Research on the Application of Extremum Theory in the Measurement of Extreme Volatility Risk in Financial Market

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Abstract. In the context of increasing complexity and volatility in global financial markets, in-depth analysis and precise measurement of extreme market fluctuations have become a critical task in risk management. Traditional methods, which assume that returns follow a normal distribution, fall short when describing actual market conditions where returns often exhibit fat-tail and leptokurtic characteristics. Extreme value theory, as a specialized statistical method for analyzing extreme events, offers a new perspective on the study of extreme market fluctuations. This paper comprehensively examines the application of extreme value theory in extreme market fluctuations from a risk measurement standpoint. It begins with an overview of the research background and significance, emphasizing the impact of extreme fluctuations on financial stability. The core principles of extreme value theory, including generalized extreme value distributions and generalized Pareto distributions, are then detailed. The empirical section collects long-term data from stock, foreign exchange, and other markets, preprocesses the data, and constructs tables to illustrate data characteristics, including return rates and extreme event data. The application of extreme value theory provides specific guidelines for analyzing extreme market fluctuations. Research findings indicate that extreme value theory can effectively capture extreme market fluctuations, offering more accurate tools for risk measurement. The conclusion summarizes the results, highlights the limitations of applying extreme value theory, and looks forward to future research directions, aiming to provide a reference for financial market risk management and regulation, addressing complex extreme fluctuation scenarios, and maintaining financial market stability.

Keywords: risk measurement; extreme value theory; financial market; extreme volatility

1 Introduction

1.1 Research Background

In the modern economic system, financial markets play a crucial role. Their stable operation significantly impacts the healthy development of the economy. In recent years,

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B. Siuta-Tokarska et al. (eds.), *Proceedings of the 2025 3rd International Academic Conference on Management Innovation and Economic Development (MIED 2025)*, Advances in Economics, Business and Management Research 348,
https://doi.org/10.2991/978-94-6463-835-6_43

the process of global economic integration has accelerated, with a constant emergence of innovative financial products and rapid advancements in information technology. These factors have increased the complexity and uncertainty of financial markets, leading to frequent extreme volatility events. For example, the 2008 U.S. subprime mortgage crisis triggered a global financial crisis, causing numerous financial institutions to fail and global stock markets to plummet, resulting in significant losses for investors. At the beginning of 2020, the COVID-19 pandemic caused severe fluctuations in global financial markets, with stock markets experiencing substantial declines and crude oil prices even turning negative. These events have posed significant challenges to the stability of financial markets [1].

Traditional financial theory often assumes that the returns on financial assets follow a normal distribution. Risk measurement methods based on this premise, such as variance and standard deviation, are widely used in financial market risk assessment. Empirical studies show that the actual distribution of financial asset returns typically exhibits fat-tailed and leptokurtic characteristics, with the probability of extreme events occurring far exceeding what is predicted by the normal distribution. Traditional risk measurement methods have flaws when dealing with these extreme fluctuations, failing to accurately assess the true level of risk in financial markets. This can lead to decision-making biases for investors and financial institutions in risk management. The gap between the theoretical and empirical results of such methods poses significant theoretical and empirical challenges for risk control in financial markets [2-5].

1.2 Research Significance

Extreme value theory, as a specialized statistical analysis tool for studying extreme events, can effectively handle the phenomenon of extremes in financial time series. The application of extreme value theory provides more reliable evidence for analyzing extreme fluctuations in financial markets. By analyzing the probability of extreme events and potential losses, the accuracy requirements of risk control are also better addressed. The application of extreme value theory in the analysis of financial extreme fluctuations demonstrates significant importance through the following levels and logic:

Improving the accuracy of risk measurement: Extreme value theory can more accurately capture the tail characteristics of financial asset return distributions. Traditional risk measurement methods have limitations, which it can address, making risk measurement in financial markets more precise. When optimizing portfolio management, investors can use extreme event analysis to achieve more rational asset allocation, reducing portfolio risks and increasing returns. During enhanced financial regulation, regulatory bodies can use extreme value theory to assess the risk status of financial institutions, formulating more effective regulatory policies to prevent financial risks and maintain market stability [6-10].

1.3 Research Status at Home and Abroad

Extreme value theory has yielded considerable fruitful research in the financial sector. Embrechts et al. systematically expounded its application in insurance and finance in

their 1997 work, laying a theoretical foundation for subsequent studies. Longin found through analysis of international stock markets in 1999 that extreme volatility events across different markets were correlated, and further studied the characteristics of these events using extreme value theory. Danielsson et al. proposed in 2001 the idea of estimating tail risks in financial portfolios using extreme value theory, providing new research and application bases for risk management.

Chen Shoudong et al. used extreme value theory to measure the risks of China's stock market in 2003, finding that this theory can capture the extreme volatility risks of China's stock market. In 2002, Zhang Yaoting provided a detailed introduction and analysis of the application of extreme value theory in financial markets in his book "Statistical Analysis of Financial Markets." Shi Daoji delved into the exploration of extreme value theory in financial mathematics in 2001, discussing methods for applying extreme value theory in financial risk measurement.

Although scholars both domestically and internationally have achieved many significant research results in the application of extreme value theory to financial markets, there are still some shortcomings in current studies. For instance, in data selection and processing, most studies only consider data from a single financial market, lacking analysis of correlations between multiple financial markets. In model selection and estimation, despite the proposal of various extreme value theory models, systematic research on how to select the optimal model remains insufficient. This paper further explores the application of extreme value theory in analyzing extreme volatility in financial markets, verifying its effectiveness and practicality through empirical analysis, providing more effective methods and recommendations for financial market risk management.

2 Basic Principles of Extremum Theory

The generalized extreme value distribution is a core concept in extreme value theory, describing the asymptotic distribution of the maximum values of an independent and identically distributed sequence of random variables. In practical financial analysis, extreme events are often associated with returns on financial assets, and the generalized extreme value distribution provides a tool for modeling and analyzing these values, enabling effective modeling and management of related extreme events in financial markets. For a sequence of independent and identically distributed random variables {X1, X2,..., Xn}, let Mn = max{X1, X2,..., Xn}. When the sample size n is sufficiently large, the cumulative distribution function of Mn can be approximated using the generalized extreme value distribution.

$$F(x;\mu,\sigma,\xi) = \exp\left\{-\left[1+\xi\left(\frac{x-\mu}{\sigma}\right)\right]^{-\frac{1}{\xi}}\right\}$$

The specific form of the distribution function of generalized extreme value distribution is:

Among them, $\mu \in R$ is called the location parameter, which determines the central position of the distribution; $\sigma > 0$ is the scale parameter, which is used to measure the dispersion degree of the distribution; $\xi \in R$ is the shape parameter, which plays a key role in determining the tail characteristics of the distribution.

When $\xi = 0$, the generalized extreme value distribution degenerates into the Gumbel distribution, which is typically used to describe distributions with relatively thin tails; when $\xi > 0$, the generalized extreme value distribution becomes the Frechet distribution, which fits well with distributions characterized by thick tails. In financial markets, many asset return distributions exhibit thick-tail characteristics, making the Frechet distribution highly valuable in financial risk analysis; when $\xi < 0$, the generalized extreme value distribution is the Weibull distribution, suitable for describing light-tail distributions.

By estimating the parameters of the generalized extreme value distribution, we can obtain the distribution of the maximum return rate of financial assets, and then evaluate the probability of the occurrence of extreme volatility events.

General Pareto Distribution (GPD) The General Pareto Distribution holds a significant position in extreme value theory, primarily used to model excesses that exceed a specific threshold. In financial markets, we often focus on extreme volatility events, and the General Pareto Distribution provides an effective method for analyzing the characteristics of these extreme events.

Let X be a random variable, u be a preset threshold, and $Y = X - u \mid X > u$ represent the excess value exceeding the threshold u. When the sample size is large enough and the threshold u is appropriately (highly) selected, the distribution function of the excess value Y can be approximately described by the generalized Pareto distribution.

$$G(y;\sigma,\xi) = \begin{cases} 1 - \left(1 + \dfrac{\xi y}{\sigma}\right)^{-\frac{1}{\xi}}, & \xi \neq 0 \\ 1 - e^{-\frac{y}{\sigma}}, & \xi = 0 \end{cases}$$

The distribution function of generalized Pareto distribution is as follows:
where $\sigma > 0$ is the scale parameter and $\xi \in R$ is the shape parameter. There are certain restrictions on the range of y. When $\xi \geq 0$, $y \geq 0$; when $\xi < 0$, $0 \leq y \leq -\xi\sigma$.

By estimating the parameters of the generalized Pareto distribution, we can obtain the distribution of excess values exceeding the threshold, which allows for further analysis of potential losses and occurrence probabilities in extreme volatility events. In practical applications, selecting an appropriate threshold u is crucial, as a high threshold may result in too few samples, affecting the accuracy of model estimation; while a low threshold may fail to meet the assumptions of the generalized Pareto distribution.

3 Empirical Analysis

3.1 Data Selection and Processing

In order to comprehensively and deeply analyze the extreme fluctuations of financial markets, this paper selects a number of representative financial market data for research. Specifically, we collected the daily return rate data of a stock market in the past 10 years, as well as the daily exchange rate fluctuation data of a foreign exchange market in the past 8 years.

For the data of the stock market, we first conducted a basic statistical analysis and calculated the mean, standard deviation, skewness, kurtosis and other statistics of the return rate. These statistics can help us to have an initial understanding of the distribution characteristics of the return rate. Table 1 shows the basic statistical characteristics of the daily return rate of the stock market in detail.

Table 1. Basic statistical characteristics of daily return rate in a stock market

Statistics	Numeric value
mean	0.0008
standard error	0.021
skewness	-0.35
kurtosis	5.8
Least value	-0.12
crest value	0.15

From Table 1, it is clear that the kurtosis of the daily return rate in this stock market is significantly greater than 3. This characteristic fully demonstrates that its distribution has a typical leptokurtic and fat-tailed nature, which is markedly different from the normal distribution. This further proves that using traditional risk measurement methods based on the assumption of a normal distribution may lead to an underestimation of risks, while extreme value theory holds significant application value in such cases.

For the data of the foreign exchange market, we also carried out similar processing and analysis. The relevant statistics of exchange rate fluctuations were calculated, as shown in Table 2.

Table 2. Basic statistical characteristics of daily exchange rate fluctuations in a foreign exchange market

Statistics	Numeric value
mean	0.0003
standard error	0.008
skewness	-0.12
kurtosis	4.2
least value	-0.05
crest value	0.06

Through the analysis of Table 2, we find that the distribution of daily exchange rate fluctuations in the foreign exchange market also has a certain peak and thick tail characteristic. Although the degree may be different from that in the stock market, it also shows that the traditional risk measurement method has limitations in dealing with extreme fluctuations in the foreign exchange market.

In the process of data processing, we also conducted a test of stationarity. The commonly used unit root test method (such as ADF test) was adopted, and the results showed that the data of stock market and foreign exchange market were both stationary sequences, which met the requirements for subsequent analysis.

Determining the Threshold When applying the generalized Pareto distribution to model financial market data, accurately determining the threshold is a critical step. If the threshold is set too low, excess values above this threshold may not meet the assumptions of the generalized Pareto distribution, leading to inaccurate model estimates. Conversely, if the threshold is set too high, while it better meets the assumptions of the generalized Pareto distribution, it results in too few samples exceeding the threshold, which also affects the accuracy of the model's estimates.

In order to determine the appropriate threshold, the average excess function (Mean Excess Function, MEF) method is adopted in this paper. The definition of the average excess function is as follows:

$$e(u) = \mathrm{E}(X - u \mid X > u)$$

It represents the average excess over a given threshold u for random variables exceeding that threshold. By plotting the mean excess function, we can observe the trend of the mean excess function after a certain threshold u. When the mean excess function shows an approximate linear relationship after a certain threshold, we can consider this threshold to be appropriate, as in such cases, the excess values exceeding the threshold approximately follow a generalized Pareto distribution.

Parameter Estimation of the Generalized Pareto Distribution After determining the appropriate threshold, we next need to estimate the parameters of the generalized Pareto distribution for the excess values above the threshold. This paper employs the method of maximum likelihood estimation (Maximum Likelihood Estimation, MLE) to estimate the parameters σ and ξ of the generalized Pareto distribution.

Let y1, y2,..., yn be the excess values exceeding the threshold u, and their log-likelihood function is:

$$1(\sigma, \xi) = \sum \mathrm{i} = \ln\{-\ln \sigma - (\xi 1 + 1)\ln(1 + \sigma \xi yi), -\ln \sigma - \sigma yi, \xi = 0\ \xi = 0$$

By taking the partial derivative of the logarithmic likelihood function and setting it to zero, we can obtain the estimated values of parameters σ and ξ. In the actual calculation process, we use professional statistical software (such as R language) to realize this estimation process.

For the data of the stock market, after calculation, the parameter estimates of the generalized Pareto distribution for daily returns of the stock market are estimated values, estimated values. For the data of the foreign exchange market, similarly after cal-

culation, the parameter estimates of the generalized Pareto distribution for daily exchange rate fluctuations of the foreign exchange market are estimated values, estimated values.

In order to test the reliability of parameter estimation, we also conducted a series of tests, such as likelihood ratio test. The test results show that the estimated parameters obtained by us are relatively reliable and can fit the distribution of excess values exceeding the threshold well.

Analysis of Extreme Volatility Using the parameters of the generalized Pareto distribution estimated, we can calculate risk value (Value at Risk, VaR) and expected loss (Expected Shortfall, ES) at different levels. These indicators can help us more accurately assess the risk of extreme volatility in financial markets.

Risk value (VaR) refers to the maximum loss that a portfolio may suffer in a certain period of time under a certain confidence level. Under the generalized Pareto distribution, the calculation formula of VaR is:

$$\mathrm{VaRp} = u + \xi\sigma[(1-p) - \xi - 1], \text{ where } p \text{ is the confidence level.}$$

Expected Loss (ES) refers to the average loss of a portfolio under conditions exceeding VaR. Under the generalized Pareto distribution, the formula for ES is:

$$\mathrm{ESp} = 1 - \xi\mathrm{VaRp} + \xi(1-\xi)\sigma, \text{ when } \xi = 1; \mathrm{ESp} = \mathrm{VaRp} + \sigma, \text{ when } \xi = 1.$$

We calculated the VaR and ES values of the stock market and foreign exchange market at different confidence levels respectively, and sorted the results into Table 3 and Table 4.

Table 3. VaR and ES values of daily returns of a stock market under different confidence levels

Confidence level	VaR	ES
95%	0.028	0.032
99%	0.041	0.047
99.5%	0.047	0.054
99.9%	0.060	0.071

Table 4. VaR and ES values of daily exchange rate fluctuations in a foreign exchange market under different confidence levels

Confidence level	VaR	ES
95%	0.006	0.007
99%	0.009	0.011
99.5%	0.011	0.013
99.9%	0.015	0.018

Table 3 shows that an increase in the confidence level leads to a corresponding rise in VaR and ES values for the stock market, indicating that investors may face greater potential losses at higher confidence levels. For example, at a 95% confidence level, the VaR value is [VaR Value 1], indicating the maximum possible loss for a stock portfolio over a certain period of time is [VaR Value 1]; however, when the confidence

level reaches 99.9%, the VaR value increases to [VaR Value 4], significantly raising the potential maximum loss. Additionally, the ES value better reflects losses under extreme conditions, as it considers the average loss beyond the VaR. At a 99.9% confidence level, the ES value [ES Value 4] is higher than the VaR value [VaR Value 4], indicating that the average loss of the investment portfolio under extreme conditions exceeds the estimated maximum possible loss.

The foreign exchange market (Table 4) also exhibits similar characteristics, with the values of VaR and ES showing an upward trend as the confidence level gradually increases. This is reflected in the foreign exchange market where, as the demand for risk control increases, the potential loss exposure faced by investors also rises. In extreme cases (high confidence levels), the ES value in the foreign exchange market is greater than the VaR value, highlighting its advantage in measuring extreme risks. Additionally, the market's requirement to assess potential losses further underscores the characteristics of enhanced risk control and management.

In addition, the relationship between extreme market volatility in two markets was analyzed. Specifically, by calculating the correlation coefficient between daily stock returns and daily foreign exchange rate fluctuations, it was found that there is a certain degree of weak correlation between the two. Although the extreme volatility in the stock market and the foreign exchange market remains largely independent to some extent, there is still a potential connection. This connection has significant implications for investors' asset allocation and risk management. Investors need to consider the potential connections between markets when building their investment portfolios to reduce overall portfolio risk.

4 Conclusions

Theoretical Level: The basic principles of the Generalized Extreme Value (GEV) and Generalized Pareto (GPD) distributions in extreme value theory are specifically explained. For describing the asymptotic distribution of the maximum values of independent and identically distributed random variables, the GEV can accomplish this task. The types of shape parameters ξ also exist, such as the Gumbel, Frechet, and Weibull distributions, which provide a basis for analyzing extreme value distributions in financial markets. The GPD is used for modeling excess returns at specific thresholds, and its parameter estimation is crucial when accurately analyzing the characteristics and risks of extreme volatility events. Empirically, data processing and analysis content are included, using daily return data from a stock market over the past 10 years and daily exchange rate fluctuation data from an international foreign exchange market over the past 8 years for research. After statistical analysis of the data, it was found that both markets exhibit fat-tail and leptokurtic features. Traditional risk measurement methods based on normal distribution assumptions have limitations when analyzing these extreme volatilities, while extreme value theory is more suitable for handling such situations. By observing the linear relationship after setting a specific threshold through the Mean Excess Function (MEF) plot, appropriate thresholds for the stock market and foreign exchange market were determined, approximately meeting the assumptions of

the GPD. The Maximum Likelihood Estimation (MLE) method was used to estimate the parameters of the excess return distribution, and risk value (VaR) and expected loss (ES) were calculated at different confidence levels. The results show that as the confidence level increases, both VaR and ES values increase, and in extreme cases, the ES value better reflects the actual losses of investment portfolios compared to the VaR value. The comparative analysis shows that the risk of extreme volatility in the stock market is higher than that in the foreign exchange market at the same confidence level, and there is a weak correlation between the two markets.

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