Home Water Heater Constant Temperature Control Based on Fuzzy PID Control

Yiming Zhang[1]

[1] The School of Mechanical and Automotive Engineering , Qingdao University of Technology, Qingdao, China
a100003478@correo.umm.edu.mx

Abstract. In contemporary society, the growing emphasis on quality of life has underscored the importance of a reliable hot water supply. In this paper, we compare the performance of home water heaters controlled by traditional proportion integration differentiation (PID) methods with those controlled by fuzzy PID methods. For home water heaters, it is crucial to minimize heating time while ensuring that the temperature remains stable at the user-defined setpoint after heating. In this study, this paper first establishes a mathematical model of the domestic water heater. MATLAB/Simulink is then used to model and simulate both conventional PID control and fuzzy PID control. It is clear from comparing the simulation results that fuzzy PID control works better than classical PID control in terms of steady-state error, overshoot, and settling time. The temperature can be stabilized and the heating time reduced with fuzzy PID control. Consequently, fuzzy PID control better satisfies the performance requirements of domestic water heaters, making it more suitable for such applications.

Keywords: Temperature, Fuzzy PID control, Traditional PID Control, Home water heater

1 Introduction

Control technology has witnessed continuous advancements in both control accuracy and control speed. The evolution of control systems can be categorized into three distinct phases: switch control, PID control, and intelligent control[1].

In contemporary society, the growing emphasis on quality of life has underscored the importance of a reliable hot water supply. Because of its affordability and straightforward design, traditional PID control is frequently found in Home water heaters. However, this method encounters significant challenges in parameter tuning and exhibits limitations when dealing with nonlinear systems. In contrast, Fuzzy PID control can effectively address these issues by managing nonlinear processes and achieving precise temperature regulation in water heaters[2].

Fuzzy logic algorithms address numerous complex challenges that are intractable using conventional control methods. There are two primary types of fuzzy logic

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controllers that are widely utilized today: Mamdani-type controller and Takagi-Sugeno-type controller[3-5].

The temperature control process involves monitoring the temperature variations within a specific space and of all objects therein, and adjusting the temperature to attain the desired level. To achieve stable temperature control in domestic water heaters, this study employs fuzzy PID control.

Fuzzy logic inference, fuzzy linguistic variables, and fuzzy set theory serve as the foundation for fuzzy control, an intelligent control methodology. This method uses a methodical framework that mimics human behaviour to simulate human reasoning and decision-making. The fuzzy controller primarily consists of three modules: fuzzy inference, fuzzy reasoning, and defuzzification[6]. The three modules are shown in Figure 1.

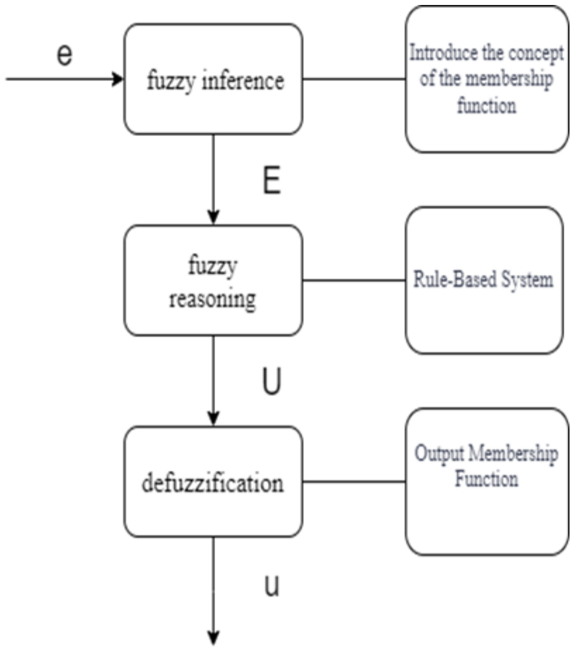

Fig. 1. Three modules of fuzzy controller. (Picture credit: Original)

2 Mathematical model of Home water heater

For a conventional storage-type electric water heater, its physical model can be simplified as follows: heating power $Q(t)$(unit: W), water temperature $T(t)$(unit: °C), and ambient temperature T_{amb}.

The water heater consists of two heat capacities and two thermal resistances, corresponding to the thermodynamic properties of the water and the heating element, respectively.

Water exhibits a specific heat capacity of C_w. The heating element's heat capacity is C_h. Water and environmental thermal resistance: R_w. R_h is the thermal resistance between the water and the heating element.

Let the water temperature be T_w and the heating element temperature be T_h. The input heating power is represented by $P(t)$.

The energy conservation equation is established according to the energy balance:

For water:

$$C_w \frac{dT_w}{dt} = \frac{T_h - T_w}{R_h} - \frac{T_w - T_{amb}}{R_w} \quad (1)$$

For heating element:

$$C_h \frac{dT_h}{dt} = P(t) - \frac{T_h - T_w}{R_h} \quad (2)$$

The initial condition is set to zero, and the differential equation is then subjected to Laplace transformation.

$$C_w s T_w(s) = \frac{T_h(s) - T_w(s)}{R_h} - \frac{T_w(s)}{R_w} \quad (3)$$

$$C_h s T_h(s) = P(s) - \frac{T_h(s) - T_w(s)}{R_h} \quad (4)$$

After rearranging the aforementioned two equations, we obtain:

$$T_w(s)\left(C_w s + \frac{1}{R_h} + \frac{1}{R_w}\right) = \frac{T_h(s)}{R_h} \quad (5)$$

$$T_h(s) = \frac{P(s) + \frac{T_w(s)}{R_h}}{C_h s + \frac{1}{R_h}} \quad (6)$$

Substitute equation (6) into equation (5) to obtain:

$$T_w(s)\left(C_w s + \frac{1}{R_h} + \frac{1}{R_w}\right) = \frac{1}{R_h} \times \frac{P(s) + \frac{T_w(s)}{R_h}}{C_h s + \frac{1}{R_h}} \quad (7)$$

Following additional simplification, the transfer function can be expressed as:

$$\frac{T_w(s)}{P(s)} = \frac{1}{C_h C_w R_h s^2 + \left(C_w + C_h\left(1 + \frac{R_h}{R_w}\right)\right)s + \frac{1}{R_w}} \quad (8)$$

Equation 8 is expressed in the standard form of a second-order system as follows:

$$G(s) = \frac{K}{\tau^2 s^2 + 2\zeta\tau s + 1} \quad (9)$$

Among these:

$$\tau = \sqrt{C_h C_w R_h R_w}$$

$$\zeta = \frac{C_w R_w + C_h(R_w + R_h)}{2\sqrt{C_h C_w R_h R_w}}$$

$$K = R_w$$

Table 1. Parameters of Home Water Heaters

Parameters	Values
C_h	2500J/℃
C_w	80000J/℃
R_h	0.01℃/W
R_w	1.23℃/W

As illustrated in Table 1, the parameters in the table are the parameters of the water heater under normal circumstances. Input the parameters into the second-order transfer function, and the following results are obtained:

$$G(s) = \frac{1.23}{2459000S^2+101500s+1} \tag{10}$$

3 Traditional PID Control

PID control is a prevalent feedback control algorithm extensively utilized in various automatic control systems, including industrial automation, robotics, temperature regulation, and motor control. In the context of PID control, P denotes the proportional component, I signifies the integral component, and D represents the derivative component[7]. Proportional control adjusts its output based on the magnitude of the current error, resulting in a rapid response. Integral control can modify the cumulative error and eliminate the steady-state error. Differential control can predict the variation trend of error, reduce overshoot and improve system stability[8]. By fine-tuning these three parameters, the controller can meet the specific requirements stipulated by users. The mathematical representation of the PID control algorithm is presented as follows[9]:

$$U(t) = K_p E(t) + K_i \int E(t)dt + K_d \frac{dE(t)}{dt}$$

$U(t)$: the controller output
$E(t)$: the error
K_p: proportional gain.
K_i: integral gain.
K_d: derivative gain.

The system's response can be accelerated by increasing the proportional gain, but oscillations may also be introduced. Increasing the integral gain can eliminate steady-state errors, yet it might result in overshoot. Elevating the derivative gain can mitigate overshoot and enhance stability, although it increases sensitivity to noise. Consequently, when tuning these three parameters, it is crucial to strike an optimal balance among response speed, overshoot, and stability, ensuring that the system responds promptly while maintaining smooth operation.

The optimal water temperature range for the human body is between 35°C and 50°C. This study determines 45°C as the optimal temperature for human physiological conditions.

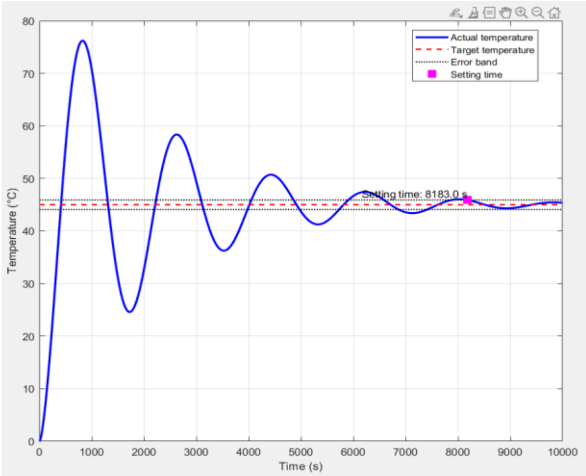

Fig. 2. Simulation results of traditional PID control. (Picture credit: Original)

Analysis of Figure 2 shows that traditional PID control takes 8183 seconds to reach the set temperature. This prolonged adjustment period makes it challenging to satisfy user requirements. Moreover, when parameters fluctuate, achieving the set temperature becomes significantly more difficult, if not impossible. Due to the numerous drawbacks of standard PID control, this study decides to employ fuzzy PID control in order to provide continuous water heater temperature management.

4 Fuzzy PID Control

One of the most popular kinds of controllers in the field of fuzzy control systems is the univariate two-dimensional fuzzy controller. It continuously employs the deviation and deviation rate as inputs to measure the dynamic nature of the output variables during the regulated process [10]. The fuzzy PID controller's construction is depicted in Figure 3.

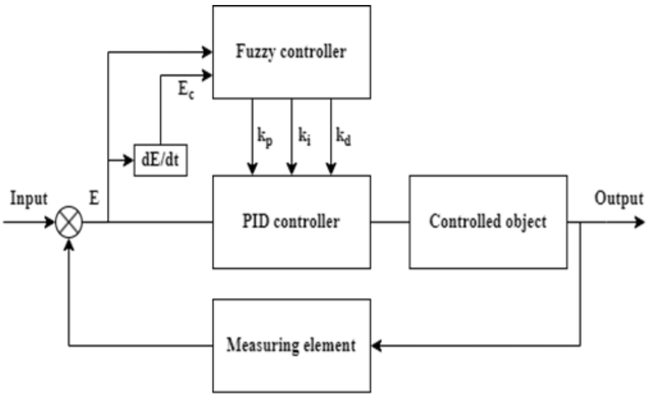

Fig. 3. The architecture of the fuzzy PID controller. (Picture credit: Original)

Fuzzy logic toolbox is a professional toolkit provided by MATLAB for the design and simulation of intelligent systems based on fuzzy logic. By simulating fuzzy concepts in human decision making, it transforms non-precise linguistic variables into a mathematically processable rule system. A rule editor, membership function editor, and fuzzy inference system (FIS) editor are the primary components of the system. A rule observer and a surface viewer make up the analysis and validation module. Figure 4 illustrates the fuzzy logic toolbox's composition.

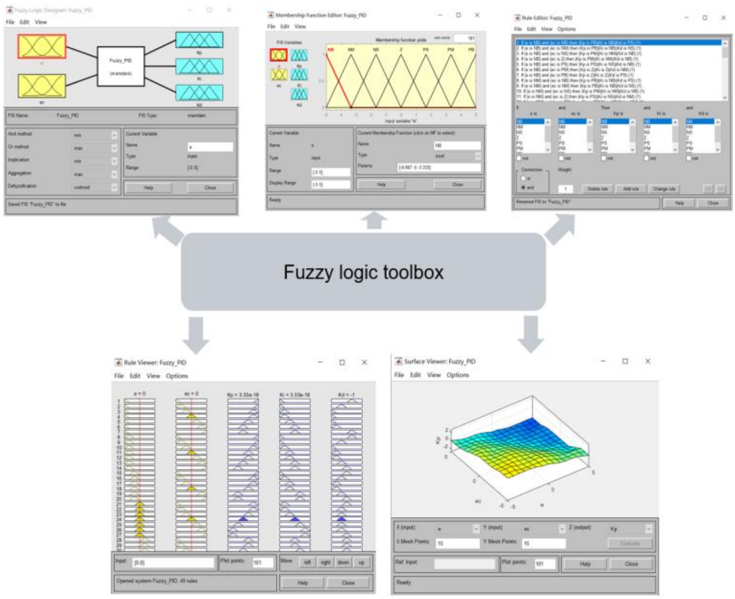

Fig. 4. Fuzzy logic toolbox. (Picture credit: Original)

In this paper, the fuzzy set domains for the error E and the rate of change of error E_c are defined within the interval [-5, 5]. The fuzzy set domains for the k_p, k_i, and k_d are established within the interval [-3, 3]. The fuzzy sets are categorized as {AB, AM, AS, Z, BS, BM, BB}. The fuzzy set's parts stand for zero, small, medium, large, negative large, negative medium, and negative small, respectively. Figures 5 and 6 show the membership functions for E and E_c. Tables 2, 3, and 4 display the fuzzy rules of k_p, k_i, and k_d.

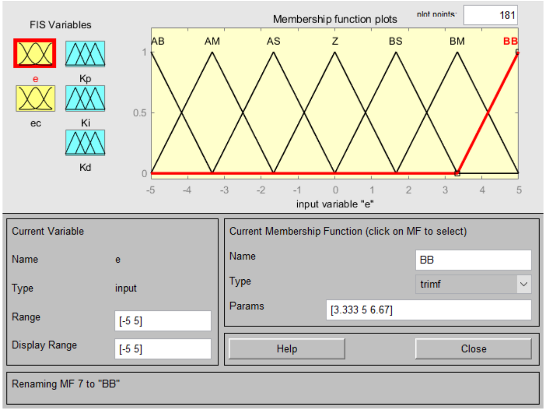

Fig. 5. The membership functions for E. (Picture credit: Original)

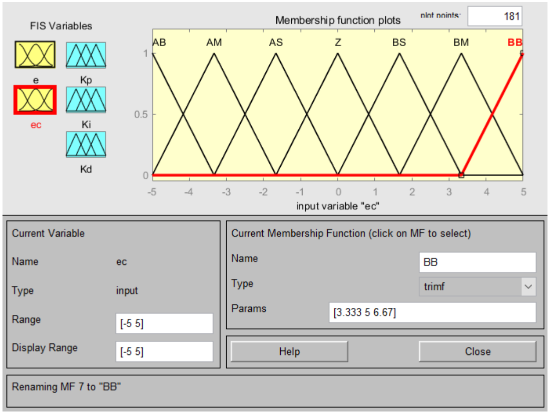

Fig. 6. The membership functions for E_c. (Picture credit: Original)

Table 2. The k_p fuzzy rules

E \ E_c	AB	AM	AS	Z	BS	BM	BB
AB	BB	BB	BB	BM	BS	BS	Z
AM	BB	BB	BM	BM	BS	Z	Z
AS	BM	BM	BM	BS	Z	AS	AM
Z	BM	BS	BS	Z	AS	AM	AM

BS	BS	BS	Z	AS	AM	AM	AM
BM	Z	Z	AS	AM	AM	AB	AB
BB	Z	AB	AS	AM	AB	AB	AB

Table 3. The k_i fuzzy rules

E \ E_c	AB	AM	AS	Z	BS	BM	BB
AB	AB	AB	AB	AM	Z	Z	Z
AM	AB	AB	AM	AM	Z	Z	Z
AS	AM	AM	AS	AS	BS	BS	BS
Z	AM	AS	AS	Z	BM	BM	BM
BS	AS	AS	Z	BS	BM	BM	BM
BM	Z	Z	BS	BM	BM	BB	BB
BB	Z	Z	BS	BM	BB	BB	BB

Table 4. The k_d fuzzy rules

E \ E_c	AB	AM	AS	Z	BS	BM	BB
AB	BS	BS	Z	Z	Z	BB	BB
AM	AS	AS	AS	AS	Z	BS	BM
AS	AB	AB	AM	AS	Z	BS	BM
Z	AB	AM	AM	AS	Z	BS	BM

BS	AB	AM	AS	AS	Z	BS	BS
BM	AM	AS	AS	AS	Z	BS	BS
BB	BS	Z	Z	Z	BB	BB	BB

Using the fuzzy table, we can derive 49 fuzzy rules, which are detailed in the subsequent statements.

$$If\,(E\ is\ A_i)\ and\ \left(E_c\ is\ A_j\right)\ then$$
$$(k_p\ is\ C_a)(k_j\ is\ C_b)(k_d\ is\ C_c)$$

where A_i and A_j are fuzzy sets of E and E_c, C_a, C_b, C_c are fuzzy sets of k_p, k_j, k_d.

Figure 7 shows how MATLAB/Simulink establishes both traditional PID control and fuzzy PID control models.

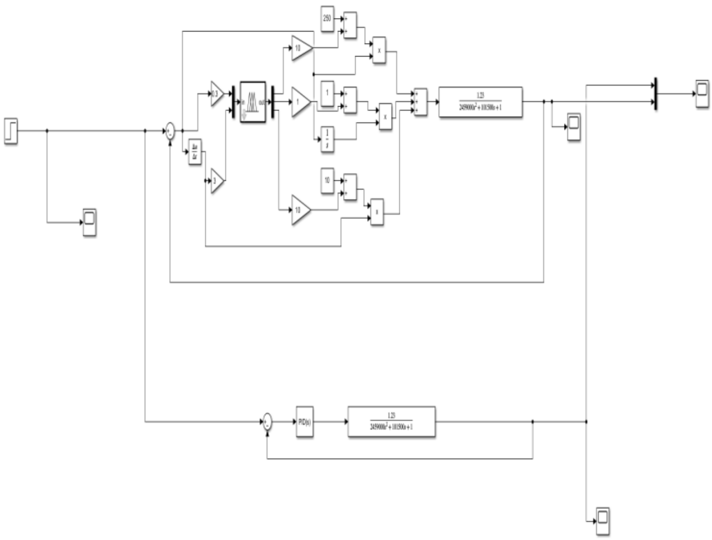

Fig. 7. Fuzzy PID control simulation model and traditional PID control. (Picture credit: Original)

The Rule viewer can display the precise values of k_p, k_i, and k_d in the case of varied E and E_c inputs, as shown in Figure 8.

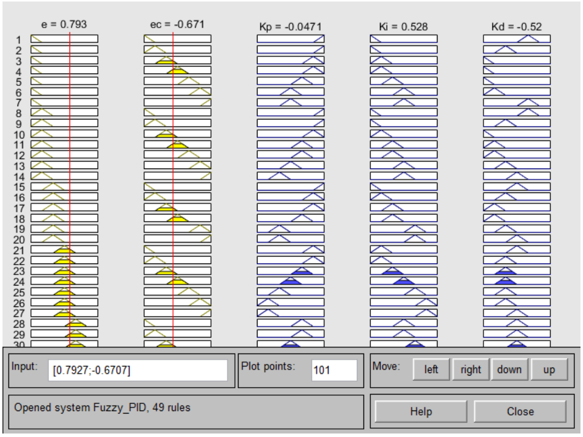

Fig. 8. Rule viewer. (Picture credit: Original)

After constructing the simulation model, the Fuzzy Inference System (FIS) is integrated into the fuzzy logic controller for simulation. The final image results are then visualized using the scope. The image output by the scope is presented in Figure 9.

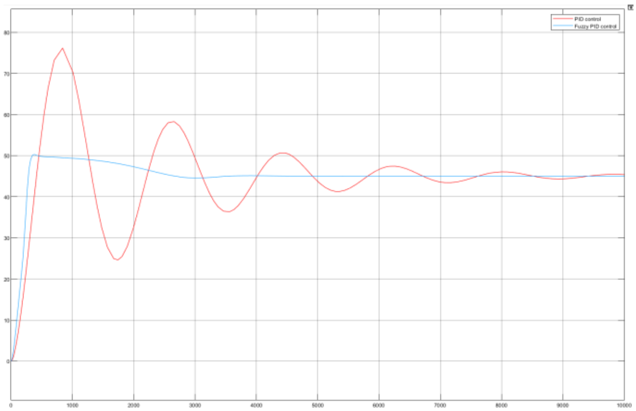

Fig. 9. Scope of output image. (Picture credit: Original)

As illustrated in Figure 9, fuzzy PID control has a shorter setting time, less overshoot, and reaches the desired temperature of 45°C faster than standard PID control. Moreover, while fuzzy PID control can stabilize the water temperature precisely at 45°C, traditional PID control only maintains the temperature fluctuating around 45°C, failing to achieve stable control.

To accurately determine the setting time for the water temperature to reach the user-defined temperature under fuzzy PID control, the recorded data was imported into MATLAB for detailed analysis. Specifically, the data from the scope was imported into the MATLAB workspace in a structure-with-time format. Subsequently, MATLAB was utilized to analyze this data to ascertain the setting time and generate the corresponding graph. The analysis results are presented in Figure 10.

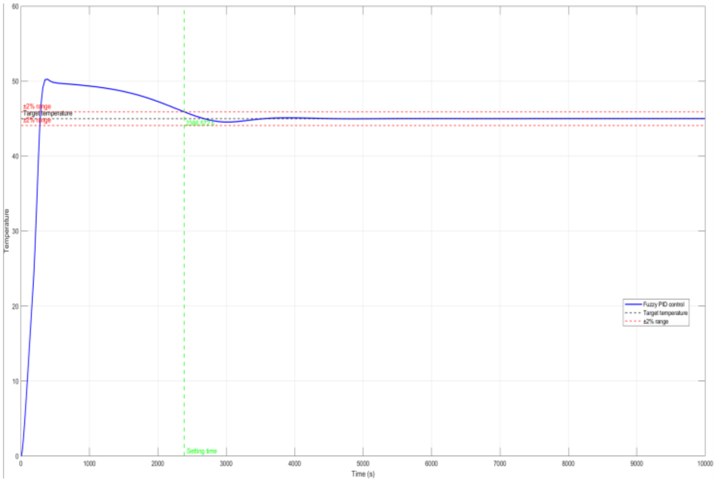

Fig. 10. Setting time under fuzzy PID control. (Picture credit: Original)

It can be observed from FIG. 10 that the setting time to reach the target water temperature under fuzzy PID control is 2386.572 seconds, significantly shorter than the 8183 seconds required under traditional PID control. Additionally, the water temperature under fuzzy PID control stabilizes precisely at 45°C, demonstrating superior stability compared to the fluctuating temperature around 45°C observed under traditional PID control.

With an overshoot of 11.69%, the maximum water temperature under fuzzy PID control is 50.2584°C. In contrast, the overshoot for traditional PID control, calculated using the same method, is significantly higher at 69.33%. This comparison clearly demonstrates that the overshoot under fuzzy PID control is substantially lower than that under traditional PID control.

5 Conclusion

Fuzzy PID produces control results that are noticeably superior to those of traditional PID, according to a comparison of the MATLAB/Simulink simulation results of fuzzy PID control and traditional PID control for home water heaters. The setting time under fuzzy PID control merely amounts to 2386.572 seconds, with an overshoot of 11.69% and zero steady-state error. Consequently, the household water heater under fuzzy PID control is more capable of fulfilling the usage demands of users.

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