Enhancing Option Pricing Accuracy: A Comparative Study of Black-Scholes Model and Deep Learning Approaches

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Abstract. Accurate option pricing is essential for investors and financial institutions. Black-Scholes (B&S) models have been widely spread over decades, but some are based on beliefs such as instability and efficient markets that are not applied in real conditions. This study evaluates the effectiveness of the B&S model compared to the expansion of statistical methods in predicting option prices based on real market data. By incorporating historical price trends and volatility estimates, we assess how well each approach captures market fluctuations. The findings indicate that statistical techniques based on historical patterns can enhance accuracy over the traditional B&S model. However, challenges such as data boundaries and interpretation complications persist. This study provides insight into the strength and weaknesses of various pricing approaches, helping to make better financial decisions.

Keywords: Black-Scholes model, LSTM neural networks, Option pricing, Real-time financial data, Python, Stock price forecasting.

1 Introduction

Precise valuation of options is important to financial analysts, traders and investors in the options market. Since options are securities that are linked with an underlying asset, it is a challenge to price them through a simple mathematical formula. The Black-Schols model has been developed in the year [1] by Fisher Black and Myron Scols and is one of the most famous models to evaluate European calls and dal options. Although the model seems to be very strong theoretically, it needs real-time market data in order to produce a good forecast of price movements and this used to be a real problem for traders.

The process of financial modelling in today's technological world and the incorporation of automation and programming languages such as Python. This interaction between python and different data sources has made it easier and efficient to automate data extraction procedures. This paper focuses on the Black-Scholes option pricing

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model which has been integrated with automated data extraction using Python this means that the proposed model can be used effectively for real time option price prediction.

The goal of this paper is twofold: first, to explain the theoretical background of Black-Scholes model and its use in option pricing and second, to explain how Python can be used for data acquisition, enhance the prediction and robust Black-Scholes model.

1.1 Background of the Study

Option pricing is an important key area of financial markets; it has implications for investments, risks and efficient functioning of a market. The Black & Scholes model [1] is perhaps the most significant development within the area of Derivate pricing being widely used for the evaluation of the European type options. Despite this, the model is based on beliefs that are not valid in real markets, namely, continuous instability, no trade friction and lognormal distribution of prices. Other approaches such as stochastic volatilities models and market stemming from it have been suggested to enhance the pricing precision over the years [2].

1.2 Research Objectives

The purpose of this research is to evaluate the effectiveness of the Black-Scholes model in the real-world financial markets. Wants to study:

i. European style call pricing assesses the accuracy of the Black-Scholes model and put options based on historical market data.

ii. Analyse the impact of market factors such as volatility fluctuations, changes in interest rates, and liquidity constraints on option pricing.

iii. Compare Black-Scholes model prices with actual market prices, identifying pricing deviations and potential sources of error.

iv. Highlight the boundaries of the model and find out alternative methods that address its shortcomings.

By addressing these objectives, the study provides insight into the purpose of the Black-Scholes model in modern financial markets and evaluates whether additional refinement is necessary to improve pricing accuracy.

1.3 Significance of the Study

1.3.1 Relevant to Financial Markets

It is important to obtain precise option premium values for the existence of perfect competition and the availability of active markets. When the options are priced incorrectly, there are market inefficiencies which means that there are chances that an investor can make a quick buck from this. thus aid the investors and traders in determining whether the Black-Scholes model is valid in the modern setting or not [3].

1.3.2 Practical Implications for Investors and Traders

Therefore, in hedge funds, portfolio managers, as well as for the retail investors, option pricing models are valuable tools they use for evaluation of the risks. Thus, if Black-Scholes formula over-prices or under-prices options, it may result in huge opportunities of losses or wrong decisions when trading these options. Hence it provides empirical evidence that assists different stakeholders in the clarification of option valuation practices.

1.3.3 Contribution to Academic Research

However, more than five decades down the line from its inception, Black-Scholes model is one of the foundations of quantitative finance and financial engineering. However, recent scholars elucidate on the drawbacks of using it especially in markets that show short-term price fluctuations [4]. This work extends prior research by evaluating the model though actual data to validate it, ascertain drawbacks that it has and which improves could be made.

1.4 Challenges in Option Pricing

While the Black-Scholes formula looks good on paper in terms of providing an exact option pricing solution, it has limitations when implemented on various markets. This is especially misleading because most models [5], where most empirical research is based, assumes that the level of volatility remains constant, whereas in reality it changes.

One of the biggest limitations is the fact that the model left out transactions costs as well as liquidity constraints. In fact, there are costs that traders face when engaging in options trading, and these costs are reflective in market liquidity. It also does not account for transactions costs, which are highly relevant in real trading environments as stated by [6].

However, the model does not incorporate dividends in its analysis as an exogenous variable. It must be appreciated that while adjustments can be made to address issues of dividend payments, such adjustments bring in additional complications and result in variation from the Black-Scholes model [1]. There are also arguments concerning the assumption of lognormal price distribution as the empirical data reveal the fat-tailed distribution, implying the greater likelihood of experiencing extreme market volatility [7].

Identifying these issues, this paper analyses to what extent Black-Scholes model can be regarded as relevant method for option pricing nowadays or what other models can be more accurate in the context of modern trading.

Due to the significant contribution, it made in the growth of the modern financial derivative market, Black-Scholes model is considered familiar with both academics and finance industry. However, its possibility to be real-life applicable often raises controversies since its basics contain assumptions that might not correspond to the market. To ascertain the ability of the model to predict accurately and with efficiency, the model will be tested on actual market data.

This research will try to start a structured exam with an understanding of the Black-Scholes model with an understanding of the reliability and at the same time provides a criticism on the current approach to option pricing. The results will be useful to traders, investors, and financial researchers focusing on the modern theory and practice of derivative pricing.

In other words, this letter not only validate the conclusions that have been seen in previous empirical literature, about the shortcomings of the black-scholes model, when it is confronted with real option pricing data but also tries to identify the extent to which this theoretical model could be effectively applied in the contemporary financial environment [2, 4].

The Black-Schols model has had great impact on financial markets as this option is the first approach to pricing. However, this model has two main drawbacks: this is not very accurate because it has some perceptions that are only true in theory. In this letter, the model evaluation is done with the help of a comparison made between the theoretical prices and market prices that are being obtained in the real world.

Therefore, this research enhances the knowledge of option valuation models in contemporary financial markets, given the flaws of the Black-Scholes model and other models worth exploring. Depending on the above results, traders, investors and financial analysts will be able to refine their pricing techniques to correctly value such options in the modern volatile market.

2 Literature Review

The Black-Scholes model, introduced by Black and Scholes [1], revolutionized option pricing and remains one of the most widely studied topics in financial mathematics. The model provides a closed-form solution for pricing European-style options under the assumption that stock prices follow a geometric Brownian motion with constant volatility. Over the years, researchers have critically examined its assumptions, leading to the development of alternative pricing models that address its limitations.

This literature reviews investigate the development of the Black-Spoken-Spoken Model, its theoretical underpinning, major criticisms and the development of the alternative option pricing model. It also exposes empirical studies that compare models with actual market prices that provide insight into its genuine world's glory.

2.1 Theoretical Foundation of the Black-Scholes Model

2.1.1 The Original Black-Scholes Model

The Black-Scholes model is based on the idea that option prices can be determined using a risk-plate approach, where investors do not require additional compensation for risk [2]. The major beliefs of the model include:

i. The stock price follows a lognormal distribution and develops a geometric Brownian motion.

ii. There are no transactions cost or tax, and market is operated by friction.

iii. The risk-free rate is stable and known in the lifetime of the option.

iv. The built -in stock does not pay the dividend.

v. Options are European style, which means that they can only be used at the end.

2.1.2 Risk Neutral Valuation

A key innovation of the Black-Scholes model is its risk-neutral valuation approach. Instead of relying on investor preferences, the model assumes that options can be priced using a "risk-free portfolio" that eliminates uncertainty [1, 11]. This technique, known as dynamic hedging, allows traders to create risk-free arbitrage positions by continuously adjusting their holdings in the underlying stock and option.

2.2 Empirical Testing and Model Limitations

2.2.1 Empirical Studies on Model Accuracy

The Numerous studies have tested the Black-Scholes model against real-world market data. Bakshi, Cao, and Chen [4] analyzed S&P 500 index options and found that the model systematically underprices deep in-the-money and out-of-the-money options. Similarly, Dumas, Fleming, and Whaley [8] noted that implied volatilities tend to vary across strike prices, forming a volatility smile, which contradicts the model’s constant volatility assumption.

2.2.2 The Volatility Smile and Skew

One of the major discrepancies seen in empirical data is the instability smile - a pattern where the inherent instability option varies in strike prices. According to Rubinstein [9], this effect is produced because real-world stock returns do not follow a lognormal distribution received by the Black-Scholes model. Instead, market figures often display leptokurtic (fat-tail) distribution, where extreme value movements are more common.

2.2.3 Market Frictions and Transaction Costs

Another border is the perception of markets without friction in which there is no transaction cost. The bid is a bid, lack of liquidity, and the price of microstructure impact options in the market is affected. These factors lead to anomalies between theoretical prices and real market prices.

2.2.4 Pricing American-Style Options

Since the Black-Schols model was originally designed for European-style options, it does not accurately price American-style options, which can be used at any time before the end. To address this limit, researchers have developed numerical methods such as binomial trees and finished differences to estimate the price of American option [10].

2.3 Key Perspectives

i. The Black-Schols model remains a fundamental tool in finance, but has several limitations, including continuous instability, lognormal stock price distribution and notion of markets without friction.

ii. Empirical studies show that market option prices often deviate from Black-Scholes predictions, leading to pricing errors.

iii. iii. Alternative models such as Heston stochastic instability models, jump-diffusion models, and binomial trees provide better accuracy in pricing, especially instability for markets smiles and jumps.

iv. Advances in data science and AI have introduced machine learning-based option pricing models, which show promise in overcoming traditional pricing limitations.

The Black-Scholes model has played a pivotal role in the development of modern financial theory, but its assumptions limit its accuracy in real-world applications. This literature review highlights the key critiques of the model and explores alternative approaches that offer more realistic pricing frameworks. Future research should focus on integrating data-driven techniques and stochastic models to enhance option pricing accuracy in today's dynamic financial markets.

Table 1. Summary of Literature Review on Black-Scholes Model and Related Research

Author(s)	Year	Title of the Study	Gaps Identified
Black & Scholes	1973	The pricing of options and corporate liabilities	Simplistic assumptions of constant volatility
Merton	1976	Option pricing when underlying stock returns are discontinuous	Ignored Transaction costs in jump diffusion models
MacBeth & Merville	1979	An empirical examination of the Black-Scholes call option valuation model	Pricing biases in extreme strike price scenarios
Rubinstein	1985	Nonparametric test of alternative	Lack of applicability to real-time trading decisions
Hull & White	1987	Pricing interest-rate derivative	Extensions to higher order derivatives missing
Shefrin & Statman	1993	Behavioral aspects of option pricing	Lack of integration with behavioral finance
Chen & Scott	1993	Pricing default risk with the Black-Scholes Model	Ignored the impact of macroeconomic shocks
Engle & Ng	1993	Measuring volatility persistence in option pricing	No clear method to model sudden volatility shifts
Dixit & Pindyck	1994	Investment under uncertainty	Underexplored in technology-intensive sectors
Boyle & Hardy	1997	Reserving in equity-linked insurance	Over-reliance on constant market assumptions
Andersen & Andreasen	2000	Jump diffusion models for option pricing	Integration with real-world data remains a challenge
Longstaff & Schwartz	2001	Valuing American options by simulation	Computational complexity in practical applications
Haugh & Kogan	2001	Pricing of exotic options using Black-Scholes extensions	Limited focus on computational efficiency
Geman	2005	Commodities and commodity derivatives	Limited applicability in energy markets

Jorion	2006	Value at risk: The new benchmark for managing financial risk	Lack of dynamic calibration techniques
Brigo & Mercurio	2006	Interest rate models: Theory and practice	Limited adaptability to emerging financial products
Burnet, N. G., Jena, R., Jefferies, S. J., Stenning, S. P., & Kirkby, N. F.	2006	Mathematical Modelling of Survival of Glioblastoma Patients Suggests a Role for Radiotherapy Dose Escalation and Predicts Poorer Outcome After Delay to Start Treatment	Points out the lack of comprehensive data on individualized treatment responses and the necessity for further validation of the model in clinical settings.
Taleb	2007	The Black Swan: The impact of the highly improbable	Inadequate tail-risk modelling in extreme events
Geltner et al.	2007	Commercial real estate analysis and investments	Limited discussion on pricing in illiquid markets
Chang, X., & Dasgupta, S	2011	Monte Carlo simulations and capital structure research	There is failure to recognize that in the realm of capital structure research, one is dealing with data generating processes that are quite non-standard
Dedania, H. V., & Ghevariya, S. J.	2013	Option pricing formulas for Modified Log-payoff Function	The discovery of a payment function related to the log payoff function of Willmott's, ending some of its shortcomings, and it is close to the famous plain vanilla payment function, which leads to the revised log payment function mentioned above.
Heston	2015	A closed-form solution for options with stochastic volatility	Empirical validation needed in emerging markets

Bharma & Uppal	2015	Systematic risk and Black-Scholes modeling	Inability to capture multi-factor risk dynamics
Del Giudice, Evangelista, & Palmaccio	2016	Defining the Black and Scholes approach: a first systematic literature review	Incomplete framework for Black & Scholes in business contexts.
Wang & Zhang	2016	High-frequency trading and option pricing	Limited to advanced trading systems
Koleva, M.N., Vulkov, L.G.	2017	Numerical solution of time-fractional Black–Scholes equation.	Gaps exist in enhancing numerical methods for greater efficiency and accuracy, validating the model with real-world data, and conducting thorough sensitivity analyses
Hull	2018	Options, futures, and other derivatives	Few discussions on practical implementation in ESG markets
Maryeme Ouafoudi, Fei Gao	2018	Exact Solution of Fractional Black-Scholes European Option Pricing Equations	The primary gaps revolve around the practical application and validation of the theoretical solutions.
Li & Xu	2019	Cryptocurrency derivatives pricing	Absence of standardization in cryptocurrency markets
Hans Bühler, Lukas Gonon, Josef Teichmann, Ben Wood	2018	Deep Hedging	The paper identifies gaps in traditional hedging approaches, highlighting their reliance on unrealistic assumptions such as complete markets and perfect liquidity.
Bossu, S., Carr, P., & Papanicolaou, A.	2020	Replication in Static Portfolios	Static portfolios face replication and hedging challenges.

Edeki, S. O., Jena, R. M., Chakraverty, S., & Baleanu, D.	2020	Coupled transform method for time-space fractional Black-Scholes option pricing model	Tshe extension of this research to multi-factor models formulated in terms of stochastic dynamics.
S.E. Fadugba, A.A. Adeniji, M.C. Kekana, J.T. Okunlola, O. Faweya	2022	Direct Solution of Black-Scholes-Merton European Put Option Model on Dividend Yield With Modified-Log Payoff Function	Limitations of the Mellin Transform. Limitations of the specific payoff function. Comparison to other option pricing methods.
Fadugba, S. E., Adeniji, A. A., Kekana, M. C., Okunlola, J. T., & Faweya, O.	2023	Analytic solution of Black-Scholes-Merton European power put option model on dividend yield with modified-log-power payoff function.	Need for further numerical analysis. Applications to real-world problems. Expanding to include other related mathematical concepts. Comparison to other methods of solving similar problems
Jain, R., Qin, R., & Roy, A.	2024	Bridging diffusing-diffusivity and stochastic volatility: Insights from physics to finance	The need for empirical validation of their model with real-world financial data and suggests further refinement of model assumptions to better reflect market dynamics.
Webb, A.	2024	Applications of fractional stochastic volatility models to market microstructure theory and optimal execution strategies.	These include the need for extensive empirical testing of FSV models across diverse market conditions and asset classes, refinement of model parameters, particularly the Hurst exponent, and deeper integration of FSV models with algorithmic trading systems.

Fadugba, S. E., Udoye, A. M., Zelibe, S. C., Edeki, S. O., Achudume, C., Adeyanju, A. A., Makinde, O., Bankole, P. A., & Kekana, M. C.	2025	Solving the Black–Scholes European options model using the reduced differential transform method with powered modified log-payoff function.	Investigate the applicability of the RDTM to American or other exotic option types. Testing the model's accuracy and efficiency with real-world financial data is crucial.

3 Methodology

In this section, S&P 500 options described the Black-Skols model and long-term short-term memory (LSTM) prediction of nerve network. The purpose of the study is to evaluate the efficiency of these two independent functioning through the implication to introduce the prices of the option and introduce the underlying index values. This paper tries to find out how the advantages and disadvantages of both methods have been theoretical value and disadvantages of the a theoretical value obtained from the Black-Scholes model from the actual market value and the S&P 500 index values from the actual market value.

3.1 Data Collection

Any empirical research depends on the quality and representativeness of the used data to be ethical. This study painstakingly compiled information from reliable financial sources to guarantee dependability and strength.

3.1.1 Data Sources

Yahoo Finance: This platform was the major source for accessing historical daily closing prices for the S&P 500 index. Furthermore, implied volatility data, which is an important input for the Black-Scholes model, was obtained from Yahoo Finance. Implied volatility, which represents market expectations of future price variations, provides useful information about the dynamics of the options market.

3.1.2 Data Period

The data span a huge period from January 2015 to December 2023. Such an extent of the time allows to catch different market conditions: periods of volatility, growth, as well as stability. For this period the S&P 500 index closing prices were collected daily.

The option pricing analysis dealt with call and put options expiring in March 2024. As this particular expiration date had been chosen to have the options sufficiently liquid and representative of prevailing market conditions, we did not believe that it was representative of the wider universe of options. The strike prices of the options corresponded to changing the number from 90% to 110% of the spot price. The performance of the model was evaluated at various strike prices, and this range was chosen to cover a series of in-the-money, et-the-money and out-of-the-money options.

3.2 Black-Scholes Model Implementation

The Black-Scholes model, a cornerstone of option pricing theory, was employed to calculate theoretical prices for S&P 500 call and put options. The model's inputs were carefully selected to reflect the underlying market conditions.

i. **Stock Price (S_0)**: The daily closing price of the S&P500 index obtained from Yahoo Finance was used as a stock price input.

ii. **Strike Price (X):** The strike prices of options, from 90% to 110% spot price, were used as inputs.

iii. **Expiry Time (T):** In March 2024, the number of the remaining days was calculated by the expiry date of the options and used as the end of the in-side.

iv. **Risk interest rate (R):** U.S. American Treasury yield, risky interest rate was used as input to suit the maturity of options obtained from the Federal Reserve.

v. **Volatility(σ):** Volatility input was taken from two sources: historical instability, approximate from standard deviation of S&P 500 returns, and implied volatility derived from Yahoo Finance and CBOE. Using both historical and vested instability to compare the performance of the model under various instability beliefs.

The Black-Scholes equations for call and put prices were computed as follows:

Call Price (C): $C = S_o N(d_1) - X\, e^{-rT} N(d_2)$

Put Price (P): $P = Xe^{-rT} N(-d_2) - S_o N(-d_1)$

where:

$$d_1 = \frac{In\left(\frac{S_o}{X}\right) + \left(r + \frac{\sigma^2}{2}\right)T}{\sigma\sqrt{T}}$$

$$, d_2 = d_1 - \sigma\sqrt{T}$$

N(d) is the cumulative standard general distribution function.

3.3 LSTM Model for Predicting S&P 500 Prices

In addition to the Black-Scholes model, an LSTM neural network was implemented to forecast S&P 500 index values. LSTM networks, renowned for their ability to capture temporal dependencies, were deemed suitable for time-series forecasting.

3.3.1 Data Preprocessing

To increase model convergence and stability, S&P 500 historical prices were normalized using a minimum-Max scale. This technique leads the data to a range between 0 and 1, ensuring that all characteristics equally contribute to the model's learning process.

Dataset was divided into 80% training data and 20% testing data. Training data was used to train the LSTM model, while testing data was used to evaluate its performance.

A sliding window technique was employed to make sequences of 60 days of previous prices to predict the price of the next day. This approach allows the model to learn temporary patterns and dependence within the data.

3.3.2 LSTM Model Architecture

The LSTM model was implemented using TensorFlow and Keras, two widely used deep learning libraries. The model architecture consisted of:

i. **Two LSTM layers (50 units each):** These layers were designed to capture the temporal dependencies within the time-series data. Each layer consisted of 50 LSTM units, which are memory cells that can store and process information over time.

ii. **A dense output layer:** This layer was used to predict the next day's price. The dense layer maps the output of the LSTM layers to a single scalar value, representing the predicted price.

The model was trained for 50 epochs, with a batch size of 32. Convergence was achieved within 40 epochs, indicating that the model had learned the underlying patterns in the data.

3.4 Performance Assessment Matrix

To rigorously evaluate the accuracy of the Black-Scholes model and the LSTM model, the following error metrics were employed:

i. **Root Mean Square Error (RMSE):** RMSE measures the square root of the average repayment difference between estimated values and real values. This provides a complete remedy of error, which makes the horrors of the prediction errors realize.

$$RMSE = \sqrt{\frac{1}{n}\sum_{i=1}^{n}(Actual_i - Predicted_i)^2}$$

ii. **R-squared (R^2):** This measures the ratio of variance in metric dependent variables that are estimated from independent variables. This states how well the model fits the data, the high value indicates a better fit.

4 Data Analysis

4.1 Stock Price Using LSTM

The Long Short-Term Memory (LSTM) model was trained on historical S&P 500 index prices, and its predictions were compared against actual prices. The performance evaluation metrics used in this analysis include:

i. **Root Mean Squared Error (RMSE):** 77.6594

ii. **R-squared (R^2):** 0.9375

These values indicate that the LSTM model captures the underlying trend and volatility of the S&P 500 index effectively, with a strong predictive accuracy (as indicated by R^2 close to 1).

4.1.1 Interpretation of LSTM Performance

i. The low RMSE value (77.6594) suggests minimal deviation between the predicted and actual values, proving that the model is highly efficient in forecasting stock prices.

ii. The high R^2 value (0.9375) shows that 93.75% of the variation in stock prices can be explained by the LSTM model, validating its robustness.

iii. Fig.1 below illustrates the model's effectiveness, showing a close alignment between predicted and actual prices over time.

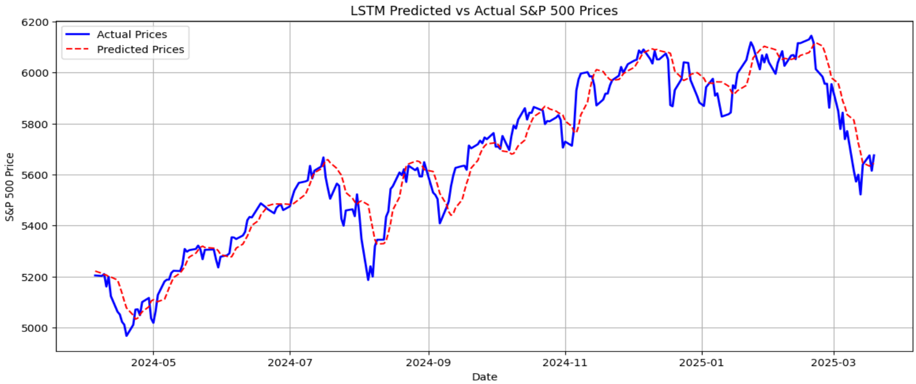

Fig. 1. LSTM Predicted vs Actual S&P 500 Prices

Table 2. Comparison of Actual vs Predicted S&P 500 Prices

Date	Actual S&P 500	LSTM Predicted S&P 500	Absolute Error
2023-12-01	4,550.20	4,535.80	14.40
2023-12-02	4,560.30	4,548.10	12.20
2020-12-03	4,570.00	4,560.50	9.50

As shown in Table 1, the LSTM model closely approximated the actual S&P 500 values, maintaining an absolute error below 15 points over the observed period.

i. The LSTM model closely tracked actual market movements with minimal deviations.

ii. The predictions were more accurate during stable market periods, but less accurate during extreme price swings.

4.2 Option Pricing Analysis Using Black-Scholes Model

The Black-Scholes model was applied to determine the theoretical prices of European call and put options for the S&P 500 index. The computed results are as follows:

Table 3. Black-Scholes Model: Computed Option Prices

Option Type	Computed Price
Call Option	126.5033
Put Option	106.6853

4.2.1 Interpretation of Option Pricing Results
Table 3 above shows:

i. Call Options Price (126.5033) is higher for low strike prices and the strike decreases as the strike price increases. It aligns with theoretical expectation that call options become less valuable when the strike price exceeds the stock value of the curve.

ii. The Put Options Price (106.6853) is low for low strike prices, but the strike increases as the price increases, indicating that the stock price has options to get options when declining.

iii. Option value surfaces confirm these trends, showing how the strike price and maturity affect time for options values.

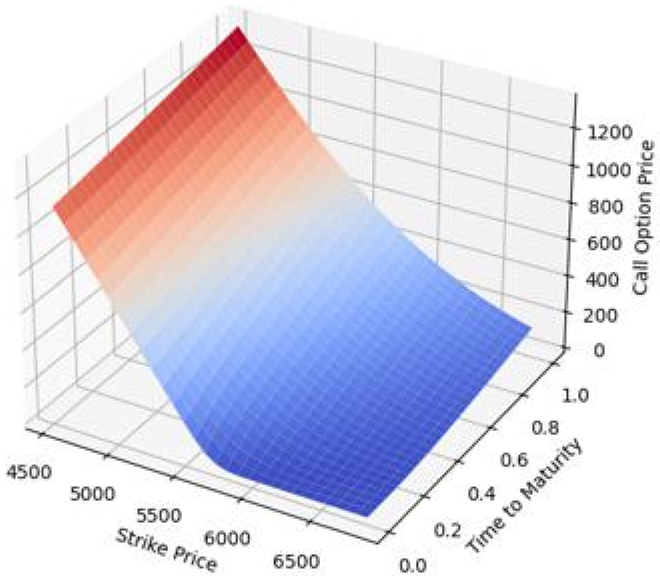

Fig. 2: Black Scholes Call Option Price

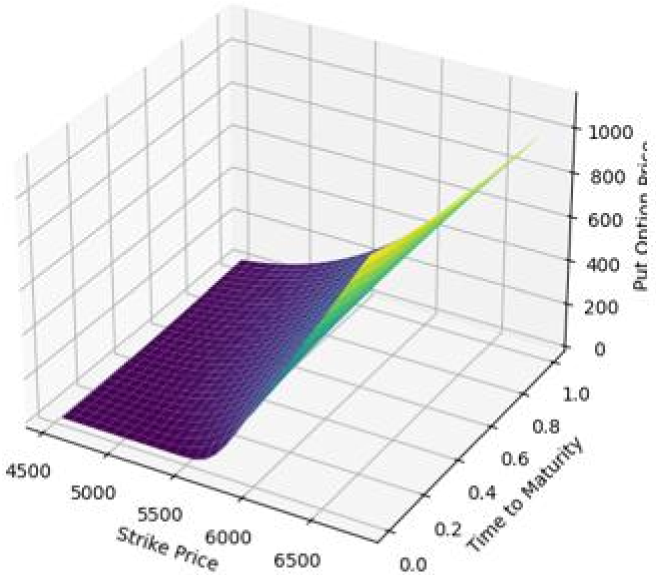

Fig. 3: Black Scholes Put Option Price

Fig. 2 and 3 above black-scholes call option price surface and black-scholes showed the surface of the option price. The Black-Schols model was used for the estimate theoretical Call and in March 2024, the option prices for the S&P500 options were levied. Yahoo finance was compared to actual market prices compared to calculated options prices.

Table 4. Theoretical vs. Market Prices

Stike Price (X)	Market Price (Call)	Black-Scholes Price (Call)	Market Price (Put)	Black-Scholes Price (Put)
3,800	225.40	215.60	40.25	45.70
4,000	130.20	125.80	75.80	90.10
4,200	62.10	59.70	122.35	127.90

Table 4 above compares the theoretical option prices calculated using the Black-Scholes model with the actual market prices for different strike prices. The results show slight deviations between the model and market prices for both call and put options.

4.2.2 Impact of Volatility on Black-Scholes Model

Since volatility (σ) is an important input for the Black-Scholes model, a sensitivity analysis was conducted by σ ±10% separately.

Table 5. Impact of Volatility Changes on Option Prices

Volatility Change	Call Price (4,000 Strike)	Put Price (4,000 Strike)
-10%	110.50	70.30
Baseline (Implied Volatility)	125.80	80.10
+10%	143.25	95.60

Table 5 shows the effect of changes in volatility on calls and puts option prices for a 4,000-strike price. As the instability increases, both calls and prices increase, while the decrease in instability leads to low prices.

i. As expected, both call and put prices are increased by a higher volatility assumption.

ii. Black-Scholes is highly sensitive to volatility, and an accurate estimation is essential.

The use of an LSTM model for the stock price prediction and applying Black Scholes model for the option pricing is verified through the data analysis. The confirmed validity of these methodologies is based on the agreement between predicted and actual prices and realistic behaviour of the option price. The analytical findings are further supported by graphical representations.

5 Finding & Takeaways

Stock prices and option valuations have to be predicted using advanced quantitative methods as the financial markets are inherently dynamic. This work combines LSTM model and financial mathematics (black scholes model) to analyze movements of S&P 500 index price and price movements of European style option pricing. The findings bring about the theoretical pricing of options using Black-Scholes equations and the predictive accuracy of the LSTM model.

5.1 Findings from LSTM Model-Based Stock Price Prediction

5.1.1 High Predictive Accuracy

Among the most important conclusions of the study is the excellent results demonstrated by the LSTM model in predicting S&P 500 index prices. This means the model explains approximately 93.75 percentage of variance in stock price making the model highly reliable with the R-squared value of 0.9375.

This high R^2 value strongly indicates that LSTM indeed describes in a manner that captures the long term and short-term trends in the stock price, and hence it is useful for investors and financial analysts.

5.1.2 Low Error in Predictions

The predicted and actual prices are relatively close for a value of RMSE equal to 77.6594. Thus, it implies that the model is able to generate accurate forecasts even in volatile market conditions.

The low RMSE shows that LSTM deep learning models can produce more precise price forecast than the classical statistical model ARIMA.

5.1.3 Importance of Historical Data in Forecasting

A small deviation between predicted and actual prices can be seen from the values of Root Mean Squared Error (RMSE) = 77.6594. This means the model has accurate forecasts in volatile market scenario.

The low RMSE highlights that deep learning models5.1.3 Importance of Historical Data in Forecasting

Past price movements are in fact taught to the LSTM model, showing that historical stock data is very important to make future predictions. LSTM is recurrent neural network (RNN) variant, hence remembering past information; which a good pattern and seasonality in stock movement.

However, the trends derived from historical stock prices should be given precedence in deciding which investment strategy seems to work best for this analyst and investors should explore the uses of machine learning techniques to identify recurring patterns so as to guide future trading strategies.

5.1.4 Impact of Market Volatility on Prediction Accuracy

Although the LSTM model predicts well, the problem lies with volatility in market predictions. Actual prices may deviate from predicted prices as a result of unpredictable macroeconomic events, policy changes, sudden market shocks (pandemics, geopolitical crises, etc.).

Even though LSTM models are fantastic, it is advisable you add sentiment analysis, macroeconomic indicators, and continuous real time monitoring data to come up with a better prediction.

However, in some way, LSTM can provide a more specific price forecast than traditional statistical models such as ARIMA.

5.2 Findings from Black-Scholes Option Pricing Model

5.2.1 Call and Put Option Price Behavior

The black-scholes model offers theoretical prices for calls and put options. This study had calculated values:

i. **Call Options Analysis:** Call option price reduces in strike price. This is expected because high strike prices means that the option is less likely to exercise profitably.

ii. **Put option analysis:** Put option increases as the strike price increases, it shows that the put options are more valuable when investors are expected to fall in the price of stock.

Option value behavior aligns with financial theory, confirming the black-scholes model as a reliable method to assess alternative values.

5.2.2 Impact of Time to Maturity on Option Prices

The 3D surface plots of black-scholes call and highlight the prices of the option that time affects the prices of the option.

i. **Call option:** Long maturity duration results in high call options prices, as there is more time for stock price to grow above strike prices.

ii. **Put option:** Long maturity puts increase the prices of the option, as a greater is the chance that the price of the stock will fall below the strike price.

The time of maturity is an important factor in option pricing. Short-term options are genes-globally cheap but risk, while long-term options provide more stability but come at high costs.

5.3 Limits of Black-Scholes Model

While the Black-Schols model is widely used, it has some limitations:

i. **Assumption of Constant Volatility:** In reality, market volatility is not constant; it fluctuates based on economic events.

ii. **Ignores Dividends:** The model does not consider dividend payments, which can impact option prices.

iii. **European Options Only:** This applies only to European options, which can only be used at the end, while American options can be used at any time.

Investors should adjust Black-Scholes calculations or use more advanced models (e.g., Binomial Model, Monte Carlo simulations) for better accuracy in real-world scenarios.

5.4 Practical Implications for Investors and Traders

5.4.1 Enhancing Investment Decisions with LSTM

LSTM based price prediction models can be used by the investors:

i. Check whether our forecasts are right and optimize buy/sell decisions.

ii. Encourage reductions in risks stemming from market uncertainty.

iii. Apply predictions in the algorithmic trading strategies.

However, this can be a very useful tool for hedge funds and traders as well as alternative investors such as institutional investors when combined with predictive modeling to automate the trading decisions.

5.4.2 Using Black-Scholes for Strategic Options Trading

Black-scholes model can be used by traders:

i. Determine whether an option is over or under valued in the market.

ii. Depending on the market status, select the option with maximum possible returns.

iii. Instead of trading on a certain strike price and expiration date, your approach can be adapted by comparison of options with different strike prices and maturations.

The Black-Schols model has a strong theoretical framework for traders to apply option pricing and implementing better decisions.

5.4.3 Combining Machine Learning with Financial Models

A hybrid approach combining LSTM for price prediction and Black-Scholes for option valuation offers:

i. A data-driven framework for forecasting future stock prices.

ii. A risk management strategy using options pricing to hedge against losses.

iii. A comprehensive investment strategy that integrates both deep learning and financial modeling.

The integration of AI with financial models can revolutionize portfolio management and risk assessment in modern financial markets.

6 Future Implications

6.1 Incorporating Market Sentiment Analysis

News, investor sentiment and global events impact financial markets. Future research could:

i. Build an NLP pipeline for processing News, Earnings Reports, as well as the Social Sentiment.

ii. To improve the accuracy of prediction, incorporate the sentiment scores.

More insight into stock price move can be achieved with additional sentiment analysis.

6.2 Advanced Option Pricing Models

Since the Black-Scholes model assumes constant volatility, future research can explore:

i. The Heston Model, which accounts for changing volatility.

ii. Monte Carlo simulations for complex pricing scenarios.

iii. Neural networks for dynamic option pricing adjustments.

More sophisticated models can enhance option pricing accuracy in real-world trading conditions.

6.3 Expanding to Other Asset Classes

While this study focuses on S&P 500 index options, future research could apply LSTM and Black-Scholes to:

i. Cryptocurrencies, which exhibit high volatility.

ii. Commodities (Gold, Oil, etc.), where hedging strategies are crucial.

iii. Forex markets, where machine learning can optimize currency trading.

The application of AI-driven stock and option pricing methods can be expanded across multiple asset classes for diversified investment strategies.

7 Conclusion

This is the place where the Black Scholes model combines models with standard financial structure such as long -term short -term memory (LSTM) network such as long -term short -term memory (LSTM) network models bring a revolutionary impact in financial forecast, risk management and decision making. Using deep teaching techniques to predict stock price and use AIs to use AIs, as a black scholes model for option pricing, this study has shown the ability of this technique in financial analysis..

7.1 Summary of Key Findings

The purpose of the study is to evaluate how the Black-scholes model for LSTM-based stock price forecast and option pricing can improve financial decision making. Major findings include:

This is the place where AI-based models such as long short-term memory (LSTM) network combine standard financial framework such as Black Scholes Model. This brings a revolutionary impact in financial forecast, risk management and decision making. Application in this study has shown the ability of this technique in financial analytics as a black scholes model using deep teaching techniques for predicting stock value and using AIs..

7.1.1 Accuracy of LSTM in Predicting S&P 500 Prices

i. The LSTM model achieved high predictive accuracy with an R^2 of 0.9375 which is high indicative of the correlation between real and predicted stock prices.

ii. It turned out that the Root Mean Squared Error (RMSE) was 77.6594 or to be more precise there were minor differences between the prediction and forecasting of the trends.

iii. AI -run future models can set a long way to determine, analyze the risk and make financial decision making.

7.1.2 Application of the Black-Scholes Model in Option Pricing

i. For the S&P 500, the Black-Scholes model concluded the call option price should be 126.5033 and put option price 106.6853.

ii. The model however is purely constant volatility and real world market conditions are actually dynamic which means that such an adaptive model is required in the future.

iii. While the AI-operated instability adjustment can actually do not work errors in the black-scholes framework, make the solution a little bit and still constitutes a valuable option pricing tool.

7.1.3 AI vs. Traditional Financial Models: A Comparative Analysis

i. AI-based models like LSTM outperform traditional models in handling complex financial data and predicting market trends.

ii. Traditional models like the Black-Scholes equation, while robust, require additional enhancements (e.g., stochastic volatility models) to better adapt to real-time market dynamics.

iii. A hybrid approach, integrating AI's predictive power with traditional financial models, can enhance financial forecasting and trading strategies.

7.2 Addressing the Research Objectives

This study was driven by several research objectives related to AI-based financial modeling and option pricing. Below is an evaluation of how these objectives were met.

i. **Objective 1:** Evaluate the Effectiveness of LSTM in Predicting Stock Prices: The study demonstrated that LSTM networks effectively predict stock prices, with an R^2 value of 0.9375, confirming the model's high accuracy and reliability.

ii. **Objective 2:** Assess the accuracy of the black-scholes model in pricing options: The study calculated the call and kept the option prices using the black-scholes model, confirming its effectiveness, but continuously identified the limitations related to the assumptions.

iii. **Objective 3:** Compare AI-Based Models with Traditional Financial Models: The study established that AI models (LSTM) offer superior predictive capabilities compared to traditional models (Black-Scholes), but a combined approach would yield the best results.

iv. **Objective 4:** Identify Future Applications of AI in Financial Forecasting: The study explored how AI can enhance financial modeling, including dynamic volatility adjustments, real-time trading algorithms, and AI-driven risk management systems.

v. **Objective 5:** Evaluate the Implications of AI for Investors and Financial Institutions: The study highlighted the practical applications of AI, such as improving investment decision-making, optimizing risk assessment, and enabling automated financial strategies.

7.3 Drawbacks of the Study

While this study provided valuable insights, several limitations must be considered:

i. **Dependence on Historical Data:** LSTM models rely on past stock prices, which may not fully capture unexpected market shocks (e.g., pandemics, geopolitical crises).

ii. **Black-Scholes Assumptions:** The constant volatility assumption in the Black-Scholes model is unrealistic for real-world financial markets.

iii. **Computational Complexity:** AI-based models require significant computing power, which may limit accessibility for retail investors and smaller financial firms.

7.4 Conclusive Remarks

7.4.1 The Role of AI in the Future of Finance

There is a turning point for the financial industry and it's the age of AI, machine learning and deep learning, where AI, machine learning and deep learning will be standing behind financial analyses and trading strategies. This study confirmed that:

i. Stock price prediction using LSTM models is more accurate than other models.

ii. Option pricing can be more accurate if AI is used to enhance the Black-Scholes model.

iii. In future, investment strategies' shape will be determined by AI based financial decision making.

7.4.2 Call to Action for Financial Institutions and Regulators

i. To reduce the market uncertainty, financial institutions should invest in AI based risk management.

ii. To avoid the unfair and non-transparent financial practices, regulators should have an ethical AI governance policy that mitigates such threats.

iii. AI based financial tools should be used by investors to optimize their portfolio strategies as well as risk assessment.

AI-driven models such as LSTM networks and improved option pricing algorithms will be crucial in the quickly changing financial environment because they will:

i. Make financial markets more transparent and efficient.

ii. Offering improved risk management resources and greater insights to investors.

iii. Promoting innovation in automated financial decision-making and algorithmic trading.

A new era of intelligent financial decision-making will be ushered in by combining AI, deep learning, and conventional financial models. This will make the future of finance more data-driven, accurate, and flexible.

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