SIRQ Mathematic Model with Fuzzy Parameter for COVID-19 Epidemic in Indonesia

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Abstract. In this research, we studied the mathematics model of COVID-19 epidemic with fuzzy parameter and the model is SIRQ (Susceptible, Infectious, Recovered, Quarantined). We defined fuzzy function for several parameters by linking several parameters with policies on handling epidemics such as quarantine and vaccination. In this research, we conducted the mathematical analysis processes of the model formed. The simulation is carried out using parameter values from various related articles. From the parameter value that are obtained, we got $0.2296 \leq R_0 \leq 1.1608$.

Keywords: mathematical model, fuzzy parameter, COVID-19

1 Introduction

Almost all infectious diseases affect daily life. Some diseases cause epidemics even pandemic in the global community. At the end of December 2019, Corona virus was first discovered in Wuhan, China and spread widely around the world. This outbreak posed a huge threat to the global public health and economics [1, 2]. The emergence of COVID-19 coincided with the largest annual human migration in the world [1].

COVID-19 entered Indonesia at the beginning of March 2020 and the Indonesian government declared the corona virus as a national disaster on April 13th, 2020 [3]. Indonesia has also made many prevention and control efforts, except for the national lockdown and regional quarantine. Indonesia initially made calls for social distancing, WFH (Work from home) and SFH (School from home). The other policies are forming the Covid-19 Task Force, PSBB (Pembatasan Sosial Berskala Besar), Transitional PSBB, New Normal, micro-scale PSBB, PPKM (Pemberlakuan Pembatasan Kegiatan Masyarakat), PPKM Level 1-4, and finally mass vaccination [4].

In this research, we studied the mathematics model of COVID-19 epidemic with fuzzy parameter. One of the studies on the Covid-19 mathematical model using fuzzy parameters was carried out by Abdy et al [5]. The model developed by Abdy et al [5] did not accommodate various kinds of government policies/actions in dealing with the Covid-19 outbreak such as quarantine for people coming from outside Indonesia and quarantine for infected people both with symptoms and without symptoms, vaccination, and the implementation of PPKM (semi lockdown). In this research a mathematical model will be built and analyzed that accommodates these various government policies.

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This research is started by conducting literature review related to the existing fuzzy epidemic models. Then new models are formed from the results of a literature review. Then the mathematics model was analyzed to determine the existence of equilibrium points and their stability. Numerical simulations were performed to verify the analysis results.

2 Model formulation and analysis

The SIRQ model is model with quarantine and vaccination strategies, without lockdown. The transfer diagram of the model can be seen at Figure 1.

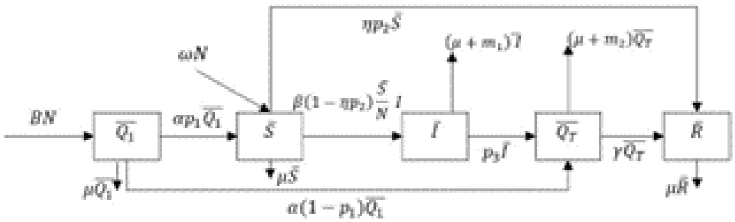

Fig. 1. The transfer diagram of the COVID-19 mathematical model without lockdown

where $\overline{Q_1}$ (the number of persons who enter population take quarantine), $\overline{S}$ (the number of susceptible persons), $\overline{I}$ (the number of infected persons with symptoms and without symptoms), $\overline{Q_T}$ (the number quarantine infected persons), $\overline{R}$ (the number of recovered persons), B (the rate of the number of persons enter population every time from immigration), p_1 (the proportion of quarantined persons free from infection), ω (the birth rates), μ (the natural death rates), p_2 (the vaccination rate of susceptible persons), $\frac{1}{a}$ (the duration of quarantines for persons from out of area), β (the infection rate of susceptible persons), η (the effectiveness of vaccination), m_1, m_2 (the death rate because of infection), p_3 (the rate of infected persons who get quarantine and treatments), and γ (the recovery rate of the quarantined person).

From Figure 1, we defined System (1)

$$\begin{aligned}
\frac{d\overline{Q_1}}{dt} &= BN - (\mu+\alpha)\overline{Q_1} \\
\frac{d\overline{S}}{st} &= \alpha p_1\overline{Q_1} + \omega N - \beta(1-\eta p_2)\frac{\overline{S}}{N}\overline{I} - (\mu+\eta p_2)\overline{S} \\
\frac{d\overline{I}}{dt} &= \beta(1-\eta p_2)\frac{\overline{S}}{N}\overline{I} - (\mu+m_1+p_3)\overline{I} \\
\frac{d\overline{Q_T}}{dt} &= \alpha(1-p_1)\overline{Q_1} + p_3\overline{I} - (\mu+m_2+\gamma)\overline{Q_T} \\
\frac{d\overline{R}}{dt} &= \gamma\overline{Q_T} + \eta p_2\overline{S} - \mu\overline{R} \\
N &= \overline{Q_1} + \overline{S} + \overline{I} + \overline{Q_T} + \overline{R}.
\end{aligned} \tag{1}$$

Then

$$\begin{aligned}
\frac{dN}{dt} &= \frac{d\overline{Q_1}}{dt} + \frac{d\overline{S}}{dt} + \frac{d\overline{I}}{dt} + \frac{d\overline{V}}{dt} + \frac{d\overline{R}}{dt} \\
&= (B+\omega-\mu)N - m_1\overline{I} - m_2\overline{Q_T}.
\end{aligned}$$

In this research, we assumed that $\omega = \mu$ then we got

$$\begin{aligned}
\frac{d\overline{Q_1}}{dt} &= BN - (\mu+\alpha)\overline{Q_1} \\
\frac{d\overline{S}}{st} &= \alpha p_1\overline{Q_1} + \mu N - \beta(1-\eta p_2)\frac{\overline{S}}{N}\overline{I} - (\mu+\eta p_2)\overline{S} \\
\frac{d\overline{I}}{dt} &= \beta(1-\eta p_2)\frac{\overline{S}}{N}\overline{I} - (\mu+m_1+p_3)\overline{I} \\
\frac{d\overline{Q_T}}{dt} &= \alpha(1-p_1)\overline{Q_1} + p_3\overline{I} - (\mu+m_2+\gamma)\overline{Q_T} \\
\frac{d\overline{R}}{dt} &= \gamma\overline{Q_T} + \eta p_2\overline{S} - \mu\overline{R} \\
N &= \overline{Q_1} + \overline{S} + \overline{I} + \overline{Q_T} + \overline{R}.
\end{aligned} \tag{2}$$

Then

$$\begin{aligned}
\frac{dN}{dt} &= \frac{d\overline{Q_1}}{dt} + \frac{d\overline{S}}{dt} + \frac{d\overline{I}}{dt} + \frac{d\overline{V}}{dt} + \frac{d\overline{R}}{dt} \\
&= BN - m_1\overline{I} - m_2\overline{Q_T}.
\end{aligned}$$

The rate of change of the total population of humans is zero (similar step is used in [6]) and given by:

$$\frac{dN}{dt} = 0 \Leftrightarrow BN - m_1\overline{I} - m_2\overline{Q_T} = 0.$$

Normalizing the equations in (2) by introducing the following normalized variables

$$Q_1 = \frac{\overline{Q_1}}{N}, S = \frac{\overline{S}}{N}, I = \frac{\overline{I}}{N}, Q_T = \frac{\overline{Q_T}}{N}, \text{ and } R = \frac{\overline{R}}{N}.$$

Because $\frac{dN}{dt} =$ then $\frac{dQ_1}{dt} = \frac{d\left(\frac{\overline{Q_1}}{N}\right)}{dt} = \frac{\frac{d\overline{Q_1}}{dt}.N - \overline{Q_1}.\frac{dN}{dt}}{N^2} = \frac{1}{N}\frac{d\overline{Q_1}}{dt}$.

For other variables, we got similar results. Because $Q_1 = \frac{\overline{Q_1}}{N}, S = \frac{\overline{S}}{N}, I = \frac{\overline{I}}{N}, Q_T = \frac{\overline{Q_T}}{N}, R = \frac{\overline{R}}{N}$ and $N = \overline{Q_1} + \overline{S} + \overline{I} + \overline{Q_T} + \overline{R}$ then $Q_1, S, I, Q_T, R \leq 1$ and we got

$$Q_1 + S + I + Q_T + R = 1 \tag{3}$$

Because R can be determined by (3), Hence, system (2) reduce into System (4)

$$\begin{aligned}
\frac{dQ_1}{dt} &= B - (\mu + \alpha)Q_1 \\
\frac{dS}{st} &= \alpha p_1 Q_1 + \mu - \beta(1 - \eta p_2)SI - (\mu + \eta p_2)S \\
\frac{dI}{dt} &= \beta(1 - \eta p_2)SI - (\mu + m_1 + p_3)I \\
\frac{dQ_T}{dt} &= \alpha(1 - p_1)Q_1 + p_3 I - (\mu + m_2 + \gamma)Q_T
\end{aligned} \tag{4}$$

We give the initial condition of every variable in System (4) such that

$$Q_1(0) \geq 0, S(0) \geq 0, I(0) > 0, \text{ and } Q_T(0) \geq 0. \tag{5}$$

2.1 The Existence and boundedness of the solution of System

In this subsection, we prove the existence and boundedness of all solutions of the System (4).

Let $f = (f_1, f_2, f_3, f_4, f_5)$ where

$$\begin{aligned}
f_1 &= B - (\mu + \alpha)Q_1, \\
f_2 &= \alpha p_1 Q_1 + \mu - \beta(1 - \eta p_2)SI - (\mu + \eta p_2)S, \\
f_3 &= \beta(1 - \eta p_2)SI - (\mu + m_1 + p_3)I, \\
f_4 &= \alpha(1 - p_1)Q_1 + p_3 I - (\mu + m_2 + \gamma)Q_T,
\end{aligned}$$

Let $D^4_+ = \{(Q_1, S, I, Q_T) \in \mathbb{R}^4 : 0 \leq Q_1, S, I, Q_T \leq 1\}$. Clearly that the function $f_i, i = 1, 2, \ldots 4$ has continues first derivative then $f \in C'(D^4_+)$. Because $f \in C'(D^4_+)$ then $f : D^4_+ \to \mathbb{R}^4$ locally Lipschitz on D^4_+.

Proposition 1. *System (4) with initial conditions (5) has solution in interval $[0, \infty)$ and the solution of system (4) are nonnegative for all $t \geq 0$.*

Proof. The Solution of system (4) with initial conditions (5) exists on $[0, \omega)$ where $0 < \omega < \infty$ because $f = (f_1, f_2, f_3, f_4) \in C'(D^4_+)$ where f_1, f_2, f_3, f_4 are right side of System (4) is locally Lipschitz on D^4_+.

By using the lower bound of every equation in System (4) we get

$$\begin{aligned}
Q_1(t) &\geq Q_1(0)e^{-(\mu+\alpha)t} \geq 0 \\
S(t) &\geq S(0)e^{-\int_0^t (\beta(1-\eta p_2)I - (\mu+\eta p_2))d\tau} \geq 0 \\
I(t) &\geq I(0)e^{-(\mu+m_1+p_3)t} \geq 0 \\
Q_T(t) &\geq Q_T(0)e^{-(\mu+m_2+\gamma)t} \geq 0
\end{aligned}$$

where the initial conditions are defined at (5).
Hence, we have completed proof.

2.2 The equilibrium points

The equilibrium points are determined by solving System (6).

$$\begin{aligned} &B-(\mu+\alpha)Q_1 &&= 0\\ &\alpha p_1 Q_1+\mu-\beta(1-\eta p_2)SI-(\mu+\eta p_2)S &&= 0\\ &\beta(1-\eta p_2)SI-(\mu+m_1+p_3)I &&= 0\\ &\alpha(1-p_1)Q_1+p_3 I-(\mu+m_2+\gamma)Q_T &&= 0 \end{aligned} \tag{6}$$

Form the 1st equation of System (5), we got

$$Q_1=\frac{\beta}{\mu+\alpha} \tag{7}$$

From the 3rd equation of System (5), we got

$$I=0 \vee S=\frac{(\mu+m_1+p_3)}{\beta(1-\eta p_2)} \tag{8}$$

The disease-free equilibrium point
This point can be determined by taking $I=0$ of equation (8). From the 2nd equation of System (6), we got

$$S=\frac{B\alpha p_1+(\mu+\alpha)\mu}{(\mu+\alpha)(\mu+\eta p_2)}.$$

From the 4th equation of System (6), we got

$$Q_T=\frac{B\alpha(1-p_1)}{(\mu+\alpha)(\mu+m_2+\gamma)}.$$

Hence, we get a disease-free equilibrium point i.e.

$$\begin{aligned} E_0 &= (Q_1^0, S^0, I^0, Q_T^0)\\ &= \left(\frac{B}{\mu+\alpha}, \frac{B\alpha p_1+(\mu+\alpha)\mu}{(\mu+\alpha)(\mu+\eta p_2)}, 0, \frac{B\alpha(1-p_1)}{(\mu+\alpha)(\mu+m_2+\gamma)}\right). \end{aligned}$$

The basic reproduction number
The basic reproduction number was determined using the next generation matrix. Form system (3), we got

$$\mathscr{F}=\beta(1-\eta p_3)SI \text{ and } \mathscr{V}=(\mu+m_1+p_3)I.$$

Hence,

$$F(E)=\frac{d\mathscr{F}}{dI}(E)=\beta(1-\eta p_2)S \text{ and } V(E)=\frac{d\mathscr{V}}{dI}(E)=(\mu+m_1+p_3)$$

where $E = (Q_1, S, I, Q_T, R)$. Then

$$F(E_0) = \beta(1-\eta p_3)S^0 \text{ and } V(E_0) = (\mu + m_1 + p_3) \Leftrightarrow V^{-1}(E_0) = \tfrac{1}{\mu+m_1+p_3}.$$

Hence,

$$\begin{aligned} R_0 = \rho(FV^{-1}) &= \tfrac{\beta(1-\eta p_2)S^0}{\mu+m_1+p_3} \\ &= \tfrac{\beta(1-\eta p_2)[B\alpha p_1+(\mu+\alpha)\mu]}{(\mu+\alpha)(\mu+\eta p_2)(\mu+m_1+p_3)}. \end{aligned}$$

The endemic equilibrium point

This point can be determined by taking $I \neq 0$ of equation (8) and we got

$$S = \frac{(\mu + m_1 + p_3)}{\beta(1-\eta p_2)}.$$

From the 2nd equation of System (6), we got

$$I = \frac{(\mu + \eta p_2)(R_0 - 1)}{\beta(1-\eta p_2)}.$$

From the 4th equation of System (6), we got

$$Q_T = \frac{\beta(1-\eta p_2)B\alpha(1-p_1) + p_3(\mu+\alpha)(\mu+\eta p_2)(R_0-1)}{\beta(1-\eta p_2)(\mu+\alpha)(\mu+m_2+\gamma)}.$$

Hence, we got an endemic equilibrium point i.e.

$$\begin{aligned} E_1 &= (Q_1^*, S^*, I^*, Q_T^*) \\ &= \left(\tfrac{B}{\mu+\alpha}, \tfrac{(\mu+m_1+p_3)}{\beta(1-\eta p_2)}, \tfrac{(\mu+\eta p_2)(R_0-1)}{\beta(1-\eta p_2)}, Q_T^*\right) \end{aligned}$$

where $Q_T^* = \frac{\beta(1-\eta p_2)B\alpha(1-p_1)+p_3(\mu+\alpha)(\mu+\eta p_2)(R_0-1)}{\beta(1-\eta p_2)(\mu+\alpha)(\mu+m_2+\gamma)}$.
Hence, we got the Theorem 2 below.

Theorem 1. *Let* $R_0 = \frac{\beta(1-\eta p_2)[B\alpha p_1+(\mu+\alpha)\mu]}{(\mu+\alpha)(\mu+\eta p_2)(\mu+m_1+p_3)}$.

1. *If* $R_0 < 1$ *then system (4) has only an equilibrium point i.e. the free-disease equilibrium point*

$$\begin{aligned} E_0 &= (Q_1^0, S^0, I^0, Q_T^0) \\ &= \left(\tfrac{B}{\mu+\alpha}, \tfrac{B\alpha p_1+(\mu+\alpha)\mu}{(\mu+\alpha)(\mu+\eta p_2)}, 0, \tfrac{B\alpha(1-p_1)}{(\mu+\alpha)(\mu+m_2+\gamma)}\right). \end{aligned}$$

2. *If* $R_0 > 1$ *then system (4) has two equilibrium points i.e.* E_0 *and*

$$\begin{aligned} E_1 &= (Q_1^*, S^*, I^*, Q_T^*) \\ &= \left(\tfrac{B}{\mu+\alpha}, \tfrac{(\mu+m_1+p_3)}{\beta(1-\eta p_2)}, \tfrac{(\mu+\eta p_2)(R_0-1)}{\beta(1-\eta p_2)}, Q_T^*\right) \end{aligned}$$

where $Q_T^* = \frac{\beta(1-\eta p_2)B\alpha(1-p_1)+p_3(\mu+\alpha)(\mu+\eta p_2)(R_0-1)}{\beta(1-\eta p_2)(\mu+\alpha)(\mu+m_2+\gamma)}$.

2.3 The local stability of equilibrium points

The Jacobian matrix of system (4) is

$$J(E) = \begin{bmatrix} -(\mu+\alpha) & 0 & 0 & 0 \\ \alpha p_1 & j_{22} & j_{23} & 0 \\ 0 & j_{32} & j_{33} & 0 \\ \alpha(1-p_1) & 0 & p_3 & j_{44} \end{bmatrix}$$

where $E = (Q_1, S, I, Q_T)$,
$j_{22} = -\beta(1-\eta p_2)I - (\mu+\eta p_2)$
$j_{23} = -\beta(1-\eta p_2)S$
$j_{32} = \beta(1-\eta p_2)I$
$j_{33} = \beta(1-\eta p_2)S - (\mu+m_1+p_3)$
$j_{44} = -(\mu+m_2+\gamma)Q_T$

The local stability of equilibrium points was given by Theorem 2.

Theorem 2. *Let* $R_0 = \frac{\beta(1-\eta p_2)[B\alpha p_1+(\mu+\alpha)\mu]}{(\mu+\alpha)(\mu+\eta p_2)(\mu+m_1+p_3)}$.

1. E_0 is locally asymptotically stable if $R_0 < 1$ and unstable if $R_0 > 1$.
2. E_1 is locally asymptotically stable if $R_0 > 1$.

Proof. The eigen values of $J(E_0)$ are

$$\begin{aligned} \lambda_1 &= -\frac{B\alpha(1-p_1)}{(\mu+\alpha)}, \\ \lambda_2 &= -(\mu+\alpha), \\ \lambda_3 &= -(\mu+\eta p_2), \\ \lambda_4 &= \beta(1-\eta p_2)S^0 - (\mu+m_1+p_3) = (\mu+m_1+p_3)(R_0-1) \end{aligned}$$

Hence, $\lambda_1 < 0, \lambda_2 < 0, \lambda_3 < 0, \lambda_4 < 0$ if $R_0 < 1$ and $\lambda_4 > 0$ if $R_0 > 1$.
Hence, E_0 is locally asymptotically stable if $R_0 < 1$ and unstable if $R_0 > 1$.
The eigen values of $J(E_1)$ are

$$\begin{aligned} \lambda_1 &= -\frac{\beta(1-\eta p_2)B\alpha(1-p_1) + p_3(\mu+\alpha)(\mu+\eta p_2)(R_0-1)}{\beta(1-\eta p_2)(\mu+\alpha)}, \\ \lambda_2 &= -(\mu+\alpha), \\ \lambda_3 &= \frac{-(\mu+\eta p_2)R_0 - \sqrt{D}}{2}, \\ \lambda_4 &= \frac{-(\mu+\eta p_2)R_0 + \sqrt{D}}{2} \end{aligned}$$

where $D = [(\mu+\eta p_2)R_0]^2 - 4(\mu+m_1+p_3)(\mu+\eta p_2)(R_0-1)$. Hence, $\lambda_1 < 0$ and $\lambda_2 < 0$ if $R_0 > 1$. For every value of D, it is easy to prove that λ_3 and λ_4 have negative real part.

2.4 The global stability of equilibrium points

The global stability of equilibrium points was given by Theorem 3.

Theorem 3. *Let* $R_0 = \frac{\beta(1-\eta p_2)[B\alpha p_1+(\mu+\alpha)\mu]}{(\mu+\alpha)(\mu+\eta p_2)(\mu+m_1+p_3)}$ *and* $4(\mu+\eta p_2) > \alpha p_1^2$.

1. E_0 is globally asymptotically stable if $R_0 < 1$.
2. E_1 is globally asymptotically stable if $R_0 > 1$.

Proof. For E_0 :
Let $V = \frac{1}{2}(Q_1 - Q_1^0)^2 + \frac{1}{2}(S - S^0)^2 + S^0 I$.
Hence $V(E_0) = 0$ and $V(E) > 0 \forall E \in D_+^4 - \{E_0\}$.
Then

$$\begin{aligned}\dot{V} &= \dot{Q}_1(Q_1 - Q_1^0) + \dot{S}(S - S^0) + S^0\dot{I}\\ &= -\beta(1-\eta p_2)(S-S^0)^2 I + IS^0(\mu+m_1+p_3)[R_0-1] - \mu(Q_1-Q_1^0)^2\\ &- a\left[(Q_1-Q_1^0) - \frac{p_1}{2}(S-S^0)\right]^2 + (S-S^0)^2\left[\frac{\alpha p_1^2}{4} - (\mu+\eta p_2)\right].\end{aligned}$$

Hence $\dot{V}(E) < 0 \forall E \in D_+^4 - \{E_0\}$ if $R_0 < 1$ and $4(\mu+\eta p_2) > \alpha p_1^2$.
Then E_0 is globally asymptotically stable if $R_0 < 1$ and $4(\mu+\eta p_2) > \alpha p_1^2$.
For E_1 :
Let $V = \frac{1}{2}(Q_1 - Q_1^*)^2 + \frac{1}{2}(S - S^*)^2 + S^*\left(I - I^* - I^*\ln\frac{I}{I^*}\right)$.
Hence $V(E_1) = 0$ and $V(E) > 0 \forall E \in D_+^4 - \{E_1\}$.
Then

$$\begin{aligned}\dot{V} &= \dot{Q}_1(Q_1 - Q_1^*) + \dot{S}(S - S^*) + S^*\dot{I}\frac{I-I^*}{I}\\ &= -\beta(1-\eta p_2)I(S-S^*)^2 - \mu(Q_1-Q_1^*)^2\\ &- a\left[(Q_1-Q_1^0) - \frac{p_1}{2}(S-S^0)\right]^2 + (S-S^0)^2\left[\frac{\alpha p_1^2}{4} - (\mu+\eta p_2)\right].\end{aligned}$$

Hence $\dot{V}(E) < 0 \forall E \in D_+^4 - \{E_1\}$ if $4(\mu+\eta p_2) > \alpha p_1^2$.
Because E_1 exists only if $R_0 > 1$ then E_1 is globally asymptotically stable if $R_0 > 1$ and $4(\mu+\eta p_2) > \alpha p_1^2$. Hence, we have completed proof.

3 Numerical simulation

The parameter value for numerical simulation is given in Table 1.

Table 1. The parameter value for numerical simulation of Model

Parameter	Parameter Value	Reference
B	3.9×10^{-5}	Estimation
p_1	0.5	Assumed
μ	0.00625	[7]
p_2	0.00019 - 0.00026	Estimation
α	0.07142 - 0.14286	Estimation
β	0.2, 0.99102	[8], [9]
η	62.1% - 95%	[10]
m_1	3.4×10^{-2}	[11]
m_2	3.4×10^{-2}	[11]
p_3	0.8	Assumed
γ	0.11624	[8]

Note :

$$B = \frac{\text{daily average of the number of individuals enter population}}{\text{total population}}$$
$$p_2 = \frac{\text{the average daily vaccination rate}}{\text{the initial number of susceptible individuals}}$$

The daily average of the number of individuals enter population is $10,355$ persons and the total population of Indonesia based on population census in 2020 is $270,203,917$ persons [12]. The average daily vaccination rate is $50,056 - 71,050$ doses [10] and the initial number of susceptible individuals is 268,757,171 persons [5]. The initial value of every variable is given $Q_1(0) = 0.00058, S(0) = 0.99464, I(0) = 0.00001$, and $Q_T(0) = 0.00001$. A requirement of global stability is $4(\mu + \eta p_2) > \alpha p_1^2$ so we choose $\eta = 95\%, p_2 = 0.00026$, and $\alpha = 0.07142$. Based on the parameter value in Table 1, we got the range of R_0 that is $0.2296 \leq R_0 \leq 1.1608$.

3.1 The simulation at $R_0 < 1$

For $R_0 < 1$, we used $\beta = 0.2$. We got $R_0 = 0.2296$ and $E_0 = (Q_1^0, S^0, I^0, Q_T^0) = (0.0005, 0.9647, 0, 0.0001)$. Figures 2 and 3 show the projection of the vector field around E_0. The solution converges to E_0.

3.2 The simulation at $R_0 > 1$

For $R_0 > 1$, we used $\beta = 0.99$. We got $R_0 = 1.1364, E_0 = (Q_1^0, S^0, I^0, Q_T^0) = (0.0005, 0.9647, 0, 0.0001)$ and $E_1 = (Q_1^*, S^*, I^*, Q_T^*) = (0.000502, 0.8489, 0.00089,$ $0.00469)$. Figures 4 and 5 show the projection of the vector field around E_0. The solution converges to E_1 and moves away from E_0.

4 Fuzzy parameter

We defined the fuzzy parameters only for β, m_1, m_2,and γ. Those parameters are determined by the corona virus load in the body that is symbolize by Ω [5]. Verma in

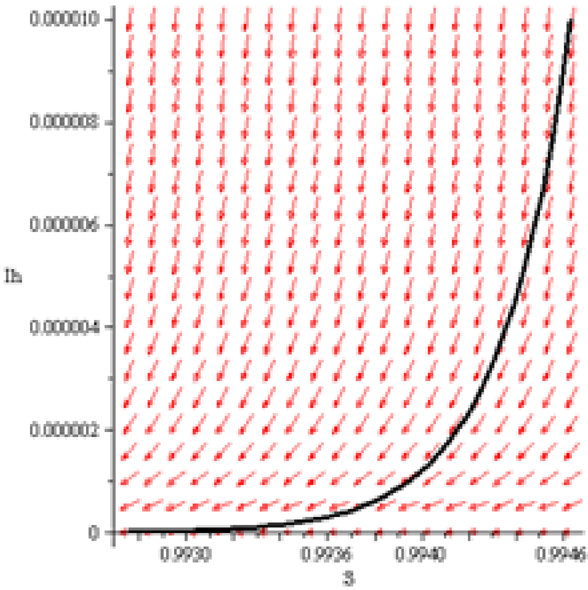

Fig. 2. The projections of vector field around E_0 (S vs I)

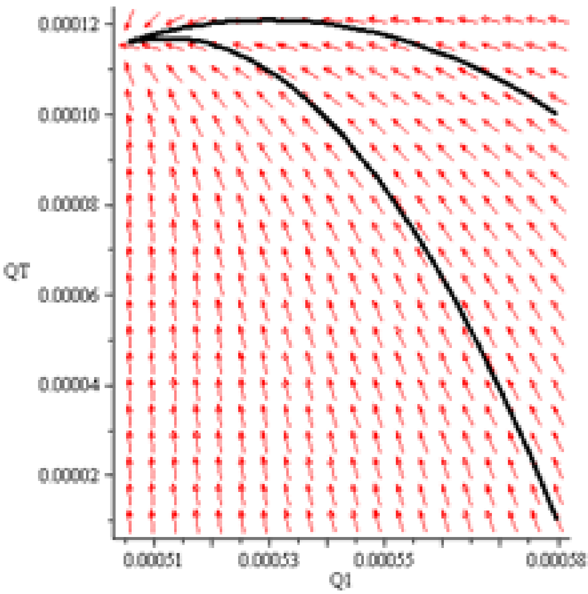

Fig. 3. The projections of vector field around E_0 (Q_1 vs Q_T)

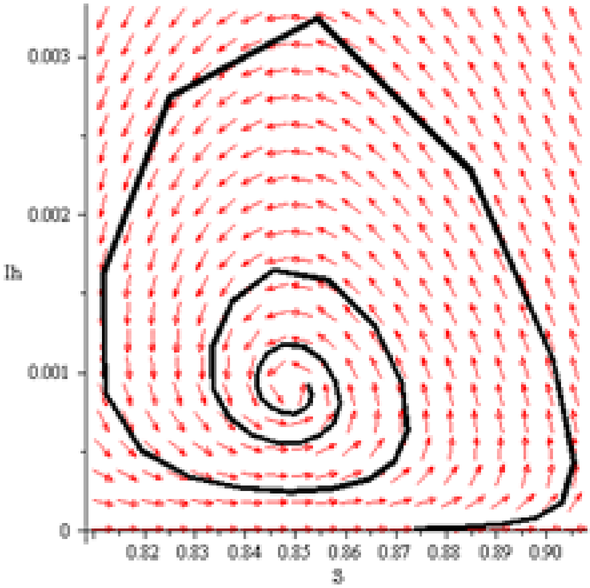

Fig. 4. The projections of vector field around E_1 (S vs I)

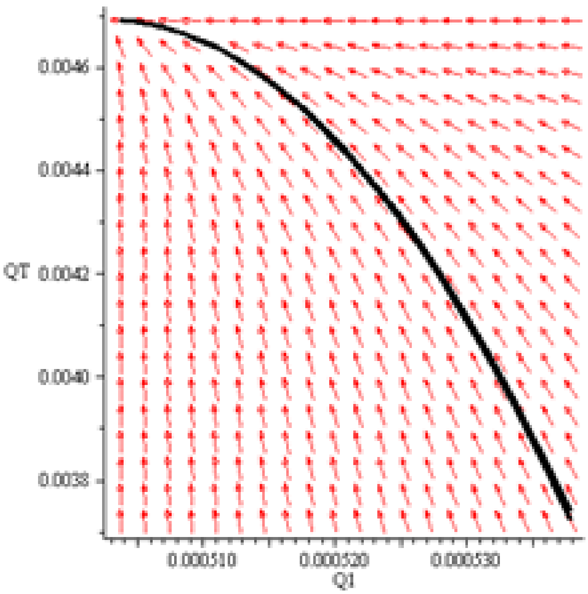

Fig. 5. The projections of vector field around E_1 (Q_1 vs Q_T)

[13] also use virus load in the body to define the formula of fuzzy parameter. We used similar notation of the bounds of Ω i.e., Ω_{min}, Ω_0, and Ω_{max} . Ω_{min} is the minimum corona virus-load in infected individual to transfer viruses to other individuals, Ω_0 is the minimum corona virus-load in infected individual that is given the maximum value of parameter, and Ω_{max} is the maximum corona virus-load in infected individual.

The formula for every parameter was given below.

$$\beta(\Omega) = \begin{cases} 0, & \text{if } 0 \le \Omega \le \Omega_{min} \\ \beta_1(1-\eta)(1-\pi).\left(\frac{\Omega-\Omega_{min}}{\Omega_0-\Omega_{min}}\right), & \text{if } \Omega_{min} < \Omega < \Omega_0 \\ \beta_1(1-\eta)(1-\pi), & \text{if } \Omega_0 \le \Omega \le \Omega_{max} \end{cases} \tag{9}$$

where β_1 is the virus transmission rate (hazard level), η of the effectiveness of vaccination, π is the proportion of susceptible person in implementing health protocols disciplinary.

$$m_1(\Omega) = \begin{cases} \frac{\Omega}{\Omega_0}(1-\vartheta)(1-p_5)+p_5, & \text{if } 0 \le \Omega < \Omega_0 \\ (1-\vartheta)(1-p_5)+p_5, & \text{if } \Omega \ge \Omega_0 \end{cases} \tag{10}$$

where p_5 is the proportion of comorbid person in the infected person who don't get treatment.

$$m_2(\Omega) = \begin{cases} \frac{\Omega}{\Omega_0}(1-\theta)(1-\vartheta)(1-p_6)+p_6, & \text{if } 0 \le \Omega < \Omega_0 \\ (1-\theta)(1-\vartheta)(1-p_6)+p_6, & \text{if } \Omega \ge \Omega_0 \end{cases} \tag{11}$$

where ϑ is the level of immunity, drugs stock, and the hospital facilities, p_6 is the proportion of comorbid person in the infected person who get treatment, and θ is the effectiveness of treatment, $0 < \vartheta \le 1, 0 < \theta \le 1, 0 \le p_5 < 1$,and $0 \le p_5 < 1$.

$$\gamma(\Omega) = \begin{cases} (\gamma_0-1)(1-\theta)(1-\vartheta)\frac{\Omega}{\Omega_0}+1, & \text{if } 0 \le \Omega < \Omega_0 \\ (\gamma_0-1)(1-\theta)(1-\vartheta)+1, & \text{if } \Omega \ge \Omega_0 \end{cases} \tag{12}$$

where $\frac{1}{\gamma_0}$ is the shortest duration of recovery.

4.1 The basic reproduction number with fuzzy parameter

Substitute fuzzy parameter into the basic reproduction number formula, we got

$$\begin{aligned} R_0 &= \frac{\beta(\Omega)(1-\eta p_2)[B\alpha p_1+(\mu+\alpha)\mu]}{(\mu+\alpha)(\mu+\eta p_2)(\mu+m_1(\Omega)+p_3)} \\ &= \begin{cases} 0, & \text{if } 0 \le \Omega \le \Omega_{min} \\ \dfrac{\beta_1(1-\eta)(1-\pi).\frac{(\Omega-\Omega_{min})}{(\Omega_0-\Omega_{min})}(1-\eta p_2)[B\alpha p_1+(\mu+\alpha)\mu]}{(\mu+\alpha)(\mu+\eta p_2)\left[\mu+\frac{\Omega}{\Omega_0}(1-\vartheta)(1-p_5)+p_5+p_3\right]}, & \text{if } \Omega_{min} < \Omega < \Omega_0 \\ \dfrac{\beta_1(1-\eta)(1-\pi)(1-\eta p_3)[B\alpha p_1+(\mu+\alpha)\mu]}{(\mu+\alpha)(\mu+\eta p_2)[\mu+(1-\vartheta)(1-p_5)+p_5+p_3]}, & \text{if } \Omega_0 \le \Omega \le \Omega_{max} \end{cases} \end{aligned}$$

The parameter value for fuzzy parameter is given in Table 2. The graph of R_0 was

Table 2. The parameter value for fuzzy parameter

Parameter	Value	Reference
β_1	1	Assumed
η	$62,1\%$	[10]
π	10%	[5]
p_5	$2,2114 \times 10^{-4}$	Assumed
ϑ	0,9	[5]
p_6	$2,2114 \times 10^{-4}$	Assumed
θ	85%	[5]
γ_0	$1,042 \times 10^{-3}$	[5]
Ω_{min}	50	Assumed
Ω_0	75	Assumed
Ω_{max}	150	Assumed

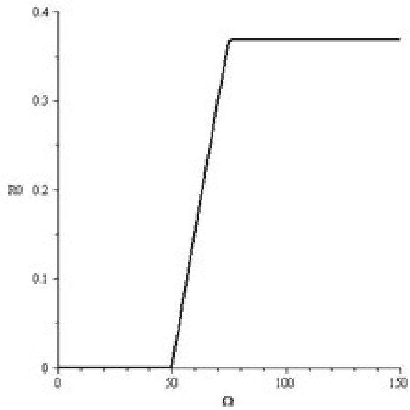

Fig. 6. The graph of R_0 vs Ω

given in Figure 6.

Using the parameter value in Table 2, we got that $R_0 < 0.5$. It happens because the effectiveness of the government policies is involved in calculating β and m_1. From this result, we can have said that the government policies have a big role in countermeasures the epidemic.

5 Conclusion

From formula of R_0 we can conclude that β, α, and p_1 have positive effect on the R_0 while η, p_2, m_1, and p_3 have negative effect on the R_0. Hence, if we want to reduce the value of R_0 then we can reduce the value of β, α, and p_1 or we can increase the value of η, p_2, m_1, and p_3. The implementation of some of those conditions can be implemented in various policies. β can be reduced by implementing the policies such that WFH, SCH, lockdown, travel ban, and physical distancing. α can be reduced by adding the quarantine periods. η can be increased by choosing vaccines that have high effectiveness, p_2 can be increased by increasing the rate of vaccine administration, and p_3 can be increased by increasing the rate of infected persons who get quarantine and treatments). To determine which parameter have the most influence on reducing R_0, we need the further research about the sensitivity analysis.

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