Inventory Models Incorporating Demand as a Function of Advertising Cost and Pricing Discount

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Abstract. Effective supply chain management relies on strong cooperation between manufacturers and retailers. Both can enhance their profits by boosting market demand through offering discounts and increasing marketing expenditure. This research aims to develop inventory models considering annual demand dependent on the discount given by the manufacturer to the customer on the maximum retailer price and the advertising expenditure of the retailer. We develop inventory models in both non-cooperative and cooperative environments to optimize profits. Inventory level and discounts are considered decision variables for the manufacturer, while EOQ and marketing expenditure are the decision variables for the retailer. Solution process and sensitivity analysis are performed using numerical examples. Concavity of the profit functions with respect to all the decision variables is discussed using analytical methods. Results show that annual market, EOQ, inventory level, demand, and profit increase in cooperative environments. A non-cooperative environment is favourable for the manufacturer.

Keywords: Manufacturer, Retailer, Price Discount, Marketing cost, Shortage.

1. Introduction

The manufacturer produces items and makes them available in the market with the help of the retailers. In the two-echelon system, cooperation between manufacturer and retailer is very important. Retailers manage inventory as per market demand. Efforts of the retailer will also affect market demand. The marketing strategy of the retailer to advertise the product will increase market demand. Market demand can be increased with the retailer's marketing expenditure. In this research, annual market demand is considered directly proportional to the retailer's marketing expenditure. Prior work modelled sellers as manufacturers and buyers as retailers for a two-echelon system. This model extends that approach by explicitly considering a supply chain for manufacturers and retailers. [12] developed models for manufacturers and retailers in competitive and cooperative environments. According to them, if the manufacturer and retailer do not want to share profit margins, then the concept of the non-cooperative model exists. They also proposed a model in a

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model in a cooperative environment where both manufacturers and retailers agree to share profit margins. [2] considered annual demand dependent on maximum retail price and marketing expenditure. They found annual demand by $D=kp^{-\alpha}m^{\beta}$, where p is the market price, m is marketing expenditure, and k is any scaling constant. [1] developed a model where the manufacturer agrees to share the profit margin. [21, 22, 29] developed inventory models considering total annual demand sensitive to marketing expenditure. [34] proposed a model considering joint lot size and order quantity for the two-echelon system as a bargaining model based on Banarjee's JELS model. As the buyers controlled the market, they considered the buyer a leader. [16] developed a model for considering payment delays and cash discounts. [35] studied a multi-echelon system chain. [15] studied a cooperative advertising cost model for manufacturers and retailers. They considered that the manufacturer will do some favour for the retailers only if certain market demand is achieved. [6] studied retailers' inventory with price-sensitive demand with the shortage. [23] developed a model for demand sensitive to the price and quantity discount. An inventory model for quantity discount was also given [26]. [31] studied price discount facilities in an EOQ model. [5] studied the importance of cooperation in the two-echelon supply chain system. They found that the manufacturer should focus on CSR activities and the final product's advertising and marketing are managed by the retailer. [14] considered demand sensitive to price, recycling factor, and service to develop the models. [17] developed an inventory model considering payment delay. [13] reviewed the coordination process in the supply chain system and found that more research is needed to examine the cooperation in the supply chain system. [25] outlined a framework in which the manufacturer offers price concessions to address demand fluctuations affected by market pricing. [9] developed a model in cooperative and non-cooperative games. They found that a cooperative environment between manufacturer and retailer increases profit, but it is very difficult to incorporate as it decreases the profit of the manufacturer. [24] studied manufacturers and retailers in cooperative and non-cooperative environments and determined the pricing, replenishment cycle, and other inventory variables. [30] found that cooperative advertisement policy among suppliers, manufacturers, and retailers is the most effective policy to increase profit in the competitive world. [27] studied the service and pricing strategy for retailers in competition and coordination with manufacturers. [37] studied joint inventory management to optimize the inventory resources. [38] studied the impact of discounts on the manufacturer and retailer supply chain system. [39] looked into the cooperative advertising issue while considering inventory decision variables in a two-layer supply chain system. [28] examined how to harmonize key decision areas—pricing, advertising, ordering, and inventory—within a multi-layer supply chain consisting of one manufacturer and several supply and distribution partners. [7] proposed an inventory model for a two-echelon supply chain comprising one manufacturer and multiple retailers, considering imperfect product quality and demand sensitivity to price. [4] examined the impact of cooperative advertising on inventory decisions in a manufacturer–retailer framework.

[3] proposed an inventory model that incorporates discounts for advance stock bookings. The study is done for price, advertising, and optimal order quantity decision variables considering total demand sensitive to price and advertising cost. [18] looked into the pricing and advertising aspects of perishable product inventory. [20] created a fuzzy economic manufacturing model tailored for imperfect production processes. [36] formulated an inventory model focused on deteriorating items, incorporating economic order policies with demand influenced by price and advertising. [19] established inventory models that account for demand based on advertising for perishable goods, both with and without shortages. [11] introduced an inventory model addressing deteriorating demand. [10] developed inventory models for two warehouses with deteriorating products. In this model, price discount policy was also considered, where the supplier offers some price discounts for advance payments. [33] developed discount policy inventory models and found that profit can be increased by 0.35 to 2.4 % with a discount of 20%. [32] developed an inventory model for optimal lot size and pricing considering price-dependent and advertising-dependent demand. [8] developed a model price affected by price and stock level. They developed a model to optimize selling price and cycle time to maximize the profit.

In this research work, annual demand is considered sensitive to marketing expenditure and discounts on the maximum retail price given by the manufacturer to the customer. We also considered that retailers should work on a fixed commission (percentage of the market retail price) basis discount by the manufacturer. In this work, decision variables EOQ and marketing expenditure will be decided by the retailer, while the inventory level of the manufacturer and discount on MRP for consumers will be decided by the manufacturer. The Stackelberg game model was developed by Heinrich Stackelberg in 1934. Stackelberg's models are based on the non-cooperation among the players. In these models, one player works as a leader and maximizes his profit by considering decisions taken by the second player, who is known as the follower.

2. Assumptions and Notations for Developing the Model

2.1 Assumptions

In developing the inventory models, the following assumptions are taken into consideration:

1. Parameters are deterministic.
2. An infinite planning horizon is assumed.
3. The manufacturer’s discount and marketing expenditure influence the annual market demand.
4. The shortage is permitted.
5. The production can occur at an infinite rate.
6. Retailer determines EOQ.
7. Advertising expenditure is decided by the retailer.

8. Discount on MRP is decided by the manufacturer.
9. Inventory level is decided by the manufacturer

2.2 Notations

In this section, all the notations are defined to develop inventory models.

2.2.1 Decision variables

z: Inventory held by the manufacturer

Q: EOQ determined by the retailer.

d: Discount of customer given by the manufacturer.

m: Retailer's marketing expenditure.

2.2.2 Input variables

k: Scaling factor used in the annual demand function, where $k>0$.

α: sensitivity constant for the discount in demand function.

β: Sensitivity constant for marketing expenditure in-demand function.

r: demand rate (units/time).

P: Maximum Retail Price

$\mathbf{O_c}$: Ordering cost of the retailer

$\mathbf{H_b}$**:** Retailer's holding cost

$\mathbf{S_c}$: Set up the cost of the manufacturer

$\mathbf{H_s}$**:** Holding cost of manufacturer

S: Shortage cost of the manufacturer (unit/time)

$\mathbf{P_c}$: Manufacturer 's Production cost (currency/unit)

x: The discount granted to the retailer by the manufacturer.

In this study, annual demand D is considered sensitive to the manufacturer's discount d offered to customers and the retailer's marketing expenditure m; therefore, an annual demand function can be formulated as $D=kd^{\alpha}m^{\beta}$

2.3 Inventory model formulations for manufacturer and retailer

***2*.3.1 Manufacturer's inventory model for profit maximization**

In this model researcher considered that the maximum retail price (MRP) P is decided by the manufacturer. The manufacturer gives a discount of x% to the retailer on MRP. Discount d% on MRP is given to the consumers and is decided by the manufacturer.

All the incurred cost are defined below:

Production cost: As production cost per unit is assumed P_c, so for yearly demand D, the total production cost for the manufacturer will be:

Production cost=P_c*D

Set up cost: Since set up cost is S_c per set up, so the total annual set up cost will be

Set up cost=$S_c * \frac{D}{Q}$

Holding cost: Average inventory per cycle* time*Number of cycles*holding cost per unit

Holding cost= $0.5 * z * \frac{z}{r} * \frac{D}{Q} * H_s$

Shortage cost: Average inventory per cycle* time*Number of cycles*shortage cost per unit

Shortage cost=$0.5 * (Q - z) * \frac{Q-z}{r} * \frac{D}{Q} * S$

Manufacturer's annual profit Π_S = Total revenue -Production cost - Setup cost - Holding cost – Shortage cost

$$\Pi s = (100 - d - x) * P * D - P_c * D - S_c * \frac{D}{Q} - 0.5 * z * \frac{z}{r} * \frac{D}{Q} * H_s - 0.5 * (Q - z) * \frac{(Q - z)}{r} * \frac{D}{Q} * S$$

By putting the $D=kd^{\alpha}m^{\beta}$ in above equation, we found following profit function for the manufacturer

$\Pi s\ = (100 - d - x) * P * kd^{\alpha}m^{\beta} - P_c * kd^{\alpha}m^{\beta} - S_c * \frac{kd^{\alpha}m^{\beta}}{Q} - 0.5 * z * \frac{z*}{r}\frac{kd^{\alpha}m^{\beta}}{Q} * H_s - 0.5 *$
$(Q - z) * \frac{(Q-z)}{r} * \frac{kd^{\alpha}m^{\beta}}{Q}S$ …………..(1)

Partially differentiating above profit function with respect to z

$\frac{\partial \Pi s}{\partial z} = -\frac{z*}{r}\frac{kd^{\alpha}m^{\beta}}{Q} * Hs + \frac{(Q-z)}{r} * \frac{kd^{\alpha}m^{\beta}}{Q}S$

By solving $\frac{\partial \Pi s}{\partial z} = 0$

We will have:

$-\frac{z*}{R}\frac{kd^{\alpha}m^{\beta}}{Q} * Hs + \frac{(Q-z)}{r} * \frac{kd^{\alpha}m^{\beta}}{Q}S = 0$

i.e.

$z = \frac{Q*S}{Hs+S}$ …………… (2)

Partially differentiating above profit function with respect to d

$\frac{\partial \Pi s}{\partial d} = \alpha kd^{\alpha-1}m^{\beta}\left[(100 - d - x)P - Pc - \frac{Sc}{Q} - 0.5\frac{z^2}{rQ}Hs - 0.5\frac{(Q-z)^2}{rQ}S\right] - kd^{\alpha}m^{\beta}P$

With the help of the first-order partial derivative i.e, $\frac{\partial \Pi s}{\partial d} = 0$

We will have

$d = \frac{\alpha}{(\alpha+1)P}\left[(100 - x)P - Pc - \frac{Sc}{Q} - 0.5\frac{z^2}{rQ}Hs - 0.5\frac{(Q-z)^2}{rQ}S\right]$ ………… (3)

To analyze the maxima and minima, second order derivatives will be analyzed for profit function

$$\Pi s\ = (100 - d - x) * P * D - Pc * D - Sc * \frac{D}{Q} - 0.5 * \frac{z^2}{r} * \frac{D}{Q} * Hs - 0.5 * \frac{(Q-z)^2}{r} * \frac{D}{Q} * S$$

(i) **Maxima and Minima for decision variable z:** Since partial differentiation will be performed with respect to z only, other terms will be considered constant so given function will be

$$\Pi s\ = constant1 - constant2 - constant3 - 0.5 * z^2 * C * Hs - 0.5 * (Q - z)^2 C * S$$

Where

$C = \frac{D}{rQ}$

Or

$\Pi s\ = constant - 0.5 * z^2 * C * Hs - 0.5 * (Q - z)^2 C * S$

So second order differentiation with respect to z will be

$\frac{\partial^2 \Pi s}{\partial z^2} = -C(H_S + S)$

Since $C = \frac{D}{rQ} > 0$, $\frac{\partial^2 \Pi s}{\partial z^2} = -C(H_S + S) < 0$ So profit function will be maxima for $z = \frac{Q*S}{Hs+S}$

(ii) **Maxima and Minima for decision variable d:** To verify the maxima and minima of the manufacturer's profit function in relation to the decision variables d, second order partiall derivatives with respect to d will be found for the function

$$\Pi s\ = (100 - d - x) * P * D - Pc * D - Sc * \frac{D}{Q} - 0.5 * \frac{z^2}{r} * \frac{D}{Q} * Hs - 0.5 * \frac{(Q - z)^2}{r} * \frac{D}{Q} * S$$

$\Pi s\ = (100 - d - x) * P * kd^{\alpha} m^{\beta} - Pc * kd^{\alpha} m^{\beta} - Sc * \frac{kd^{\alpha} m^{\beta}}{Q} - 0.5 * \frac{z^2 *}{r} \frac{kd^{\alpha} m^{\beta}}{Q} * Hs - 0.5 * \frac{(Q-z)^2}{r} * \frac{kd^{\alpha} m^{\beta}}{Q} S$

Since partial differentiation will be performed with respect to d only, other terms will be considered constant so given function will be

$\Pi s\ = kd^{\alpha} m^{\beta} [(100 - d - x) * P - Pc - \frac{Sc}{Q} - 0.5 * \frac{z^2}{rQ} * Hs - 0.5 * \frac{((Q-z)^2)}{rQ} * S]$

Or

$\Pi s\ = kd^{\alpha} m^{\beta} [(100 - d - x) * P - \mathrm{C}]$

Where $C = -(Pc + \frac{Sc}{Q} + 0.5 * \frac{z^2}{rQ} * Hs + 0.5 * \frac{((Q-z)^2)}{rQ} * S]$

So, second order differentiation with respect to d will be

$\frac{\partial^2 \Pi s}{\partial d^2} = kd^{\alpha-2} m^{\beta} [\alpha(\alpha - 1)((100 - \mathrm{d} - \mathrm{x})\mathrm{P} + \mathrm{C}) - 2\alpha \mathrm{Pd}]$

For d>0 and α>2, $kd^{\alpha-2} m^{\beta}$ will be positive so maxima and minima depends on sign of

$[\alpha(\alpha-1)((100-d-x)P+C)-2\alpha Pd]$ For α>1, and assuming the drop in (100-d-x) isn't steep or d is not large, second order derivative will be due to the sign of -2αPd, so profit function will be concave for

$$d=\frac{\alpha}{(\alpha+1)P}[(100-x)P-Pc-\frac{Sc}{Q}-0.5\frac{z^2}{rQ}Hs-0.5\frac{(Q-z)^2}{rQ}S].$$

2.3.2 Retailer's inventory model for profit maximization: This section focuses on developing a retailer inventory model to optimize profit.

Total generated revenue from market= (100-d)*P*D

As x% discount on MRP is given to the retailer so purchase cost = (100-d-x)*P*D

Two types of cost are considered for retailers one is ordering cost and another is holding cost

Ordering cost=$O_c*\frac{D}{Q}$

Holding cost=$0.5*Q*\frac{Q}{r}*\frac{D}{Q}*H_b$

Marketing cost=Marketing cost per unit*Number of units

Marketing cost=$m*D$

Retailer's profit=Total Revenue-Purchase cost-Ordering cost-Holding cost-Marketing cost

$$\Pi b=(100-d)*P*D-(100-d-x)*P*D-O_c*\frac{D}{Q}-0.5*Q*\frac{Q*}{r}\frac{D}{Q}*H_b-m*D$$

Or it will be

$$\Pi b=x*P*D-O_c*\frac{D}{Q}-0.5*\frac{Q^2*}{r}\frac{D}{Q}*H_b-m*D.. \ldots\ldots.. (4)$$

By putting the D=$kd^{\alpha}m^{\beta}$ in equation (1), we found following profit function for the manufacturer

$$\Pi b=x*P*kd^{\alpha}m^{\beta}-O_c*\frac{kd^{\alpha}m^{\beta}}{Q}-0.5*\frac{Q^2*}{r}\frac{kd^{\alpha}m^{\beta}}{Q}*H_b-kd^{\alpha}m^{\beta+1}$$

As decision variables for the retailer are EOQ i.e. Q and marketing expenditure per unit m so we can find Q and m by

$$\frac{\partial\Pi b}{\partial Q}=0 \quad , \quad \frac{\partial\Pi b}{\partial m}=0$$

$$Q=\sqrt{\frac{2rOc}{H_b}} \qquad \ldots\ldots\ldots\ldots..(5)$$

$$m = \frac{\beta}{\beta+1}\left[x * P - \frac{O_c}{Q} - 0.5 * \frac{Q}{r} * H_b\right]$$
…………(6)

To analyze the retailer's profit function for its maxima and minima in relation to the decision variables Q and m, second order derivatives will be found for the function

$$\Pi b = x * P * kd^{\alpha}m^{\beta} - O_c * \frac{kd^{\alpha}m^{\beta}}{Q} - 0.5 * \frac{Q^2 *}{r}\frac{kd^{\alpha}m^{\beta}}{Q} * H_b - kd^{\alpha}m^{\beta+1}$$

(i) **Maxima and minima for the decision variable Q:** Since partial differentiation will be performed with respect to Q only other terms will be considered constant so retailer's profit function will be

$$\Pi b = Constant - O_c * \frac{kd^{\alpha}m^{\beta}}{Q} - 0.5 * \frac{Q^2 *}{r}\frac{kd^{\alpha}m^{\beta}}{Q} * H_b - Constant$$

So second order differentiation with respect to Q will be

$$\frac{\partial^2 \Pi b}{\partial Q^2} = -2O_c * \frac{kd^{\alpha}m^{\beta}}{Q^3}$$

Or $\frac{\partial^2 \Pi b}{\partial Q^2} = -2O_c * \frac{D}{Q^3}$

Since Oc, D and Q are more than zero, $\frac{\partial^2 \Pi b}{\partial Q^2} =< 0$ So buyers profit function will be maxima for

$$Q = \sqrt{\frac{2rO_c}{Hb}}$$

(ii) **Maxima and Minima for decision variable m:** To verify the maxima and minima of profit function of retailer with respect to decision variables m, second order derivatives will be found for the function

$$\Pi b = x * P * kd^{\alpha}m^{\beta} - O_c * \frac{kd^{\alpha}m^{\beta}}{Q} - 0.5 * \frac{Q^2 *}{r}\frac{kd^{\alpha}m^{\beta}}{Q} * H_b - kd^{\alpha}m^{\beta+1}$$

Since partial differentiation will be performed with respect to m only other terms will be considered constant so given function will be

$$\Pi b = kd^{\alpha}m^{\beta}(xP - \frac{O_c}{Q} - 0.5 * \frac{QH_b}{r}) - kd^{\alpha}m^{\beta+1}$$

So second order differentiation with respect to z will be

$$\frac{\partial^2 \Pi b}{\partial m^2} = -kd^{\alpha}m^{\beta-1}(\beta+1)$$

Or

$$\frac{\partial^2 \Pi b}{\partial Q^2} = -2Oc * \frac{D}{Q^3}$$

Since d, m and β+1 are more than zero, $\frac{\partial^2 \Pi b}{\partial m^2} =< 0$ So buyers profit function will be maxima for

$$m = \frac{\beta}{\beta+1}\left[x * P - \frac{O_c}{Q} - 0.5 * \frac{Q}{r} * H_b\right]$$

3. Mathematical models

3.1 Non-Cooperative model: This section considers the manufacturer and retailer non-cooperative models. In Stackelberg games, one player will be a leader and initiate to maximize his profit, and the second player will be the follower. The objective of the leader will be to maximize his profit after considering all the moves taken by the follower.

3.1.1 Manufacturer model for profit maximization in a non-cooperative environment: In a non-cooperative environment, the manufacturer aims to maximize profit after accounting all actions taken by the retailer. The manufacturer's profit maximization model can be expressed as follows:

Max

$$\Pi s = (100 - d - x) * P * kd^{\alpha}m^{\beta} - Pc * kd^{\alpha}m^{\beta} - Sc * \frac{kd^{\alpha}m^{\beta}}{Q} - 0.5 * \frac{z^2 *}{r}\frac{kd^{\alpha}m^{\beta}}{Q} * Hs - 0.5 * \frac{(Q-z)^2}{r} * \frac{kd^{\alpha}m^{\beta}}{Q} S \quad \text{………………(7)}$$

s.t.

$$Q = \sqrt{\frac{2rOc}{Hb}} \quad \text{………………. (8)}$$

$$m = \frac{\beta}{\beta+1}\left[x * P - \frac{Oc}{Q} - 0.5 * \frac{Q}{r} * Hb\right] \quad \text{…………….. (9)}$$

3.1.2 Retailer's model for profit maximization in a non-cooperative environment: In a non-cooperative environment retailer will maximize his profit, considering the moves taken by the manufacturer, so retailers' model for profit maximization will be

Max

$$\Pi b = x * P * D - Oc * \frac{D}{Q} - 0.5 * \frac{Q^2 *}{r}\frac{D}{Q} * H_b - m * D$$ ……………….(10)

s.t.

$$z = \frac{Q*S}{H_s+S}$$ ………………..(11)

$$d = \frac{\alpha}{(\alpha+1)P}[(100-x)P - Pc - \frac{Sc}{Q} - 0.5\frac{z^2}{rQ}H_s - 0.5\frac{(Q-z)^2}{rQ}S]$$ ……………..(12)

3.2 Cooperative model: Collaboration between the manufacturer and retailer is consistently advantageous in a supply chain. This study considers a cooperative model in which both work together to maximize profit.

Pareto efficiency can be achieved by focusing on maximizing total profit:

Π=λΠs+(1-λ)Πb where λ<1

i.e.

Max $$\Pi c = \lambda[(100-d-x) * P * kd^{\alpha}m^{\beta} - P_c * kd^{\alpha}m^{\beta} - S_c * \frac{kd^{\alpha}m^{\beta}}{Q} - 0.5 * \frac{z^2 *}{r}\frac{kd^{\alpha}m^{\beta}}{Q} * H_s - 0.5 * \frac{(Q-z)^2}{r} * \frac{kd^{\alpha}m^{\beta}}{Q}S] + (1-\lambda)\left[x * P * kd^{\alpha}m^{\beta} - O_c * \frac{kd^{\alpha}m^{\beta}}{Q} - 0.5 * Q * \frac{Q*}{r}\frac{kd^{\alpha}m^{\beta}}{Q} * H_b - m * kd^{\alpha}m^{\beta}\right]$$

……………….(13)

To find the decision variables, using first order partial differentiation

$\frac{\partial \Pi c}{\partial z} = 0$, $\frac{\partial \Pi c}{\partial Q} = 0$, $\frac{\partial \Pi c}{\partial d} = 0, \frac{\partial \Pi c}{\partial m} = 0$, we find

$$z = \frac{Q*S}{H_s+S}$$………… (14)

$$Q = \sqrt{\frac{\lambda z^2(H_s+S) + 2r(1-\lambda)O_c + 2r\lambda S_c}{(1-\lambda)H_b + \lambda S}}$$…………… (15)

$$d = \frac{\alpha}{\lambda(\alpha+1)P}[\lambda(100-x)P - \lambda P_c - \lambda\frac{S_c}{Q} - 0.5\lambda\frac{z^2}{rQ}H_s - 0.5\lambda\frac{(Q-z)^2}{rQ}S + (1-\lambda)xP - (1-\lambda)\frac{O_c}{Q} - 0.5(1-\lambda)\frac{Q}{r}H_b - (1-\lambda)m]$$ ………… (16)

$m = \frac{\beta}{(\beta+1)}\frac{\lambda}{(1-\lambda)}\left[(100-d-x)*P-P_c-\frac{S_C}{Q}-0.5*\frac{z^2}{rQ}*Hs-0.5*\frac{(Q-z)^2}{rQ}*S\right]+\frac{\beta}{\beta+1}\left[x*P-\frac{O_C}{Q}-0.5*\frac{Q}{r}*H_b\right]$

……………….(17)

4. Computational results

To test the mathematical models, a given hypothetical data set is used:

p=5; O_c=.2; Hb=.1; Pc=1; S_c=2; H_s=.2; S=.1; k=2000; r=1500; x=25;

4.1Manufacturer's Model: manufacturer's model provided the following results for α=0.7, β= 0.03

Objective value: (Profit of manufacturer (Πs=5037547), d=30.79, m=3.64, Q=77.45, z=25.82, D= 22899

4.2Retailer's Model: The retailer's model provided the following results for α=0.6, β= 0.03

Objective value: (Profit of Buyer (Πb=1864939), d=28.04, m=3.64, Q=125, z=41.7, D=15367

4.3 Cooperative model: In a cooperative model, Pareto efficient solution is considered where the profit is defined by:

Total profit=λ*profit of manufacturer+ (1-λ)*profit of the retailer.

The cooperative model provided the following results for α=0.6, β= 0.05 and λ=.6

Objective value: (Total Profit (Πc=3118314), d=33.88, m=8.28, Q=219, z=73, D=18405

5.Sensitivity Analysis: This section covers the sensitivity analysis for each decision variable d, m, Q, z, annual demand (D). Two ways are considered for all three models, where first α changes and β is fixed, and when β changes and α is fixed. The results are given below in tables 1-6.

Table 1. Manufacturer-Stackelberg model when α varies and β=0.03

A	**0.1**	**0.2**	**0.3**	**0.4**	**0.5**
D	6.799	12.465	17.26	21.37	24.93
Z	25.82	25.82	25.82	25.82	25.82
Q	77.46	77.46	77.46	77.46	77.46
M	3.63	3.63	3.63	3.63	3.63
D	2518	3443	4886	7075	10380
Πs	856129	1073116	1405579	1890066	2588018

Table 2. Manufacturer-Stackelberg model when β varies and α =0.2

B	0.01	0.02	0.03	0.04	0.05
D	12.465	12.465	12.465	12.465	12.465
Z	25.82	25.82	25.82	25.82	25.82
Q	77.46	77.46	77.46	77.46	77.46
M	1.235	2.447	3.634	4.799	5.942
D	3319	3372	3443	3527	3621
Πs	1034549	1051005	1073116	1099212	1128577

Table 3. Retailer-Stackelberg model when α varies and β=0.03

A	0.1	0.2	0.3	0.4	0.5
D	6.799	12.47	17.26	21.37	24.93
Z	29.399	32.46	35.14	37.55	39.72
Q	88.19	97.38	105.44	112.64	119.15
M	3.64	3.64	3.64	3.64	3.64
D	2518	3443	4886	7076	10381
Πb	305610	417892	592974	858702	1259787

Table 4. Retailer-Stackelberg model when β varies and α =0.2

B	0.01	0.02	0.03	0.04	0.05
D	12.465	12.465	12.465	12.465	12.465
Z	32.57	32.517	32.46	32.4	32.35
Q	97.72	97.55	97.38	97.21	97.05
M	1.237	2.45	3.64	4.8	5.95
D	3319	3372	3443	3527	3621
Πb	410838	413287	417892	423945	431132

Table 5. Cooperative model when α varies and β=0.03

A	0.1	0.2	0.3	0.4	0.5
D	7.72	14.17	19.62	24.3	28.35
Z	79.67	79.67	79.67	79.67	79.67
Q	239	239	239	239	239

M	5.69	5.49	5.32	5.18	5.06
D	2585	3577	5136	7529	11181
Πc	699379	887220	1176274	1601201	2219683

Table 6. Cooperative model when β varies and α =0.2

B	**0.01**	**0.02**	**0.03**	**0.04**	**0.05**
D	14.22	14.19	14.17	14.15	14.12
Z	79.67	79.67	79.67	79.67	79.67
Q	239	239	239	239	239
M	1.87	3.7	5.49	7.26	8.985
D	3422	3490	3577	3677	3790
Πc	852015	867234	887220	910618	936858

5.1 Analysis through graphs of decision variables d, m, Q, z, and profits: In this section changes are shown for all decision variables and profit for all three models using graphs.

5. 1.1 Effect of parameter α on d, m, Q, z, Πs, Π$_b$, Π$_c$ considering β= 0.03 constant:

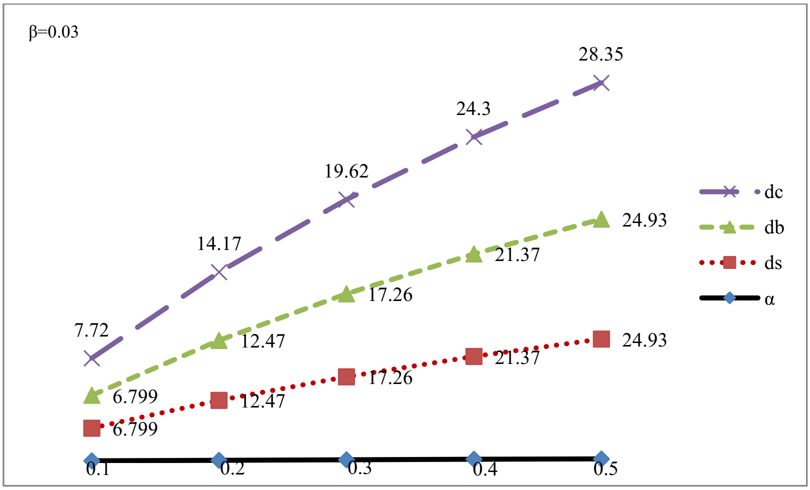

Fig 1. The effect of parameter α on d

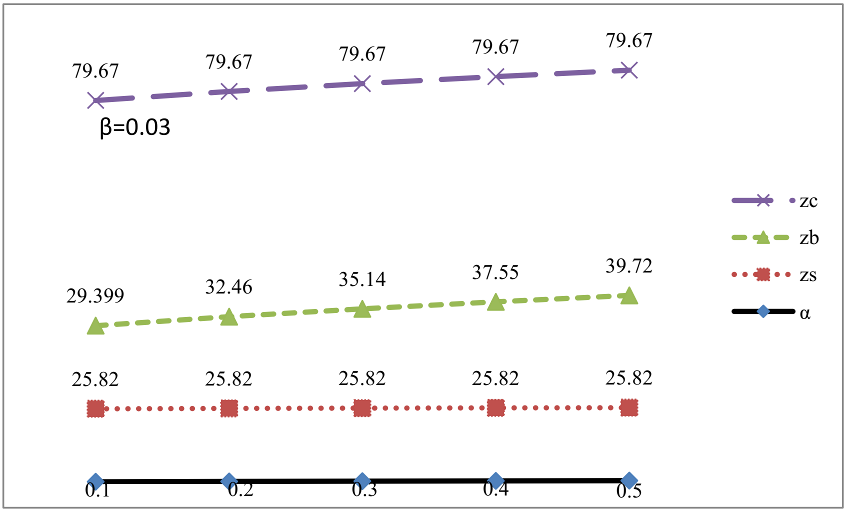

Fig. 2. The effect of parameter α on z

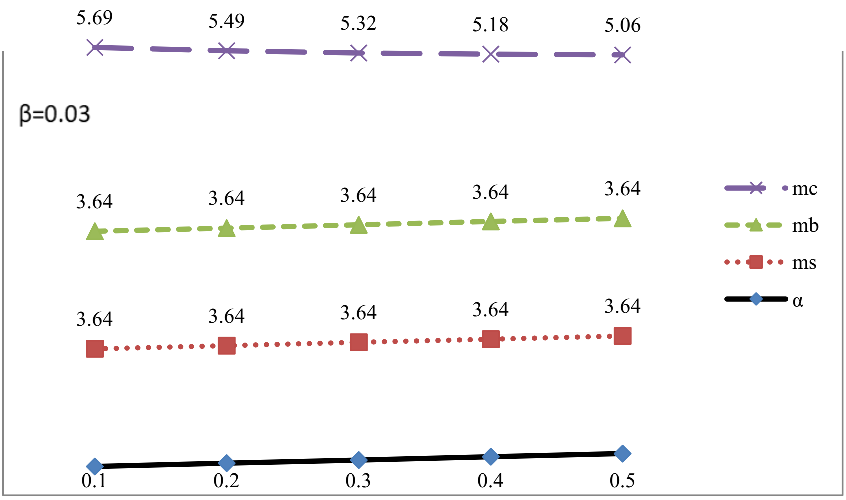

Fig. 3. The effect of parameter α on m

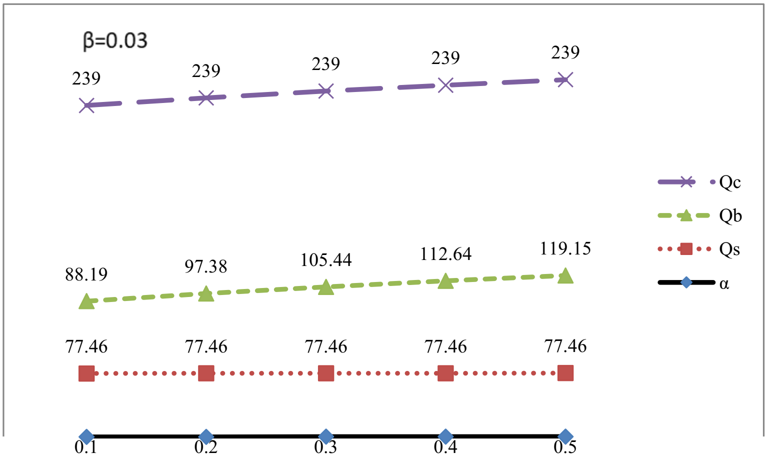

Fig. 4. The effect of parameter α on Q

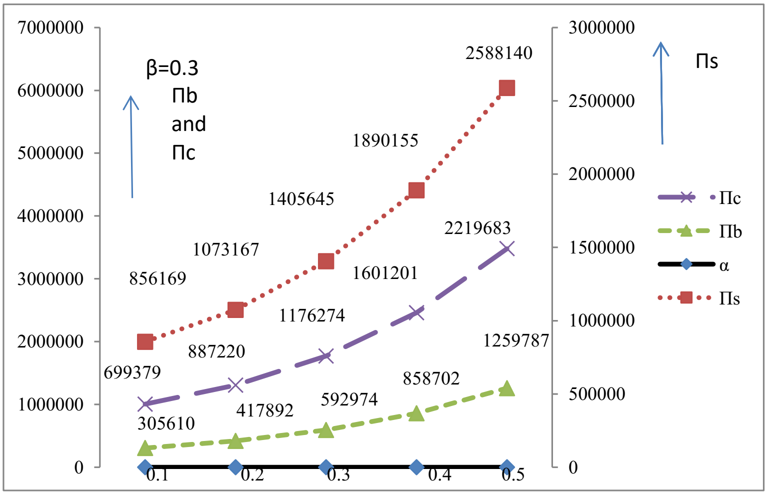

Fig 5. The effect of parameter α on profits

From figures 1-5 we can see that as parameter α increases, the discount to price increases in the cooperative model compared to the non-cooperative model. For the manufacturer-Stackelberg model, the inventory level z does not change, while it increases in cooperative and retailer-Stackelberg models. Marketing expenditure does not change in the non-cooperative model, while it decreases in the cooperative model. Economic order quantity Q does not change in the cooperative and Manufacturer-Stackelberg model, but it increases for the Retailer-Stackelberg model. Profit increases in both cooperative and non-cooperative models, while the manufacturer's profit is having a high growth rate in a non-cooperative environment.

5.1.2 Graphical analysis of the effect of the β parameter on each decision variable d, m, Q, z, and Πs, Πb, Πc considering α=0.2 constant:

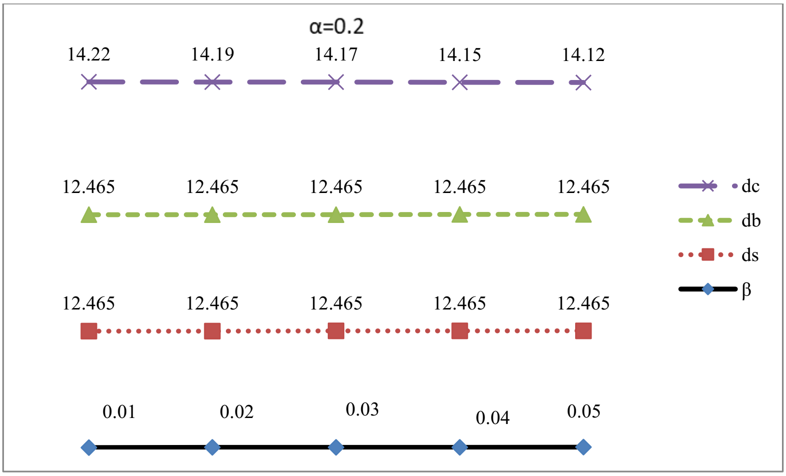

Fig 6. The effect of parameter β on d

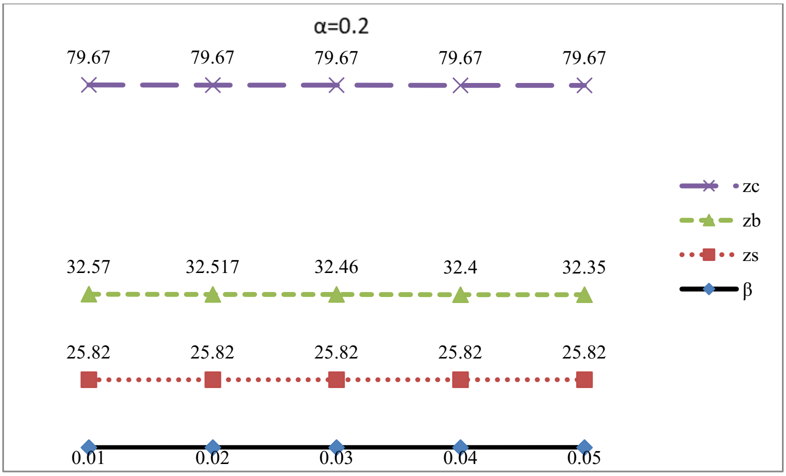

Fig. 7. The effect of parameter β on z

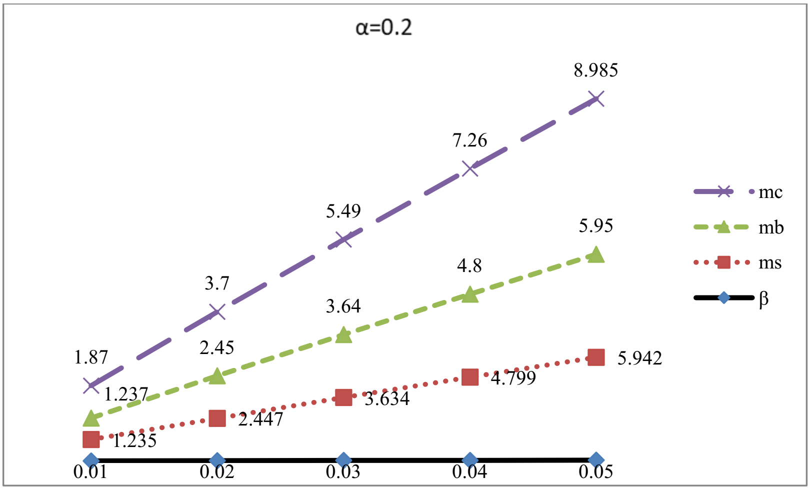

Fig. 8. The effect of parameter β on m

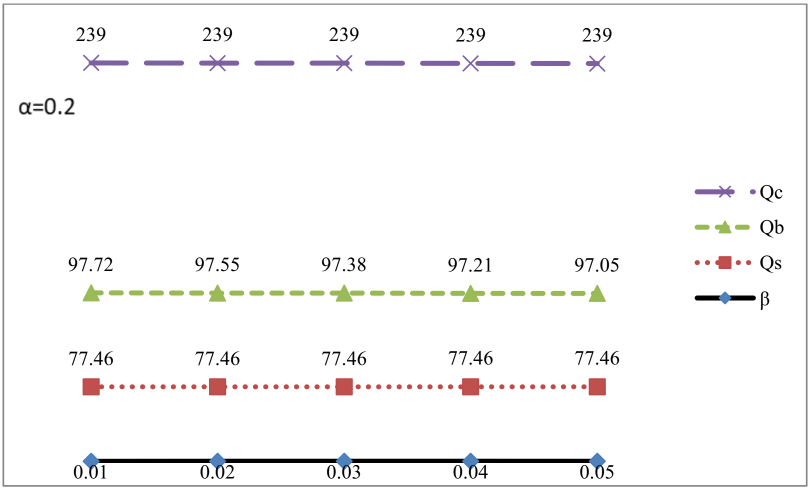

Fig. 9. The effect of parameter β on Q

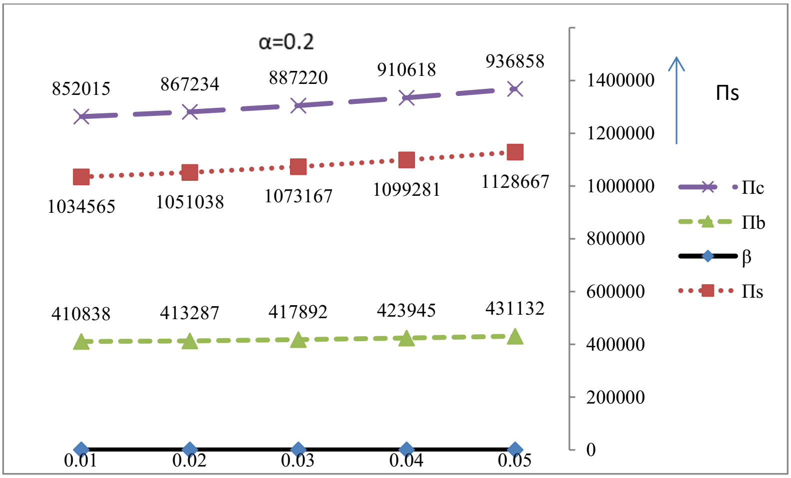

Fig. 10. The effect of parameter β on profits

From figure 6-10 we can see that with the increasing value of β, the discount does not change in a non-cooperative environment, while it decreases in a cooperative environment. The inventory level (z) does not change for the Manufacturer-Stackelberg in a cooperative environment, while it decreases for the Retailer-Stackelberg model. In both cooperative and non-cooperative models, marketing expenditure has high growth. Economic order quantity is fixed for the Manufacturer-Stackelberg model and cooperative model, while it decreases for the Retailer-Stackelberg model. Profits increase in both scenarios; however, the manufacturer earns greater profit in the non-cooperative model.

Conclusion

In this study, the researchers developed inventory models for both the manufacturer and retailer, taking into account annual demand sensitive to the manufacturer's customer discount and the retailer's marketing expenditure. It was assumed that the manufacturer produces the product while the retailer promotes the product in the market. Two decision variables, inventory level (z) and discounts on MRP (d), are considered for the manufacturer, and two decision variables, economic order quantity (Q) and marketing expenditure (m), are considered for the retailer. Mathematical models are developed in non-cooperative (Stackelberg) and cooperative environments. An optimal feasible solution is examined by considering a numerical example with the help of LINGO software. Annual market demand, EOQ, and inventory increases in a cooperative environment in comparison to the non-cooperative models. The manufacturer's profit is more in a non-cooperative environment comparatively cooperative environment. Future studies can be performed considering annual market demand dependent on market price, price discounts, and marketing expenditure. Stackelberg models can also be developed for manufacturers and retailers with and without shortages. In this work, demand is considered deterministic, while demand can also be considered probabilistic. Marketing expenditure can also be shared by the manufacturer and retailer.

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Disclosure of Interests.

The authors declare that they have no financial or personal conflicts of interest that could have affected the outcomes of this research.

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