Plane Departure Schedule to Depati Amir Airport Using Max-Plus Algebra

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Abstract. Nusantara is the nickname for Indonesia a country that has many islands intertwined with a very wide ocean. The ocean area has an area of 5,877,879 square kilometers which is almost three times that of the land area, which is 2,012,402 square kilometers. The Indonesian mainland is divided into large islands and small islands totaling 17,508 islands. In addition to having vast land and sea areas, Indonesia is also a country with a very high population. Based on data from the Badan Pusat Statistik or BPS, the total population of Indonesia in the 2020 population census is 270.20 million people. The increasing mobility of the population between islands certainly requires adequate transportation, starting from sea, land, and air transportation. Until 2023, it is recorded that Indonesia has 340 airports spread throughout Indonesia and 32 of them are international airports. One of the airports located on the island of Bangka is Depati Amir Airport. Based on BPS data, the number of flights recorded in 2021 is 4,686 per year and will increase to 5,126 per year in 2022. One of the scheduling techniques known in mathematics is Max-plus algebra which began to be used in 1970 for project scheduling or land and air transportation systems. this research will model existing flight schedules at Depati Amir Airport using Max-plus Algebra. Scheduling with max-plus algebra is of course better because it has calculated the fastest departure schedule so it is more on time.

Keywords: max-plus algebra, flight scheduling, depati amir airport

1 Introduction

Nusantara is the nickname for Indonesia a country that has many islands intertwined with a very wide ocean. The ocean area has an area of 5.877.879 square kilometers which is almost three times that of the land area, which is 2.012.402 square kilometers. The Indonesian mainland is divided into large islands and small islands totaling 17.508 islands [1].

In addition to having vast land and sea areas, Indonesia is also a country with a very high population. Based on data from the Badan Pusat Statistik or BPS, the total population of Indonesia in the 2020 population census is 270.20 million people [2]. Environmental, economic, and welfare conditions are certainly influenced by population mobility [3]. The increasing mobility of the population between islands certainly requires adequate transportation, starting from sea, land, and air transportation. Of the

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three types of transportation, airplanes as air transportation are in great demand because they are fast, efficient, and relatively moderate in cost [4]. More and more airlines are popping up indicating that air transportation is advancing rapidly and giving people many choices in choosing airlines. In addition, the construction of airports in Indonesia is increasingly being carried out and changes in the status of airports to international airports are carried out in almost every region.

Until 2023, it is recorded that Indonesia has 340 airports spread throughout Indonesia and 32 of them are international airports. One of the airports located on the island of Bangka is Depati Amir Airport. This airport was established in 1942 as an air harbor after the Japanese occupation. Depati Amir Airport officially became an International Airport in 2019, progressing more and more, especially from the increasing number of flights. Based on BPS data, the number of flights recorded in 2021 is 4.686 per year and will increase to 5.126 per year in 2022. Meanwhile, the number of passengers in 2021 was recorded at 310.571 people per year and will continue to increase in 2022 to 593.869 people per year.

The increasing number of flights and passengers certainly requires setting the right flight schedule. One of the scheduling techniques known in mathematics is Max-plus algebra which began to be used in 1970 for project scheduling or land and air transportation system [5, **?**,**?**,**?**,**?**]. Based on this, the research will model existing flight schedules at Depati Amir Airport using Max-plus Algebra.

2 Max-plus Algebra

Max-plus Algebra has been known since 1970 to solve rail system problems. Since the 1990s it has begun to grow rapidly and is used to solve network problems, project scheduling, queues, and others [6]. Max-plus algebra is a structure built using the operations addition $\oplus$ and multiplication $\otimes$ which are defined as

Definition 1. *Given $\square_\varepsilon = \square \cup \varepsilon$ with $\varepsilon = -\infty$. The set $\square_\varepsilon$ over the operation $\oplus$ and $\otimes$ are define as follows*

$$x \oplus y = x + y \textit{ end } x \otimes y = \max\{x, y\}$$

where $x, y \in \square_\varepsilon$. Furthermore, max-plus algebra is denoted as $\square_{max} = (\square_\varepsilon, \oplus, \otimes)$.

In max-plus algebra, commutative properties and identity elements for multiplication and addition operations are explained in the following properties [7].

Lemma 1.

1. *The additive identity $e = 0$, $x \otimes e = e \otimes x = x + 0 = x$, for $x \in \square_{max}$*
2. *The multiplicative identity $\varepsilon = -\infty$, $x \oplus \varepsilon = \varepsilon \oplus x = \max\{x, -\infty\} = x$, for $x \in \square_{max}$*
3. *Commutative $x \otimes (y \oplus z) = x + \max\{y, z\} = \max(x + y, x + z) = (x \otimes y) \oplus (x \otimes z)$, for $x, y, z \in \square_{\max}$*

2.1 Matrices Over Max-Plus Algebra

Definition 2. *The set* $\square_{max}^{m\times n}$ *is defined as* $\{A=(A_{ij})\mid A_{ij}\in\square_{max}^{\mathrm{mxn}}, i=1,2,3\cdots,m$ *and* $j=1,2,3,\cdots,n\}$

Furthermore, matrix operations on max-plus algebra are defined aa

Definition 3.

1. *a If* $A,B\in\sqcup_{\mathrm{max}}^{m\times n}$ *is known defined*
   $A\oplus B=(A\oplus B)_{ij}=A_{ij}\oplus B_{ij}$
2. *b If* $A\in\square_{max}^{m\times k}, B\in\square_{max}^{k\times n}$ *is known defined*
   $A\otimes B=(A\otimes B)_{ij}=\overset{p}{\oplus} A_{k=1}A_{ik}B_{kj}$

2.2 System Of Linear Equations On Max-Plus Algebra

A system of linear equations over max-plus algebra has the form $A\otimes x=b$ with $A\in\square_{\mathrm{max}}^{m\times n}, x\in\square_{\mathrm{max}}^{n}$ and $b\in\square_{\mathrm{max}}^{m}$. The system of Linear Equations does not always have a solution, so solution $A\otimes x=b$ is weakened by the definition of a sub-solution as follows [6], [8].

Definition 4. *Let* $A\in\square_{max}^{m\times n}$ *and* $b\in\square_{max}^{m}$. *Vector* $x'\in\square_{max}^{n}$ *is called a sub-completion of a system of linear equations* $A\otimes x=b$ *if it satisfies* $A\otimes x'\leq b$.

The form of matrix $A\otimes x=b$ is described as

$$\begin{bmatrix} a_{11} & a_{12} & \cdots & a_{1n} \\ a_{21} & a_{22} & \cdots & a_{2n} \\ \vdots & \vdots & \ddots & \vdots \\ a_{m1} & a_{m2} & \cdots & a_{mn} \end{bmatrix} \otimes \begin{bmatrix} x_1 \\ x_2 \\ \vdots \\ x_n \end{bmatrix} = \begin{bmatrix} b_1 \\ b_2 \\ \vdots \\ b_n \end{bmatrix}$$

$$(a_{11}\otimes x_1)\oplus(a_{12}\otimes x_2)\oplus\cdots(a_{1n}\otimes x_n)=b_1$$
$$(a_{21}\otimes x_1)\oplus(a_{22}\otimes x_2)\oplus\cdots(a_{2n}\otimes x_n)=b_2$$
$$\vdots$$
$$(a_{m1}\otimes x_1)\oplus(a_{m2}\otimes x_2)\oplus\cdots(a_{mn}\otimes x_n)=b_m$$

So it is obtained

$$\max\{(a_{11}+x_1),(a_{12}+x_2),\cdots,(a_{1n}+x_n)\}=b_1$$
$$\max\{(a_{21}+x_1),(a_{22}+x_2),\cdots,(a_{2n}+x_n)\}=b_2$$
$$\vdots$$
$$\max\{(a_{m1}+x_1),(a_{m2}+x_2),\cdots,(a_{mn}+x_n)\}=b_m$$

Equation $A \otimes x = b$ has a solution if and only if $\bar{x}$ is a solution of $A_1 \otimes \bar{x} = \bar{b}$ and the solution of

$A \otimes x = b$ is $\bar{x} = \begin{bmatrix} \bar{x} \\ \varepsilon \\ \vdots \\ \varepsilon \end{bmatrix}$. Next, if $A \otimes x = b$ has a solution then $a_{ij} + x_j \leq b_i$.

3 Result and Discussion

Scheduling using Algebra Max-plus starts by looking for flight departure schedule data. Furthermore, data on travel time to the destination city and the transfer duration of each aircraft are needed. Depati Amir Airport starts operating at 05.00 and the first flight starts at 07.00.

There are 3 departure destinations from Depati Amir Airport, namely Jakarta, Tanjung Pandan and Palembang. The possible synchronization rules are as follows

1. Departing to Jakarta does not require waiting for a plane from Palembang or Tanjung Pandan because it already has a departure schedule in each city.
2. Departure to Palembang awaiting arrival from Tanjung Pandan
3. Departure to Tanjung Pandan awaiting arrival from Palembang

The following is the flight departure schedule from Depati Amir Airport.

Table 1. Plane Departure Schedule from Depati Amir Airport

Variable	Departure	Destination City
b1	07.00	Jakarta
b2	08.15	Tanjung Pandan
b3	08.55	Jakarta
b4	09.20	Palembang
b5	10.45	Jakarta
b6	10.45	Jakarta
b7	11.05	Palembang
b8	11.10	Jakarta
b9	11.20	Jakarta
b10	11.55	Jakarta
b11	12.05	Tanjung Pandan
b12	16.25	Jakarta
b13	17.30	Jakarta

Source: IG Depati Amir Airport

Based on the table above, the following graph is obtained.

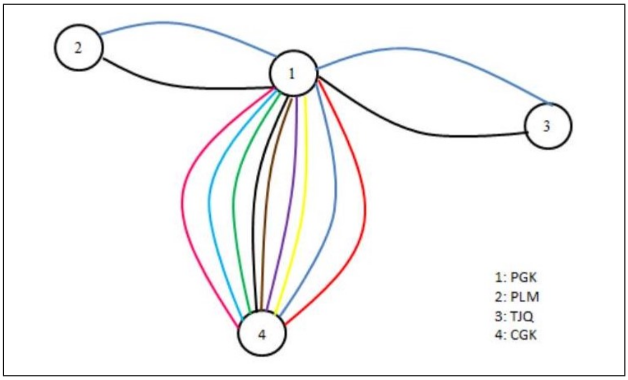

Fig. 1. Graf of Plane Departure Schedule from Depati Amir Airport

There are 13 flights to three cities, namely Jakarta, Palembang, and Tangjung Pandan. The flight duration from Depati Amir Airport is calculated from the time passengers enter the plane until the passengers leave the plane in the destination cities presented in the following table

Table 2. Travel Time Data

Variable	Hometown	Travel Time (minutes)
d1	Jakarta	70
d2	Palembang	42
d3	Tanjung Pandan	43

To continue the journey to the next city the plane has a waiting time to be used again. This waiting time is referred to as the transfer duration which will be different for each airline. Five airlines are operating at Depati Amir airport, namely Lion, Citilink, Sriwijaya, Garuda, and Batik. Garuda and Sriwijaya transfer duration are 45 minutes, Lion transfer duration is 40 minutes, Citilink transfer duration is 30 minutes and Batik transfer duration is 50 minutes.

The model that will be used to design the flight departure schedule is $A \otimes x = b$. Matrix A are the sum of transit duration and flight time. The value of x is the flight departure time from the city of origin, while b is the departure time.

$$\begin{bmatrix} a_{11}+d & a_{12}+d & \cdots & a_{1n}+d \\ a_{21}+d & a_{22}+d & \cdots & a_{2n}+d \\ \vdots & \vdots & \ddots & \vdots \\ a_{m1}+d & a_{m2}+d & \cdots & a_{mn}+d \end{bmatrix} \otimes \begin{bmatrix} x_1 \\ x_2 \\ \vdots \\ x_n \end{bmatrix} = \begin{bmatrix} b_1 \\ b_2 \\ \vdots \\ b_n \end{bmatrix}$$

$$((a_{11}+d)\otimes x_1)\oplus((a_{12}+d)\otimes x_2)\oplus\cdots((a_{1n}+d)\otimes x_n)=b_1$$
$$((a_{21}+d)\otimes x_1)\oplus((a_{22}+d)\otimes x_2)\oplus\cdots((a_{2n}+d)\otimes x_n)=b_2$$
$$\vdots$$
$$((a_{m1}+d)\otimes x_1)\oplus((a_{m2}+d)\otimes x_2)\oplus\cdots((a_{mn}+d)\otimes x_n)=b_m$$

so that

$$\max\{((a_{11}+d)+x_1),((a_{12}+d)+x_2),\cdots,((a_{1n}+d)+x_n)\}=b_1$$
$$\max\{((a_{21}+d)+x_1),((a_{22}+d)+x_2),\cdots,((a_{2n}+d)+x_n)\}=b_2$$
$$\vdots$$
$$\max\{((a_{m1}+d)+x_1),((a_{m2}+d)+x_2),\cdots,((a_{mn}+d)+x_n)\}=b_m$$

If there is no route to a particular destination then the element in the matrix is written as ε. The x value obtained is then converted to time units so that the design of the flight departure schedule is obtained which is presented in the following table.

Table 3. Departure Schedule Design from Origin Airport

Variable	Hometown	Departure
x1	Jakarta	05.10
x2	Tanjung Pandan	06.52
x3	Jakarta	07.15
x4	Palembang	07.53
x5	Jakarta	08.55
x6	Jakarta	08.50
x7	Palembang	09.43
x8	Jakarta	09.30
x9	Jakarta	09.20
x10	Jakarta	10.00
x11	Tanjung Pandan	10.37
x12	Jakarta	14.35
x13	Jakarta	15.35

4 Conclusion

Based on the results and discussion it can be concluded that scheduling using Max-Plus Algebra can provide input in scheduling airlines at Depati Amir airport. Scheduling with max-plus algebra is of course better because it has calculated the fastest departure schedule so it is more on time.

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