



Embedded Options in Green Bonds and Designing Investment Strategies: A Study Based on the Black-Scholes Model

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Abstract. As sustainable development becomes a global consensus, green bonds have emerged as a critical instrument for financing environmentally beneficial projects. In addition to their environmental focus, the increasing adoption of embedded options—such as callable and puttable structures—has introduced new complexities into green bond pricing and investment analysis. This study aims to explore the theoretical pricing of embedded put options in green bonds using a modified Black-Scholes (BS) model and to derive investment implications based on model outputs. Taking "22 Tonglu 01," a municipal green bond with a put option, as the empirical subject, the research calibrates model parameters from actual market data, including valuation prices, government bond yields, and index-based volatility. The embedded option is priced using a BS framework adapted for fixed-income features. Sensitivity analysis confirms that volatility significantly affects the option value, indicating the importance of accurate parameter estimation. The findings validate the feasibility of applying a calibrated BS model to green bond valuation and demonstrate that embedded options, even with limited magnitude, contribute meaningful risk management value. This approach offers theoretical guidance and methodological tools for pricing complex green bond structures in low-liquidity environments.

Keywords: Green bonds, Embedded options, Black-Scholes model, Volatility calibration, Sustainable finance.

1 Introduction

In recent years, with the increasing importance of environmental protection and sustainable development in the world, the green finance market has flourished, and green bonds, as an essential part of it, have continued to expand in scale. Green bonds are designed to provide financial support for environmental projects or sustainable development projects with significant environmental benefits. The introduction of embedded options, such as callable options and sellable options, has made the structure of green bonds more complex, which can not only meet the financing needs of issuers but also provide investors with more risk management tools and investment choices. However, accurate pricing of embedded options for green bonds and formulating

reasonable investment strategies have become the key issues facing the current market.

Previous studies have examined the pricing, market dynamics, and financial implications of green bonds, focusing on their role in sustainable finance and investment strategies. Many of these studies suggest that green bonds serve as an effective financial instrument for funding environmentally friendly projects, while others highlight challenges related to pricing premiums, market transparency, and investor behavior. Baker et al. explored the pricing and ownership structures of green bonds in the U.S. market, comparing them with conventional bonds. Their findings suggest that green bonds are often issued at a premium, particularly in the secondary market, where investor demand for sustainable financial products leads to higher valuations [1]. However, Flammer argued that while green bond issuance enhances corporate environmental performance, it does not necessarily translate into lower financing costs. The study, using a difference-in-differences approach, demonstrated that green bond issuers tend to attract long-term investors but do not always benefit from a pricing advantage over traditional bonds [2]. Extending the discussion to the broader green finance ecosystem, Zhang and Umair analyzed the interconnectedness of green bonds, renewable energy stocks, and carbon markets. By applying vector autoregressive models, they found significant spillover effects among these financial assets, indicating that green bonds play a complementary role in stabilizing sustainable investment portfolios [3]. Similarly, Chatziantoniou et al. employed a quantile frequency connectedness approach to investigate the interactions between green bonds, sustainable investments, and clean energy markets. Their results suggest that green bonds and clean energy stocks act as net receivers of market shocks, while sustainability indices are key drivers of financial spillovers [4]. Beyond asset price dynamics, several studies have focused on the impact of green bond issuance on corporate financing strategies. Zhong applied a double-difference model to assess the effect of green bonds on corporate debt financing costs. The study found that green bonds significantly lower borrowing costs by improving firms' environmental transparency and promoting green technology innovation. However, the effect varies across industries, with a more pronounced impact in highly polluting sectors and in China's eastern regions [5]. Fatica et al. further examined the role of financial institutions in the green bond market, demonstrating that while non-financial corporations and supranational entities benefit from green bond issuance, financial institutions do not experience a significant yield advantage. The study attributes this to investor uncertainty regarding the direct link between financial institutions' green bond issuance and actual green investment projects [6]. Despite the growing body of research, gaps remain in understanding the mechanisms through which green bonds influence financial stability, investment decisions, and corporate sustainability. While some studies emphasize the positive role of green bonds in promoting environmental finance, others point to inconsistencies in pricing effects and the challenges of regulatory standardization [7]. Addressing these gaps requires further exploration of how green bonds interact with broader financial markets, as well as comparative studies across different economic contexts. This review highlights the complexities of green bond markets, underscoring the need for

continued research into their pricing mechanisms, risk-return trade-offs, and long-term sustainability impacts.

2 BS Model and the Theoretical Foundation of Embedded Option Pricing for Green Bonds

2.1 Fundamentals of the BS Model

In this study, the Black-Scholes (BS) model serves as the analytical basis for pricing the embedded European-style put option in a green bond. Although the BS model was originally developed for pricing equity options, its core structure is applicable to fixed-income instruments with embedded option features, provided that certain market assumptions are adjusted or interpreted flexibly.

The model assumes that the underlying asset price follows a geometric Brownian motion and that markets are frictionless, with constant volatility and interest rates. These assumptions allow the model to provide a closed-form solution for the value of European options. In the context of this research, the embedded put option grants bondholders the right to sell the bond back to the issuer at a predetermined price before maturity. The pricing objective is therefore to estimate the theoretical value of this right, using a BS framework adapted to bond market conditions.

The pricing formula used in this study is:

$$P = Ke^{-rT} \cdot N(-d_2) - S_0 \cdot N(-d_1) \quad (1)$$

Where,

$$d1 = \frac{\ln(S_0/K) + (r + \frac{1}{2}\sigma^2)T}{\sigma\sqrt{T}}, d2 = d1 - \sigma\sqrt{T} \quad (2)$$

Here, S_0 represents the bond's estimated market value, K is the repurchase (put) price specified in the bond terms, r is the risk-free rate derived from the government bond yield curve, σ is the annualized volatility estimated from a green bond index, and T is the remaining time to maturity expressed in years.

Since direct price and volatility data for individual green bonds are often unavailable due to limited secondary market activity, proxies and calibration techniques are employed [8]. Specifically, S_0 is taken from the bond's official valuation on the pricing date, while σ is computed from the historical returns of the ChinaBond Green Bond Index. The BS model does not account for credit spreads or liquidity risk; however, it offers a transparent and analytically tractable method for estimating option value, especially when inputs are carefully calibrated based on observable market indicators. The accuracy of the pricing results therefore hinges on the reliability of these calibrated parameters, which are detailed in the empirical section of this paper.

2.2 Green Bonds and Embedded Options

Green bonds finance environmentally sustainable projects, following strict regulatory guidelines like the Green Bond Principles. Unlike conventional bonds, their proceeds are allocated exclusively to green initiatives such as renewable energy and climate adaptation. Many green bonds include embedded options, which provide flexibility to issuers and investors. A call option allows issuers to redeem bonds early to reduce financing costs when interest rates decline, whereas a put option enables investors to sell bonds back to issuers, mitigating credit risk. Convertible options also exist, allowing conversion into equity. These features influence pricing and risk management strategies.

2.3 Applicability and Adjustments of the BS Model in Green Bond Embedded Option Pricing

Applying the BS model to green bond embedded options presents unique challenges due to non-financial factors such as regulatory incentives and environmental benefits. Green bonds often carry policy-driven risk premiums, and investor preferences for sustainability can affect pricing beyond traditional market risks. Credit risk in green bonds differs significantly from conventional bonds due to heightened transparency and strict use-of-proceeds requirements, complicating risk assessment.

Despite these challenges, the BS model remains a useful tool for pricing green bond options, but modifications are necessary. Volatility assumptions should incorporate the impact of green certification and market sentiment, while risk-free rates should reflect the cost of capital in sustainable finance. Additionally, liquidity premiums must be considered, as green bonds often exhibit lower liquidity than traditional bonds. The BS model may also require integration with stochastic interest rate models to better capture real-world pricing dynamics.

A more refined approach would involve incorporating state-dependent variables, reflecting the impact of policy interventions and investor sentiment shifts. Hybrid models that integrate sustainability-driven risk factors with traditional financial modeling techniques could enhance pricing accuracy. These adjustments ensure a more precise valuation of green bonds with embedded options, aligning financial modeling with the evolving characteristics of green finance markets.

3 Empirical Study on Embedded Option Pricing in Green Bonds

This study selects the municipal green bond “22 Tonglu 01” as the empirical subject, issued in 2022 by Tonglu Investment Construction Development Group Co., Ltd. The bond is aligned with green financing principles, with certified environmental use of proceeds, and includes an investor put option prior to maturity. To estimate the theoretical value of the embedded put option, a modified Black-Scholes model is em-

ployed. Accurate parameter calibration is crucial to ensure the practical validity of the pricing results, particularly in the context of low-liquidity fixed-income securities.

3.1 Data Sources and Processing Methodology

The bond's valuation price on the evaluation date, March 25, 2025, is adopted as the current market value. As secondary market trading for "22 Tonglu 01" is limited, the estimated price published by Eastmoney.com, equal to 100.00 CNY, serves as the proxy for the spot price S . According to the official prospectus, the exercise price K for the put option is set at 100.00 CNY, consistent with the par value typically used in investor repurchase clauses.

The risk-free rate r is extracted from the Chinese government bond yield curve released by ChinaBond on the same date. Given that the bond's remaining maturity is approximately 0.4 years, the one-year treasury yield—reported at 1.5354%—is selected to match the time horizon of the option and minimize discounting bias. Time to maturity T is computed by calculating the day-count fraction between March 25 and the bond's maturity date of August 22, resulting in a value of 0.4 years.

Volatility σ , a key input in the Black-Scholes model, cannot be estimated directly from the bond price series due to the absence of active trading. To resolve this, the study uses the ChinaBond Green Bond Index as a representative proxy for green bond market volatility. Weekly closing values over the past 30 weeks are collected, and logarithmic returns are computed. The standard deviation of the weekly returns is then annualized using the square root of 52 to estimate market volatility. The resulting annualized volatility is approximately 3.9825%, which is considered a stable and conservative estimate of systemic price fluctuations in the green bond segment.

3.2 Modified BS Model and Option Pricing Process

Once all parameters are calibrated from reliable sources or appropriate proxies, the Black-Scholes option pricing framework is applied to estimate the theoretical value of the embedded European-style put option. Although the original Black-Scholes model was developed for equity options, its functional structure is adaptable to fixed-income derivatives, particularly for pricing embedded options in bonds under simplified assumptions such as constant volatility and frictionless markets.

The pricing process begins by substituting the calibrated values— $S=100.00$, $K=100.00$, $r=0.015354$, $\sigma=0.039825$, and $T=0.4$ —into the standard model. The intermediate variables d_1 and d_2 are computed using:

$$d_1 = \frac{\ln(S/K) + (r + \frac{1}{2}\sigma^2)T}{\sigma\sqrt{T}}, d_2 = d_1 - \sigma\sqrt{T} \quad (3)$$

The theoretical value P of the embedded put option is then calculated as:

$$P = Ke^{-rT} \cdot N(-d_2) - S \cdot N(-d_1) \quad (4)$$

The result is a theoretical value of approximately **0.7252 CNY**. This means that, under current market conditions, the embedded put option in “22 Tonglu 01” contributes a non-trivial value to the bond’s payoff structure, especially in downside scenarios. The fact that the spot price equals the strike price and the time to maturity is short helps explain why the option is near-the-money and moderately valued. Despite its low magnitude, the existence of such a protective floor in bond repayment adds measurable risk-reduction value to investors.

3.3 Pricing Results and Sensitivity Analysis

The pricing result demonstrates the viability of using a well-calibrated Black-Scholes framework to evaluate embedded option value in green bonds. The calculated value of 0.7252 CNY represents 0.73% of the bond’s par value and is a rational estimate given the option’s neutral position and short time horizon. This valuation reflects the limited but real financial protection provided by the put feature in case of issuer-specific or market-wide distress [9]. From a fixed-income investor’s perspective, such embedded rights can serve as built-in insurance, especially under deteriorating credit or liquidity conditions.

To assess model robustness and explore how parameter changes affect option value, a sensitivity analysis is conducted, focusing on the volatility input. The model is re-estimated for a range of σ values from 1% to 10%. Results show a strong positive relationship: as volatility increases, the value of the put option rises in a convex pattern. At 1% volatility, the option is valued at around 0.10 CNY, while at 10%, the value grows to approximately 2.30 CNY. This behavior is consistent with classical option theory, where greater market uncertainty enhances the intrinsic and time value of options, particularly those nearing the money, as shown in Figure 1.

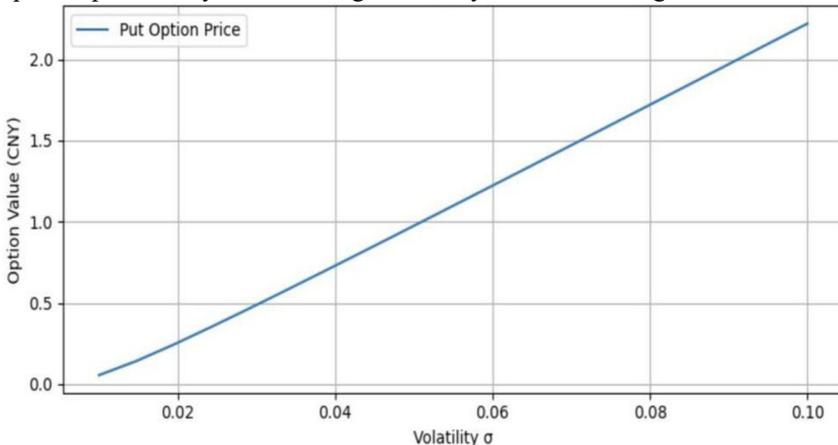


Fig. 1. Sensitivity of put option value to volatility

This sensitivity finding also supports the use of proxy volatility from a broad index. Although index-based estimation might underestimate idiosyncratic credit risk, it

provides a conservative baseline and avoids model instability in illiquid environments. Overall, the empirical pricing process, grounded in realistic parameter calibration and confirmed by sensitivity results, validates the effectiveness of the modified Black-Scholes model in quantifying embedded option value in green bonds. This methodology can be extended to other fixed-income instruments with structured features, offering a practical and replicable framework for sustainable finance analysis.

4 Conclusion

The pricing mechanism of embedded put options in green bonds was examined using a modified Black-Scholes model adapted to the fixed-income context. Using “22 Tonglu 01” as a representative case, the analysis involved calibrating key model parameters—including bond valuation, risk-free rate, time to maturity, strike price, and volatility—based on real market data [10]. Volatility was estimated from historical returns of the ChinaBond Green Bond Index, allowing for a stable proxy in the absence of liquid trading data.

The pricing output suggests that the embedded option holds a modest but tangible theoretical value (0.7252 CNY), providing limited downside protection to investors under current market conditions. Sensitivity tests show a clear positive relationship between volatility and option value, reinforcing the importance of precise parameter estimation and the relevance of embedded protections in green bond structures.

These findings highlight the practical feasibility of applying classical option pricing tools to the valuation of structured fixed-income products, even in low-liquidity settings. Proper calibration methods enhance the reliability of such models, offering investors and market participants a useful reference for assessing risk and return in sustainable debt instruments. The integration of embedded option analysis into green bond evaluation contributes to more accurate pricing and informed investment decision-making in the evolving field of sustainable finance.

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