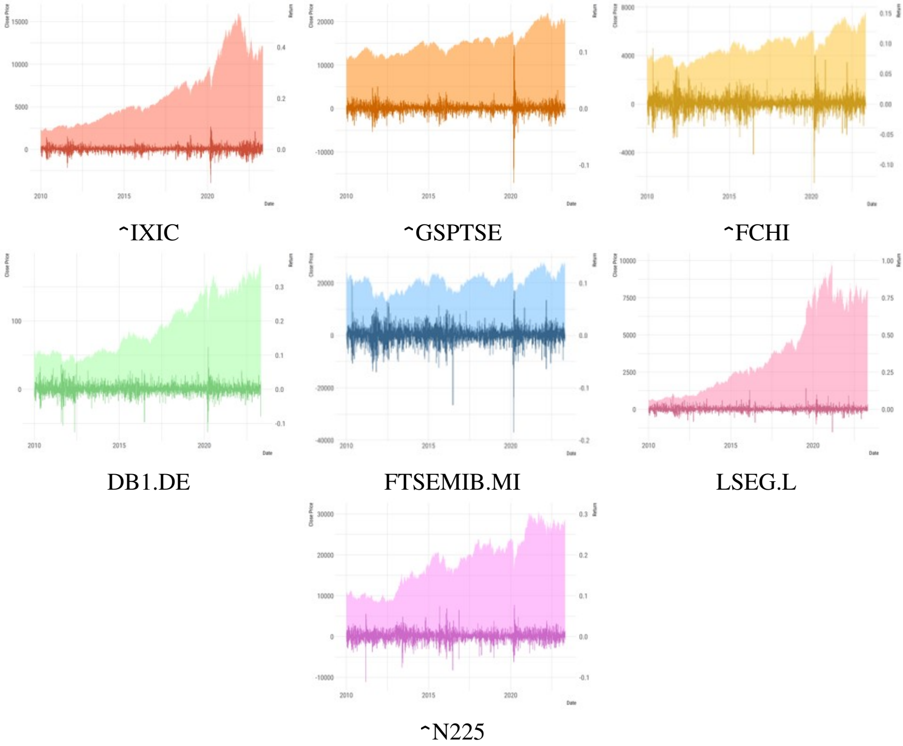


Fig. 1. Development of G7 Stock Indices and Its Returns

Table 1. Descriptive Statistics of Stock Return

	Mean	Max	Min	Std.Dev
^IXIC	0.000	0.089	-0.131	0.013
^GSPTSE	0.000	0.113	-0.132	0.009
^FCHI	0.000	0.092	-0.131	0.013
DB1.DE	0.000	0.123	-0.126	0.016
FTSEMIB.MI	0.000	0.107	-0.185	0.016
LSEG.L	0.000	0.143	-0.155	0.017
^N225	0.000	0.077	-0.112	0.013

Table 1 shows that all stock returns have averages around zero and this information supports the fact that the returns data are stationary at the mean. It is noted that over the observation time horizon, the highest daily return was achieved by the UK market (LSEG.L) at 14.3%, followed by the German market (DB1.DE) 12.3% and the Canadian market (^GSPTSE) 11.3%. Although the UK achieved the highest daily return, it can be seen from Table 1 that the UK market has the highest instability with a standard deviation of 1.7%, followed by the German market and the Italian market at 1.6%.

Table 2. Characteristics of Stock Return

	Skew	Kurt	AD Test	Lillf Test	AugDF Test
^IXIC	-0.60	10.67	49.17	0.09	-15.64
			(0.00)	(0.00)	(0.01)
^GSPTSE	-1.35	36.33	68.86	0.09	-15.56
			(0.00)	(0.00)	(0.01)
^FCHI	-0.49	10.68	42.65	0.08	-14.73
			(0.00)	(0.00)	(0.01)
DB1.DE	-0.39	9.66	27.79	0.06	-15.16
			(0.00)	(0.00)	(0.01)
FTSEMIB.MI	-0.91	13.58	33.67	0.07	-14.56
			(0.00)	(0.00)	(0.01)
LSEG.L	0.09	10.52	31.49	0.06	-15.82
			(0.00)	(0.00)	(0.01)
^N225	-0.41	7.92	23.22	0.06	-14.93
			(0.00)	(0.00)	(0.01)

The value in bracket is the p-value of the test results.

The characteristics of the return data can be seen from Table 2 which shows that the distribution of sample returns of all stocks except the UK market has negative skewness with a very high kurtosis (> 3), so it can be concluded that all observed returns are leptokurtic and fat-tailed. Based on Table 2, it can also be seen that from the Anderson-Darling and Lilliefors normality tests, the return data of all markets are not normally distributed ($p - value < \alpha = 0.05$). In addition, the Augmented Dikkey-Fuller stationarity test ($p - value < \alpha = 0.05$) in Table 2 supports the stationarity identification in Figure 1 and Table 1 that all stock returns studied are stationary.

Based on information on the characteristics of this return data, this study used the MS-GARCH model with the maximum-likelihood method based on several conditions of error distribution assumptions from the ARMA(p,q) mean model, namely normal and t-student. In addition, skewness information is accommodated with the MS-AssymetricGARCH model.

3.2 Mean Model and Residual Diagnostics

Of the seven G7 stocks, it is known that German stock returns are not serially correlated so they are constant (Random Walk). As for the other six countries, stock returns are modeled as in Table 3. Based on Table 3, although German returns are not serially correlated, the LM test shows that all countries including Germany have conditional heteroscedasticity effects at almost all lags.

3.3 Estimation of The MS-GARCH-Family Models

As explained in subchapter 2.2, that in this study regime change is identified based on 3 schemes namely MS-GARCH, MS-EGARCH, and MS-GJRGARCH. The MS-GARCH scheme assumes normal distribution in regime 1 (low volatility regime) and

Table 3. Mean Model and Test of ARCH/GARCH Effect

	Model	LM
^IXIC	AR(1)	0.00
^GSPTSE	ARMA(3,3)	0.00
^FCHI	AR(1,1)	0.00
DB1.DE	RW	0.00
FTSEMIB.MI	AR(1)	0.00
LSEG.L	MA(1)	0.00
^N225	AR(1)	0.00

t-student distribution in regime 2 (high volatility regime). In the MS-EGARCH scheme, both models are estimated assuming normal distribution with GARCH in regime 1 and EGARCH in regime 2. As for the MS-GJRGARCH scheme, both models are estimated under the assumption of normal distribution for GARCH in regime 1 and GJRGARCH in regime 2.

Table 4 shows the parameter estimation results of scheme 1, MS-GARCH with regime 1 (low volatility) assumed under normal distribution and regime 2 (high volatility) assumed under t-student distribution. It is found that $\alpha_{11}+\beta_1<1$ and $\alpha_{12}+\beta_2<1$ for all G7 stocks indicate that the process is stationary in both regimes. With values ranging from 0.831 to 0.971, it shows that the volatility persistence captured by the MS-GARCH model is very high.

Table 4. Estimation Results of MS-GARCH Model

	^IXIC	^GSPTSE	^FCHI	DB1.DE	FTSEMIB.MI	LSEG.L	^N225
	Regime 1 – Low Volatility Regime						
α_{01}	0	0.000001*	0	0.000001*	0.000001*	0.00002*	0
α_{11}	0.083*	0.048	0.075*	0.018	0.051*	0.12*	0.037
β_1	0.867*	0.924*	0.882*	0.953*	0.916*	0.711*	0.885*
$\alpha_{11}+\beta_1$	0.95	0.972	0.957	0.971	0.967	0.831	0.922
	Regime 2 – High Volatility Regime						
α_{02}	0	0	0.00004*	0.0001*	0.0001*	0.003	0.00002*
α_{12}	0.216	0.00001	0.274	0.151	0.302	0.212	0.189*
β_2	0.775*	0.201	0.716*	0.835*	0.692*	0.767*	0.794*
$\alpha_{12}+\beta_2$	0.991	0.20101	0.99	0.986	0.994	0.979	0.983

*) significant at level 5%

In the MS-EGARCH model, the persistence regime is shown by 2 conditions, namely $\alpha_{11}+\beta_1$ and $\alpha_{12}+\beta_2+\frac{1}{2}\alpha_{22}$. Based on Table 5, it is known that the persistence regime 1 (low volatility regime) is stationary for all stock returns. However, in regime 2 (high volatility regime) it can be seen that only the Canadian and Japanese markets are stationary. The non-stationarity of the process will have an impact on the shorter duration of regime 2 period and soon move to regime 1. This is different from the MS-GJRGARCH model approach (Table 6) where both models of regime 1 and regime 2 periods have stationary processes indicated by $\alpha_{11}+\beta_1<1$ and $\alpha_{12}+\beta_2+\frac{1}{2}\alpha_{22}<1$.

Table 5. Estimation Results of MS-EGARCH Model

	^IXIC	^GSPTSE	^FCHI	DB1.DE	FTSEMIB.MI	LSEG.L	^N225
	Regime 1 – Low Volatility Regime						
α_{01}	0.00001*	0.00001*	0.00001*	0.00001*	0.00001*	0.00002*	0.00001*
α_{11}	0.006*	0.065*	0.034*	0.018	0.016*	0.12*	0.053*
β_1	0.993*	0.908*	0.91*	0.951*	0.984*	0.701*	0.944*
$\alpha_{11}+\beta_1$	0.999	0.973	0.944	0.969	1	0.821	0.997
	Regime 2 – High Volatility Regime						
α_{02}	−0.41*	−0.51*	−0.329*	−0.222*	−0.564*	−0.407*	−1.226*
α_{12}	0.157*	0.022*	0.138*	0.17*	0.167*	0.181	−0.125*
α_{22}	−0.237*	−0.415*	−0.216*	−0.133*	−0.276*	−0.412*	−0.351*
β_2	0.955*	0.949*	0.96*	0.964*	0.938*	0.928*	0.865*
$\alpha_{12}+\beta_2$	1.112	0.971	1.098	1.134	1.105	1.109	0.74
$\alpha_{12}+\beta_2+\frac{1}{2}\alpha_{22}$	0.9935	0.7635	0.99	1.0675	0.967	0.903	0.5645

*) significant at level 5%

Table 6. Estimation Results of MS-GJRGARCH Model

	^IXIC	^GSPTSE	^FCHI	DB1.DE	FTSEMIB.MI	LSEG.L	^N225
	Regime 1 – Low Volatility Regime						
α_{01}	0.00001	0.00001*	0.00001*	0.00001*	0	0.00002*	0
α_{11}	0.038	0.036*	0.053*	0.015	0.025*	0.12*	0.023*
β_1	0.654*	0.952*	0.857*	0.958*	0.946*	0.709*	0.907*
$\alpha_{11}+\beta_1$	0.692	0.988	0.91	0.973	0.971	0.829	0.93
	Regime 2 – High Volatility Regime						
α_{02}	0.00001	0.00001*	0.00001*	0.00003*	0.00002*	0.0002*	0.00001*
α_{12}	0.0001*	0.0004*	0.001*	0.00002	0.00001	0.0004	0.05*
α_{22}	0.201*	0.305*	0.285*	0.181*	0.319*	0.349	0.23*
β_2	0.868	0.795*	0.847*	0.897*	0.829*	0.809*	0.81
$\alpha_{12}+\beta_2$	0.8681	0.7954	0.848	0.89702	0.82901	0.8094	0.86
$\alpha_{12}+\beta_2+\frac{1}{2}\alpha_{22}$	0.9686	0.9479	0.9905	0.98752	0.98851	0.9839	0.975

*) significant at level 5%

Table 7. Regime-Switching Properties of MS-GARCH Model

	P_{11}	P_{22}	π^1	π^2	δ_1	δ_2
^IXIC	0.58	0.24	0.64	0.36	2.4	1.3
^GSPTSE	0.96	0.85	0.79	0.21	24.4	6.7
^FCHI	0.75	0.24	0.75	0.25	4	1.3
DB1.DE	0.88	0.53	0.8	0.2	8.3	2.1
FTSEMIB.MI	0.77	0.22	0.77	0.23	4.4	1.3
LSEG.L	0.93	0.00	0.94	0.06	14.7	1
^N225	0.34	0.54	0.41	0.59	1.5	2.2

Table 9. Regime-Switching Properties of MS-GJRGARCH Model

	P_{11}	P_{22}	π^1	π^2	δ_1	δ_2
^IXIC	1.00	1.00	0.11	0.89	250	2000
^GSPTSE	0.84	0.89	0.4	0.6	6.2	9.1
^FCHI	0.23	0.60	0.34	0.66	1.3	2.5
DB1.DE	0.87	0.62	0.75	0.25	7.9	2.6
FTSEMIB.MI	0.44	0.50	0.47	0.53	1.8	2
LSEG.L	0.92	0.00	0.93	0.07	13.2	1

Table 8. Regime-Switching Properties of MS-EGARCH Model

	P_{11}	P_{22}	π^1	π^2	δ_1	δ_2
^IXIC	0.58	0.90	0.19	0.81	2.4	10
^GSPTSE	0.97	0.96	0.6	0.4	37	25
^FCHI	0.27	0.55	0.38	0.62	1.4	2.2
DB1.DE	0.85	0.54	0.76	0.24	6.7	2.2
FTSEMIB.MI	0.35	0.68	0.33	0.67	1.5	3.1
LSEG.L	0.90	0.00	0.9	0.1	9.5	1
^N225	0.95	0.95	0.48	0.52	18.2	20

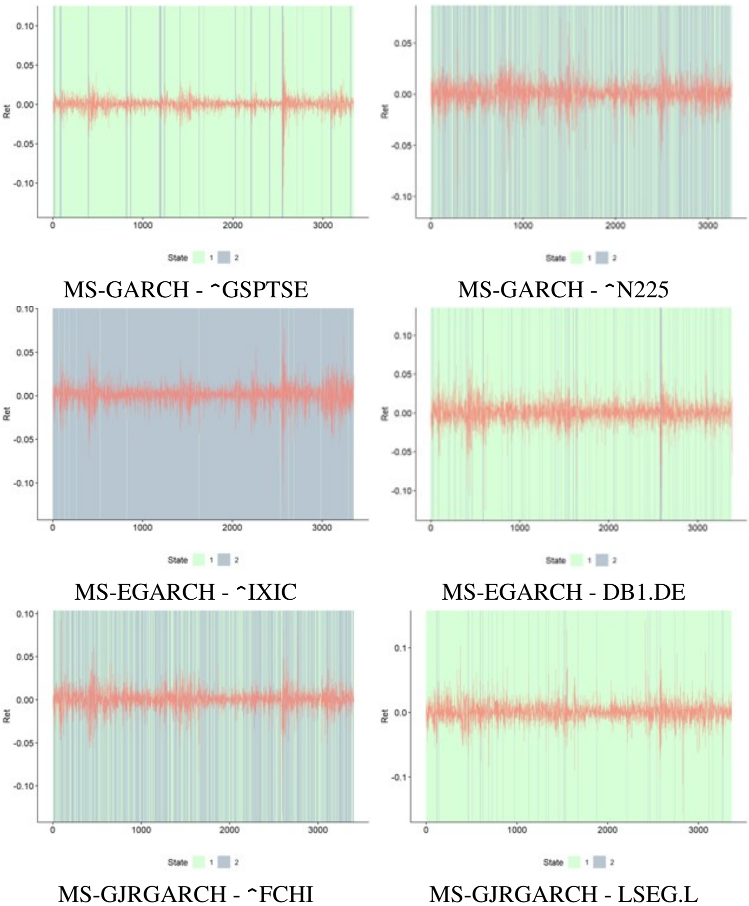
Table 7 - Table 9 show the characteristics of the regime-switching i.e. unconditional probabilities $(\pi^j = \frac{1-P_{ii}}{2-P_{ii}-P_{jj}}, j = 1,2 \text{ and } i \neq j)$ and the duration of regime survival $\left(\delta_j = \frac{1}{1-P_{jj}}\right)$ for each G7 stock for each model used [64]. In aggregate, in the MS-GARCH Model, the unconditional probability of regime 1 is much higher than regime 2. All stocks except Japan have longer expected duration of staying in regime 1 (low volatility regime) compared to the duration of staying in regime 2.

Similarly, it can be seen from the regime-switching characteristics of the MS-EGARCH model in Table 8 that in aggregate the unconditional probability of staying in regime 1 is greater than staying in regime 2. However, for practical purposes, the use of aggregates in the MS-EGARCH model is not appropriate because markets between G7 countries have different chances of staying in a particular regime. For example, in the US market, it is known that the probability of surviving in regime 2 (high volatility regime) of 81% with an expected duration of 10 days is much higher than the probability of surviving in regime 1 (19%). This condition is much different from the German market which has a chance of staying in regime 1 of 76% compared to staying in regime 2 which is 24% with an expected duration of staying in regime 1 for 7 days. Figure 2 shows some graphs of regime period 1 and regime period 2 in some G7 markets for each regime-switching model approach.

3.4 Performance of Models in Forecasting VaR

To determine the performance of the model in forecasting VaR, Kupiek test with 5% alpha is used in this study. Table 10 shows the p-value of the Kupiek test, the model is said to be suitable for predicting VaR values if the p-value is greater than 0.05, vice versa. However, to determine which final model is the most appropriate in forecasting VaR, it is also necessary to examine other requirements such as model stationarity and significance of parameter estimates (see Table 4 - Table 6).

Based on these, for the US and Italian markets, the most appropriate model is MS-GJRGARCH. As for the Canadian market, none of the models applied in this study is suitable in forecasting VaR. Unlike the US market, the French market is more suitable to be modeled with MS-EGARCH, while the German market is more suitable to be modeled with MS-GJRGARCH than MS-EGARCH despite its higher Kupiek test p-value. This is because the regime-switching process in EGARCH for Germany is not stationary. For the UK market, although the MS-GARCH and MS-GJRGARCH mod-



MS-GARCH - ^GSPTSE

MS-GARCH - ^N225

MS-EGARCH - ^IXIC

MS-EGARCH - DB1.DE

MS-GJRGARCH - ^FCHI

MS-GJRGARCH - LSEG.L

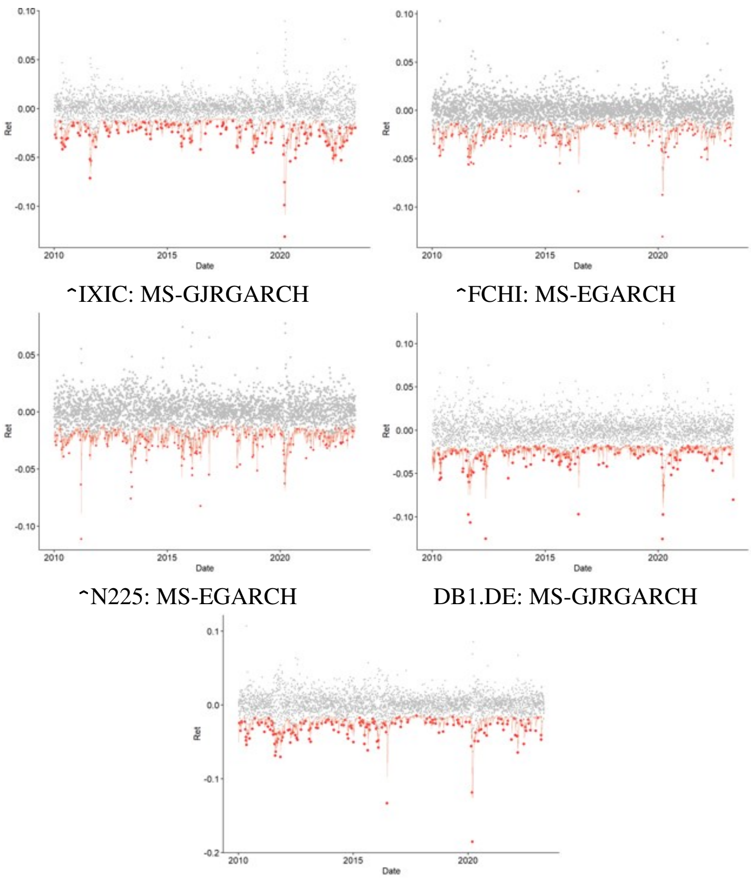
Fig. 2. Period Regime-Switching

els show very high p-values for the Kupiek test, the parameters of the models are not significant. Similarly, for the MS-EGARCH model with high Kupiek test p-values, it is known from Table 5 that the regime-switching process is not stationary. Similarly, for the Japanese market, the p-value is high for MS-GARCH but it is known from Table 4 that the parameters are not significant, so the suitable model for forecasting VaR is MS-EGARCH.

Figure 3 is the VaR forecast accuracy graph of the best model selected based on Table 10. The red dots on the graph indicate the inaccuracy of the model in forecasting VaR, while the gray dots indicate the ability of the model in forecasting VaR.

Table 10. p-Value of Kupiek Test

	MS-GARCH	MS-EGARCH	MS-GJRGARCH
^IXIC	0.112	0.152	**0.614**
^GSPTSE	0.007	0.014	0
^FCHI	0.013	**0.792**	0.407
DB1.DE	0.703	0.947	**0.803**
FTSEMIB.MI	0.008	0.221	**0.359**
LSEG.L	0.987	0.924	0.862
^N225	0.571	**0.421**	0.333



^IXIC: MS-GJRGARCH

^FCHI: MS-EGARCH

^N225: MS-EGARCH

DB1.DE: MS-GJRGARCH

FTSEMIB.MI: MS-GJRGARCH

Fig. 3. VaR Forecasting

4 Conclusion

Of the seven stocks from the G7 financial markets, it can be seen that all returns have non-normally distributed, leptokurtic, stationary at the mean, and fat-tailed properties.

In addition, the time-dependency properties are also evident from the stock returns studied except for the DB1.DE stock return of Germany which is random walk.

Based on the 3 schemes used, each market has different characteristics in the regime-switching process, so the model approach used to forecast VaR is different. It is also known that although based on the backtesting test shows that the model is suitable in forecasting VaR, but this cannot be used as the main reference because it is possible that the estimated model parameters are not significant and the regime-switching process is not stationary.

Another thing that needs to be observed in the results of this study is the determination of the distribution assumptions used between regimes. The inconsistency between the backtesting results and the parameter estimates and stationarity of the regime-switching process can also be caused by the initial determination of the residual distribution assumptions. Likewise, the determination of the scheme that changes between regime 1 and regime 2 is very likely to be the cause of the inconsistency of VaR forecast results.

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