Residual Waves of War: A Novel Model for Economic Shock and Recovery Using Oscillating F-Distribution Functions

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Abstract. As we observe serious rise in global crises like frontal and civil conflicts, natural disasters, global pandemics etc. economic systems (commodity, money and labor market equilibriums) are highly prone to disruptions and shocks. The shocks travel vast distances, both temporally and spatially, and the path or trajectory of both the impact and the recovery are very difficult to predict or model. Traditional economic models frequently fail to capture the nonlinear and residual dynamics of such events. Thus, policy-makers too find it difficult to foresee or mitigate long-term damages, because there are hardly any reliable tool for such forecasts. This paper proposes a unique theoretical model that conceptualizes economic shocks as wave-like phenomena. We draw principles from traditional wave interference and statistical distribution theory, too simulate the economic shock behavior. Specifically, we consider the sinusoidal interference function to define any economic shock, then integrate it within an F-distribution envelope. This enables us to predictably represent both immediate impact and residual oscillations of such ripples over time. The magnitude and duration of the shock are encoded into the degrees of freedom of the F-distribution, while the embedded sinusoidal component captures the constructive and destructive interferences that projects multi-layered economic disturbances. We propose empirical validation (fit) of the model on Russia's GDP over their invasion in Ukraine. The findings reveal that such hybrid models not only enhance predictive accuracy but also provide a meaningful foundation for designing dynamic policy responses post-shock. This framework, while currently applied to war-related disruptions, is designed for broader adaptability to other economic shocks, offering a powerful new tool for economic diagnostics and forecasting.

Keywords: Economic shock, economic recovery, economic impact of war, wave-interference model of economic disturbance

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1 Introduction

Unforeseen economic shocks often suddenly cause the market (commodity, money or labor) to deviate from the equilibrium. The essential indices tend to show abnormal rise or fall that does not reflect normal economic patterns. These shocks can be caused by conflicts, natural calamities, pandemics, financial crises etc.. These shocks can resemble wave-like disturbances that may ripple across economic dimensions that often require chaotic and non-linear recovery paths [1, 2]. One of the most complex challenges of macroeconomic prediction (forecasting) is to model such recovery patterns, while understanding the temporal dynamics [3; 4]. Traditional econometric models rely majorly on structural variables and static assumptions. They often fail to provide a workable framework of such economic echoes that are mostly dynamic and multi-scalar [5]. We find economic shocks to follow a pattern: there exists an initial sharp deviation in terms of any key macroeconomic indicators like GDP growth, capital flows, stock index fluctuation, inflation etc. followed by a series of oscillations before stabilizing (IMF, 2021). This stabilizing force is either based on market corrections or policy interventions, but is rarely linear or symmetric. There exists several interruptions of subsequent events. It is almost always affected by forces like global contagion, or delayed governmental reactions. This makes the stabilizing path nearly impossible to predict with conventional econometric tools [6, 7]. In this paper, we propose an alternate dynamic model, grounded majorly in the theory of physical systems – wave interference – to predict the behavior of any such economic indicators after a single point shock, extendable to multiple temporal shocks too.

We conceptualize the economic disturbance of a shock similar to the behavior of transverse waves on a disturbed surface. It is like a sudden jolt causing a ripple effect radiating outwards; then, due to market forces or policy interventions, gradually attenuates [8]. We suggest to build and model any economic shock on this analogy where even multiple ripples can interact and result in constructive or destructive interference patterns. If sufficient economic information is fed into the model, it should help us predict the magnitude of the shock, as well as, the time it might take for the economy or the indicator to return to equilibrium or a stable state.

One of the first attempt to conceptualize the economic system as a wave-ripple happen to be during the 2008 financial crisis, with the concept of 'Lehman Wave' (refer to Fig. 1). The model theoretically proposed the mechanisms of production and consumption ripples in the form of damped oscillations across the economy that leads to large-scale disturbances.

Although the model did not attempt to provide the mathematical rigor, it was conceptually intuitive, but not useful for predictive econometrics. [9] developed a more formal approach that attempted to use the more practical agent-interaction system that defines the economy. The authors combined the Kuramoto model and Ising-like spin interaction that defined the synchronization among micro agents, essentially during economic crisis. A somewhat complementary approach was proposed a year earlier, where the economic shock propagation over global networks was conceptualized [10] leveraging epidemic-like contagion mechanisms, where both direct and indirect economic shock effects were studied. In another research, similar network structure was used to visualize inter-sectoral shock propagation using input-output network designs [11]. However, both the models suffered practical inapplicability. The later assumed static inter-sectoral relationships that oversimplified the dynamic agent-interaction. The former laid too stringent assumptions over network stability and contagion rates that can rarely be applied in real-world.

Fig. 1. LEHMAN, WEEKLY. This chart of Lehman Brothers shows the sharp market plunge.

Despite the limitations, the models have their own advantages in dealing with localized ripple effects. For example, such agent-based model was integrated with input-output tables (multiregional) to simulate and analyze the impact of flood disasters in Enshi, China [12]. It worked well to capture recovery mechanisms however, it could not be conveniently extended to all disaster types. Thus, we propose a generalized model where each economic shock or disturbance is projected as a wave initiation – a ripple that moves through time and space, interact with other disturbances that leads to positive or negative interferences and can attenuate after a significant period of time. The proposed model is expected to have a higher predictability of sudden shocks and dampening time, i.e. the time it is likely to take for the economic indicator to stabilize, or return to its equilibrium state.

2 Theoretical Idea

Suppose, we have one disturbance caused by a sudden conflict or war. The wave behavior will resemble Fig. 2 (a). Note that at A_1, the shock is felt the most, and over time, it fades out to A_n.

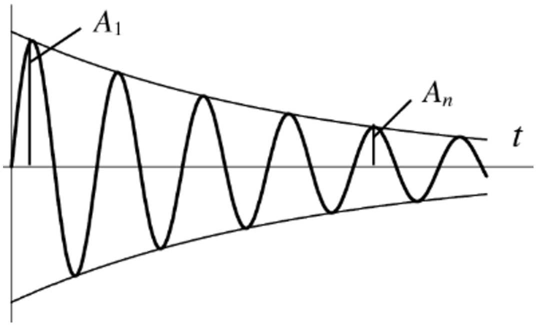


Fig. 2(a): Damped Sine-wave resembling shock

Based on the trajectory and the dampening effect, it is possible to predict the time when the effect will subside. Theoretically, the amplitude never reaches zero (zero is the limiting value), but for all practical purposes, the effect becomes negligible after a certain point in time. If the conflict-duration is longer, the wavelength will also increase. But, in real life, it is never one shock that causes this ripple. There are multiple waves that disseminates at the same time (starting early or late). Figure 2(b) shows this interference effect.

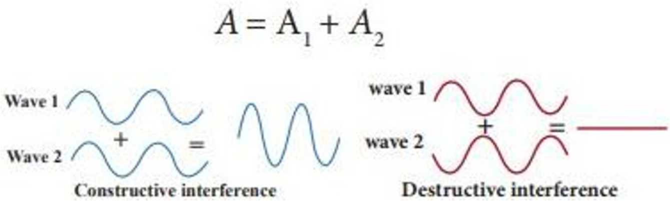


Fig. 2(b): Constructive and destructive interference

The interference can be positive, where multiple shocks combine to create a larger shock; or negative, where a shock is negated by some form of market adjustments or policy interventions. Thus, the intensity of the wave (shock) at any time (t), can be predicted not only based on the initial amplitude and frequency of a shock in isolation, but as the combined effect of all the shocks. The model is mathematically structured in the following section.

3 Initial Model

The simplest way to express a wave function, or an oscillation, is with a sinusoid. Let $'t'$ be time (represented on the x-axis), and the axis itself representing the equilibrium state. Note that the equilibrium state is not a fixed value of the indicator, rather is the general increase or decrease in the value or the relevant index that is projected without any abnormal shock. Hence, let

$$s_0(t) = Asin(\omega t) \qquad (1)$$

where A > 0 is the amplitude and $\omega > 0$ is angular frequency. Thus function captures repeated overshoot/undershoot (cycles) with period $2\pi/\omega$.

But to model a typical economic shock and the consequent recovery, the sinusoid alone seems to be insufficient. First of all $s_0(t)$ crosses zero and becomes negative at every cycle. Interpreting a negative deviation magnitude is meaningless. If we consider the x-axis to be the equilibrium state, and the positive peak as the stock, the negative peaks fails to contribute to it in any credible sense. Secondly, real stocks typically decay: the first overshoot in large and subsequent oscillations shrink in amplitude due to stabilizing forces (markets or policy intervention). Finally, real stocks are rarely symmetric simple sinusoids. They have a sharp onset and then a longer tail (an F-shaped or skewed amplitude envelop). Hence, the model must (i) ensure positively (ii) add damping (decay), and (iii) shape the initial impulse.

Positivity: A minimal modification to keep positivity is to add a positive baseline and keep oscillations as relative perturbations. Define

$$S(t) = 1 + \alpha \sin(\omega t),\ 0 < \alpha < 1 \tag{2}$$

Then,

$S(t) \in (1-\alpha, 1+\alpha) \subset (0, \alpha)$.

Multiplying any positive envelop by $S(t)$ yields a strictly positive $y(t)$. The choice $0 < \alpha < 1$ ensures 'trough' remain positive and the sinusoid only modulates magnitude around a positive baseline (not sign). 'α' is the relative oscillation depth, and 'ω' remains the frequency of oscillatory relaxation.

Introducing damping: an exponentially decaying envelop. As we have assumed earlier that economic recovery can stem from either market correction or policy interventions, we multiply $S(t)$ by a strictly positive envelop $E(t)$. The simple envelop expressing continuous proportional decay is exponential:

$$E_1(t) = Ae^{\lambda t}, \lambda > 0 \tag{3}$$

where λ is the decay rate (higher λ signifies faster recovery). Economically, λ combines the net stabilizing strength of markets and the combined interventions. But, a pure exponential function forces the initial maximum amplitude to equal A_0 and lacks the flexibility to shape the initial rise, that is, how sharply the shocks reaches its main peak.

4 Extension: F-distribution envelop

We finally need an envelop that can (a) create a single pronounced peak whose height we can adjust and (b) control how gradual the shock rises and tails off. A univariate PDF with two shape parameters is thus necessary. The Fisher-F distribution PDF has exactly the structure we want:

$$f_{d_1 d_2}(u) = \frac{\left({}^{d_1}/_{d_2}\right)^{d_1/2}}{B(d_1/2, d_2/2)} u^{\frac{d_1}{2}-1}\left(1 + \frac{d_1}{d_2}u\right)^{-\frac{d_1+d_2}{2}}, \quad u > 0 \tag{4}$$

where $d_1, d_2 > 0$ are the degree of freedom and $B(.,.)$ is the Beta function. d_1 (numerator dof) affect the location and sharpness of the initial rise. We can think of d_1 controlling how concentrated mass is, near the early times. In our economic interpretation this corresponds to how large and concentrated the initial shock is. d_2 (denominator dof) affect the tail thickness, and how gradual the decay is; or economically, how gradual or long running shock's influence is. Hence, the envelop:

$$E(t) = e^{-\lambda t}\left(\alpha + Cf_{d_1 d_2}(kt + \mu)\right) \tag{5}$$

combines exponential stabilization $\left(e^{-\lambda t}\right)$ with an F-shaped impulse scaled by C and shifted up by $\alpha > 0$. The offset $\alpha > 0$ ensures the envelop never vanishes; the factor C is chosen so that F-term is normalized to an interpretable range (e.g., so its maximum is 1 on the time window of interest). The Python simulation of two combinations of the parameters are shown in Appendix 1. The conceptual plot of the model is presented in Appendix 2.

Interpretation Summary:
C - Scales the envelop contribution of the F-shape (shock intensity).
k - Compresses/expands time into the F-domain (speed of the shock).
μ - Shifts the F-argument so mode occurs at desired calendar time.
λ - Speed of aggregate stabilization (market pulse policy).

Economically, damping arises from two generic channels. (i) Market forces: Price adjustments, reallocation of factors, inventory corrections, automatics stabilizers in wages/prices reduce the deviation at a natural rate. (ii) Policy intervention: Fiscal stimulus/transfer, market easing, targeted bailouts accelerate recovery beyond market-only dynamics. We model both aggregated into an effective exponential decay parameter $\lambda > 0$. Higher λ reflects stronger or faster-stabilizing combined effect.

We now derive an ODE satisfied by $y(t) = E(t)S(t)$. This ODE must be linear with time-dependent forcing determined by the F-PDF and its derivatives

1. Envelope identities: Defined $u = kt + \mu$ and $f(u) = f_{d_1 d_2}(u)$.
By definition, from (5), $E(t) = e^{-\lambda t}\left(\alpha + Cf(u)\right)$
Differentiating once, we get
$$E'(t) = -\lambda e^{-\lambda t}\left(\alpha + Cf(u)\right) + e^{-\lambda t} C f'(u) k$$
$$= -\lambda E(t) + R(t), \tag{6}$$

where $R(t) = Cke^{-\lambda t} f'(u)$.

Differentiating again, we get:
$$E''(t) = -\lambda E'(t) + R'(t)$$
$$= -\lambda\left(-\lambda E(t) + R(t)\right) + R'(t) = \lambda^2 E(t) - \lambda R(t) + R'(t). \tag{7}$$

2. Oscillatory identities: Let $S(t) = 1 + \alpha\sin(\omega t)$. Then
$$S'(t) = \alpha\omega\cos(\omega t) \text{ and } S''(t) = -\alpha\omega^2\sin(\omega t) = \omega^2(1 - S(t)) \tag{8}$$
Hence, S satisfies the constant –coefficient ODE
$$S''(t) + \omega^2 S(t) = \omega^2.$$
3. Differentiate $y(t) = E(t)S(t)$, using product rule
$y'(t) = E(t)S'(t) + E'(t)S(t)$ and $y''(t) = E''(t)S(t) + 2E'(t)S'(t) + E(t)S''(t)$,
Substituting the expressions for $E'(t), E''(t)$ and $S''(t)$, we get

$$y''(t) = [\lambda^2 E(t) - \lambda R(t) + R'(t)]S(t) + 2[-\lambda E(t) + R(t)]S'(t) + E(t)[\omega^2(1 - S(t))] \tag{9.1}$$

$$y''(t) = \lambda^2 E(t)S(t) - \lambda R(t)S(t) + R'(t)S(t) - 2\lambda E(t)S'(t) + 2R(t)S'(t) + \omega^2 E(t) - \omega^2 E(t)S(t), \tag{9.2}$$

Rearranging left hand grouping terms in $y(t)$, $y'(t)$ and constants. Observe $E(t)S(t) = y(t)$ and $E(t)S'(t) = y'(t) - E'(t)S(t)$. After collecting terms in $y''(t)$, $y'(t)$ and $y(t)$, we obtain the compact linear second-order ODE:

$$y''(t) + 2\lambda y'(t) + (\lambda^2 + \omega^2)y(t) = R'(t)S(t) + 2R(t)S'(t) + \lambda R(t)S(t) + \omega^2 E(t). \tag{10}$$

This identity is exact, given the definitions of $E(t), R(t)$ and $S(t)$.

4. Now we express the RHS in terms of f, f' and f''. Substituting $R(t) = Cke^{-\lambda t}f'(u)$ and its derivative $R'^{(t)} = Cke^{-\lambda t}[kf''(u) - \lambda'(u)]$; and after grouping terms the final explicit ODE becomes:

$$y''(t) + 2\lambda y'(t) + (\lambda^2 + \omega^2)y(t) = e^{-\lambda t}[Ck^2 f''(u)(1 + \alpha\sin(\omega t)) + 2Ck\alpha\omega f'(u)\cos(\omega t) + \omega^2(\alpha + cf(u))], \tag{11}$$

with $u = kt + \mu$. This is linear in $y(t)$ with a time dependent forcing expressed by f, f', f'' and it fully encodes the F-shaped impulse, the exponential damping and the sinusoidal modulation.

Closed from of $f, f', f''$with $d_1, d_2 > 0$, the F-PDF can be written in factorized form

$$f(u) = Au^{P-1}(1 + bu)^{-q} \tag{12}$$

where $P = {d_1}/{2}$, $q = {(d_1 + d_2)}/{2}$, $b = {d_1}/{d_2}$ and $A = {b^P}/{B\left(P, {d_2}/{2}\right)}$: Beta normalization constant.

Define, $A(u) = \frac{P-1}{u} - \frac{qb}{1+bu}$.

Then, $f'(u) = f(u)A(u), f''(u) = f(u)[A(u)^2 + A'(u)]$, with $A'(u) = -\frac{P-1}{u^2} - \frac{qb^2}{(1+bu)^2}$.

These closed-form factors make numeric evaluation stable and avoid finite difference differentiation.

Now since all algebraic steps above are exact identifies, the unique solution $y(t)$ of the derived ODE with initial conditions $y'(0) = E(0)S'(0) + E'(0)S(0)$ is exactly:

$$y(t) = e^{-\lambda t}\left(\alpha + Cf_{d_1,d_2}(kt + \mu)\right)(1 + \alpha\sin(\omega t)) \tag{13}$$

This is verifiable by substitution. Compute y', y'' from the closed form, substitute in LHS of the ODE and simplify using the identifies for E', E'' and S''. The result equals the RHS, confirming exact equivalence.

5. Case

In our specific case, war causes abrupt and severe disturbance to economic equilibrium like, sudden supply-chain collapse, output contractions, inflation spikes etc. that then gradually dampens or attenuates as market forces and policy interventions take effect. For example, Germany during World War I (1913 – 1928) shows heavy initial disruptions, hyperinflation era, and slow recovery – ideally fits the asymmetric, long tail case of the proposed model. During World War II, the US and Western Europe (1939 – 1952) shows dramatic mobilization (GDP boost in war industries), then postwar reconstruction and damped recovery (Marshall Plan). And more recently – Russia (2022 – 24): commodity and sanction shocks.

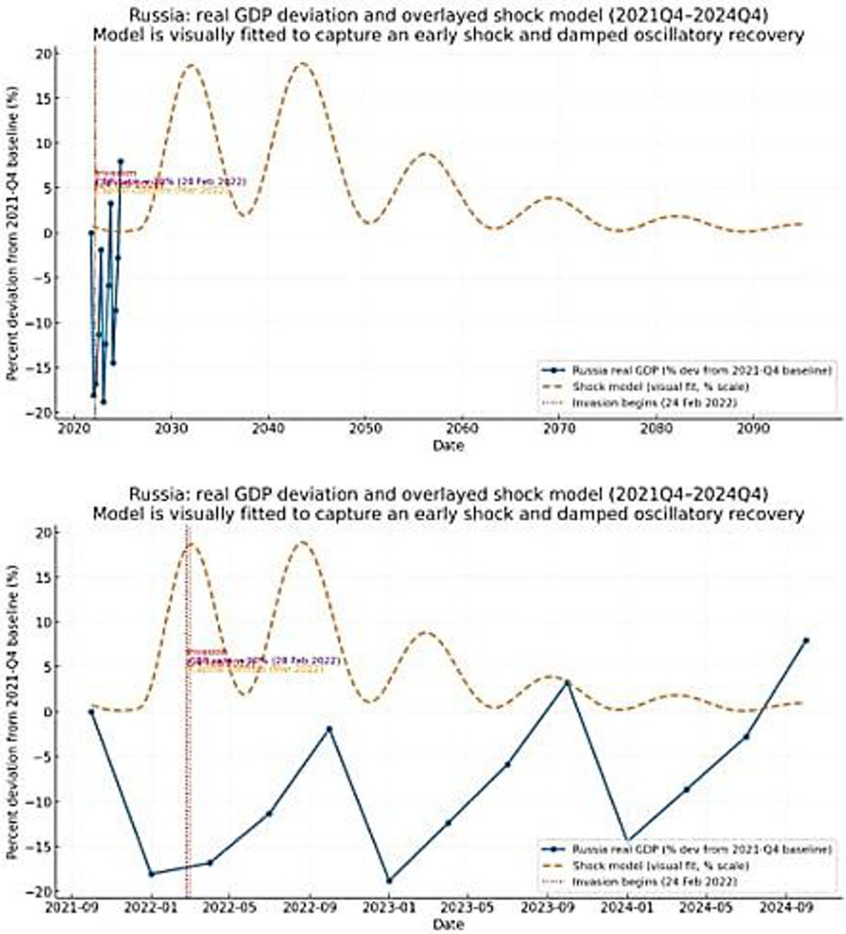


Fig. 3: Russia's Real GDP shock

In Fig. 3, the blue line represents Russia's real GDP expressed as percent deviation from 2021 – Q4 baseline, using IMF/FRED quarterly series. Orange dashed curve is the locked shock model $y(t) = e^{-\lambda t}(\alpha + Cf_{d_1,d_2}(kt + \mu)(1 + \alpha \sin \omega t)$ visually scaled to percent units, so that peak magnitudes are comparable. And the vertical annotations mark three key events around the shock: 1. Invasion start (24 February 2022), 2. Bank of Russia emergency rate hike (28 February 2022), and 3. The capital control restrictions implemented in early March 2022.

6. Future Scope: Residual Wave

Now, waves that trace out the temporal path for a long time, subsides gradually and the amplitude tends to zero. The effect never entirely runs out. There would always be some residue that spreads over time and space all across. Now, there are 'n' number of such shocks or ripples that leave the residue after sufficiently long time. But it is to be noted that when such residual waves merge in form of constructive interference the peak goes further up that leads to fresh economic shock or ripple. We call the phenomenon – 'Residual wave effect'.

Let $\{t_i\}i \geq 1$ be shock arrival times drawn from a Poisson process with rate λ, and for each shock i let A_i (initial amplitude), ω_i (angular frequency), $\emptyset_i$(phase), and $\partial_i > 0$ (decay rate) be random draws from specified distributions $F_A, F_\omega, F_\emptyset, F_\partial$. We define the cumulative wave-signal on the economy at time t by the linear superposition

$$S(t) = \sum_{t_i<t} A_i e^{-\partial_i(t-t_i)} \cos(\omega_i(t - t_i) + \emptyset_i) \quad (14)$$

and the baseline (equilibrium) economic variable of interest (e.g., GDP deviation) by $X(t) = \mu + S(t) + \varepsilon(t)$ where μ is a long-run mean and ε(t) is zero-mean idiosyncratic noise (e.g., Gaussian white noise or an ARMA process). To capture sudden, nonlinear crisis amplification from constructive interference, introduce a threshold Θ >0, a crisis-amplification exponent γ≥0 , and intensity κ≥0, and define the observed impact

$$Y(t) = X(t) \left[1 + \kappa \mathrm{H}(|S(t)| - \Theta) \left(\frac{|S(t)|}{\Theta}\right)^{\gamma}\right] \quad (15)$$

where *H(·)* is the Heaviside step function (1 when its argument is positive, 0 otherwise). The probability of a systemic shock at time 't' is $P_{Crisis}(t) = \Pr(|S(t)| > \Theta$, which can be approximated analytically under Gaussian/small-dependence assumptions or estimated numerically by Monte Carlo sampling of the Poisson arrivals and parameter distributions; model calibration proceeds by estimating λ, μ and the distributions F, Θ, κ, γ and the noise process from historical series (e.g., via likelihood or simulation-based methods). It is one scenario that leads to high unpredictability of shock and recovery trajectories. The proposed model predicts individual patterns of disturbances, but the combined interference effect is to be considered as a promising avenue of future research and validation.

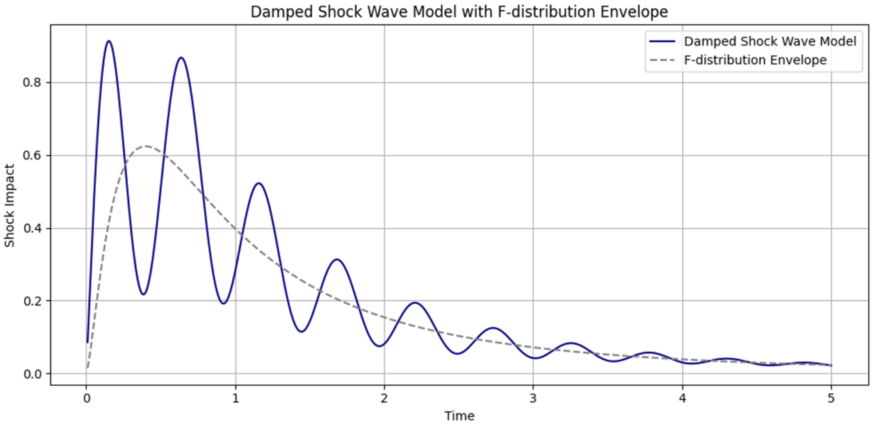
Combined with this, remains the fact that the disturbances rarely remain confined to one or two economic or financial indices. In other words, the wave interferences are not local to the plane of reference. And the way they spread to other planes should have a separate model for prediction. Both these models can be tested after adequate simulations in MATLAB that we keep for future study.

Appendix 1: Simulation results of two different combinations of the parameters

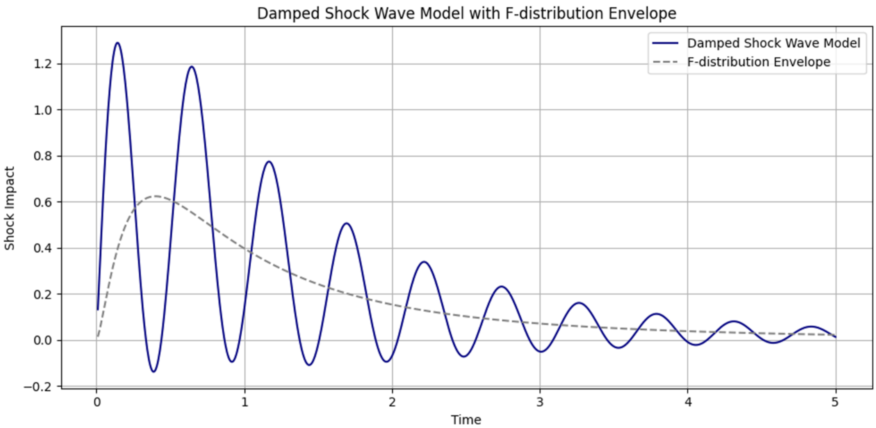
Python Code:

```
import numpy as np
import matplotlib.pyplot as plt
from scipy.special import beta

# Parameters
alpha = 5
beta_param = 4
A = 1
lambda_ = 1
omega = 12

# F-distribution envelope function
def f_distribution(t, alpha, beta_param):
    numerator = (alpha / beta_param) * (alpha / 2) * t * (alpha / 2 - 1)
    denominator = beta(alpha / 2, beta_param / 2) * (1 + (alpha * t / beta_param)) ** ((alpha +
beta_param) / 2)
    return numerator / denominator

# Damped oscillation function
def damped_sine(t, A, lambda_, omega):
    return A * np.exp(-lambda_ * t) * np.sin(omega * t)

# Total shock wave model
def shock_model(t):
    return f_distribution(t, alpha, beta_param) + damped_sine(t, A, lambda_, omega)

# Time range
t = np.linspace(0.01, 5, 1000)  # Avoid t=0 due to division

# Plot
plt.figure(figsize=(10, 5))
plt.plot(t, shock_model(t), label='Damped Shock Wave Model', color='navy')
plt.plot(t, f_distribution(t, alpha, beta_param), '--', label='F-distribution Envelope',
color='gray')
plt.xlabel('Time')
plt.ylabel('Shock Impact')
plt.legend()
plt.grid(True)
plt.title('Damped Shock Wave Model with F-distribution Envelope')
plt.tight_layout()
plt.show()
```

$$\alpha = 5, \beta = 4, A = 0.6, \lambda = 1, \omega = 12$$



$$\alpha = 5, \beta = 4, A = 1, \lambda = 0.6, \omega = 12$$



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