Ecological Protection for Slope Based on Fractal Theory

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Abstract. This paper takes a tailings accumulation slope as a case study and systematically explores the structural characteristics of plant roots and their impact on slope stability by combining fractal theory with numerical simulation. The box-counting dimension was introduced to quantify the spatial complexity of root systems, and the fractal characteristics of two typical plant roots, Pittosporum tobira and Vetiveria zizanioides, were analyzed. Slope stability simulations under multiple conditions (no rainfall, light rainfall, heavy rainfall) were conducted using Midas GTS NX. The results indicate that a higher root fractal dimension corresponds to greater shear strength of the root-soil composite. Vegetation can slightly increase the safety factor, but its controlling effect on deep-seated landslides is extremely limited. ` primarily suppress surface erosion, while woody roots provide deep anchoring and shear support. Roots within the slope experience both tension and compression. The state of stress (tension or compression) is related not only to the root's position on the slope but also to its growth direction, even for roots at the same location.

Keywords: Slope; Fractal Theory; Ecological Slope Protection; Root System; Herbaceous Roots; Woody Roots;Axial Force

1 Introduction

Traditional slope protection methods, such as rubble masonry and concrete facing, are ecologically destructive and difficult to harmonize with the natural environment. In recent years, ecological slope protection technology has been increasingly widely used in mountain road slope management due to its dual functions of environmental protection and engineering reinforcement[1]. Ecological slope protection enhances the anti-erosion capacity and stability of slopes through the interception effect of the plant canopy[2], the soil fixation effect of roots[3], and the composite reinforcement effect of the root network[4], while also promoting ecosystem restoration and protecting biodiversity.

Plant roots, as the core component of ecological slope protection, directly affect slope stability through their morphological characteristics and mechanical properties. Traditional research often uses single indicators such as root density, root length, and

K. Bellitürk et al. (eds.), *Proceedings of the 2025 2nd International Symposium on Agricultural Engineering and Biology (ISAEB 2025)*, Advances in Biological Sciences Research 50,
https://doi.org/10.2991/978-94-6463-910-0_31

root diameter to quantitatively analyze the effect of roots on slope stability, but a holistic description and quantification of the complex root structure remains insufficient[5]. Fractal theory, as a mathematical tool for studying complex geometric shapes, can effectively describe the fractal characteristics of plant roots, reveal their spatial distribution patterns and structural complexity, and provide a new perspective for the quantitative evaluation of ecological slope protection effects[6].

This paper takes an accumulation tailing slope as a case to study. Combining fractal theory, it systematically analyzes the relationship between the fractal dimension of plant roots and slope stability, and explores the mechanism of composite vegetation ecological slope protection on tailings slopes. An experimental and numerical simulation combined method is used to quantify the impact of root fractal dimension on overall slope stability.

2 Project Overview

The ecological restoration area is underlain mainly by tailings gravel, limestone, and marl. No surface water is present, groundwater is poorly developed, and atmospheric precipitation drains away rapidly. Occasional detachments of loose debris from the middle to upper parts of the slope affect traffic safety on the adjacent road. After ecological slope stabilization, rockfalls have been completely eliminated and the overall ecosystem has improved dramatically.

(a) Full view before treatment (b) Tailings accumulation (c) Full view after treatment

Fig. 1. Slope Before and After Treatment

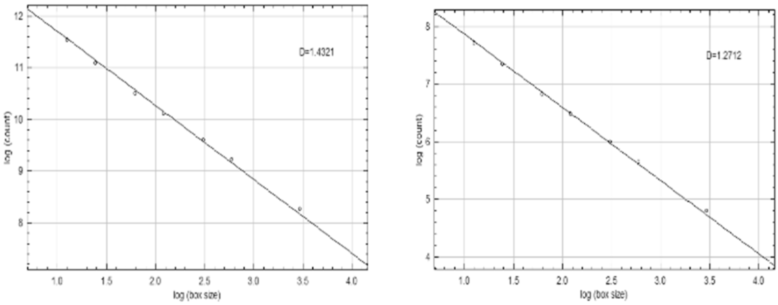
3 Analysis of Overall Characteristics of Plant Roots Based on Fractal Theory

Fractal Theory was proposed by mathematician Benoit Mandelbrot[7] as a mathematical theory to study complex geometric shapes and natural phenomena. It describes structures that have self-similarity and a fractional dimension (fractal dimension).

Plant root systems are intensively and irregularly distributed in the soil, making them suitable for description using fractal theory to characterize their overall structure[8]. Studying the complex morphological characteristics of plant root systems from a fractal perspective allows understanding their geometry using the fractal dimension as a simple representation of the root system state. Box dimension calculates the fractal dimension through the relationship between the number of boxes covering the fractal shape and the box size.

(a) Pittosporum tobira (b) Vetiveria zizanioides

Fig. 2. Vegetation Root Systems



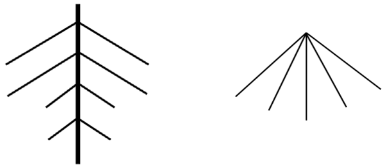
(a) Pittosporum tobira (b) Vetiveria zizanioides

Fig. 3. Root System Box Dimension Calculation Graphs

This slope plans to use a combination of herbaceous and woody plants for comprehensive ecological management. Figure 2 shows images of the root systems of Pittosporum tobira and Vetiveria zizanioides after two years of growth. Using ImageJ to analyze the two root systems, the box dimensions were obtained as 1.4321 and 1.2712, respectively.

Based on the physical images, Pittosporum tobira has a taproot system with a distinct main root from which lateral roots of various orders emerge, clearly defined with longer upper lateral roots and shorter lower lateral roots. Vetiveria zizanioides has a fibrous root system composed of adventitious roots without a clear distinction between main and lateral roots. The shear strength of the root-soil composite is positively correlated with the box dimension of the root system [7]. As the root box dimension increases, the distribution of roots in the soil becomes more complex and dense, better fixing soil particles and thus significantly improving the shear strength of the root-soil composite. Since root length, morphology, and structure change during growth, affecting the box dimension, the reinforcement effect on the slope stabilizes after two years of normal vegetation growth when the root length reaches a certain level.

Numerical simulations generally simplify root systems into truss structures. The principle of simplification is that the length, diameter, morphology, and box dimension of the simplified roots are similar to the actual roots after two years of growth. Thus, the simplified root diagrams were obtained, as shown in Fig. 4. Specific parameters of the simplified roots are listed in Table 1.



(a) Pittosporum tobira (Box Dimension 1.4015) (b) Vetiveria zizanioides (Box Dimension 1.2688)

Fig. 4. Simplified Root System Diagrams Based on Box Dimension

4 Numerical Simulation of Slope Stability Under the Influence of Plant Roots

4.1 Numerical Simulation Model and Calculation Parameters

This paper uses Midas GTS NX for numerical simulation of ecological slope protection with plant roots.

The slope geotechnical body uses the Mohr-Coulomb model. The slope model is simplified from the measured topography of a typical profile in the study area after leveling and soil covering. The slope model height is 16m, width is 55m, with an average slope angle of 24°.

The strata in this area are limestone and marl. After partial mining of the limestone, slag was directly piled on the concave excavation ground. To ensure normal growth of the revegetation plants, the slope body was covered with 0.5m of soil. This paper will compare and analyze the slope displacement and strain contours, safety factor changes, and root axial forces under three conditions (original bare slope, soil-covered slope, and revegetated slope after soil covering) and under three working conditions (no rainfall, light rainfall: 20mm/day * 5 days, and heavy rainfall: 100mm/day * 1 day) to reveal the mechanism of ecological slope protection with roots. The physical and mechanical parameters of the calculation materials are shown in Table 2.

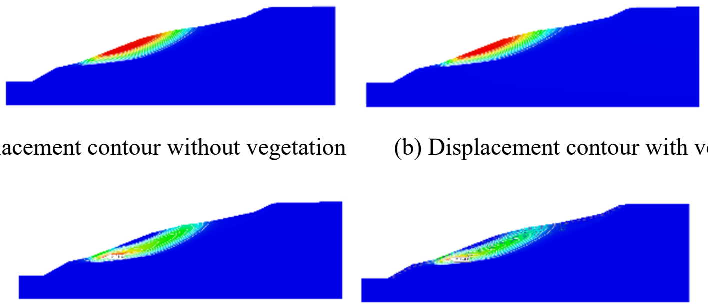
Table 2. Physical and Mechanical Parameters of Materials

Material	Natural Unit Weight/$kN \cdot m^{-3}$	Stiffness/ $kN \cdot m^{-2}$	Cohesion/kPa	Internal friction angle/(°)	Poisson's Ratio
Casing soil	18.6	6000	15	0	0.35
Gravelly Soil	19	40000	0	22	0.25
Limestone	23.5	62000	70	35	0.25

Marlstone	23	53000	60	30	0.25
	Unit Weight /kN·m^{-3}	Poisson's Ratio		Elastic Modulus /kN·m^{-2}	
Herb Roots	5	0.35		8000	
Woody Main Roots	8	0.35		32000	
Woody Lateral Roots	6	0.35		16000	

4.2 Analysis of Numerical Simulation Results

Fig. 5 shows the total displacement and strain increment contours of the slope under a typical working condition (100mm/day * 1day). Fig. 8 shows the variation of the safety factor with and without considering vegetation for various working conditions.



(a) Displacement contour without vegetation (b) Displacement contour with vegetation

(c) Strain increment without vegetation (d) Strain increment with vegetation

Fig. 5. Total Displacement and Strain Increment Contours Under Typical Condition (100mm/day * 1day)

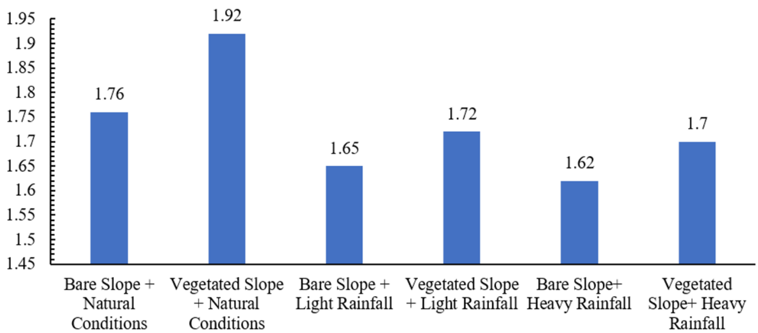
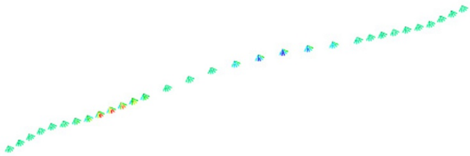


Fig. 6. Safety Factor with and Without Vegetation Influence for Various Conditions

From Fig. 5, it can be seen that under the 100mm/day * 1day condition, the displacement of the sliding surface and the scale of the landslide show almost no difference whether vegetation is considered or not. This is because the sliding surface of this slope is located within the gravel layer, which is beyond the depth of the woody plant roots. This indicates that the presence of vegetation can hardly prevent the continuous development of deep-seated landslides.

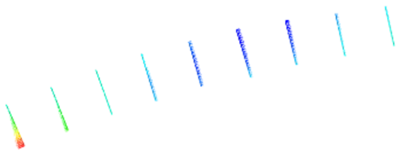
From Fig. 6, it can be seen that vegetation increases the safety factor by an average of about 0.08. Rainfall gradually reduces slope stability, with heavy rain posing the greatest threat to the bare slope. Vegetation can noticeably buffer the adverse effects of rainfall. In summary, whether under natural or rainfall conditions, vegetation can slightly increase the safety factor, but its hindering effect on the continuous development of deep-seated sliding is extremely limited.

Fig. 7 shows the axial force distribution of herbaceous roots, woody lateral roots, and woody main roots in the slope stability calculation under the 100mm/day * 1day condition. Vegetation roots are simulated using a "truss element" model.



(a) Herbaceous roots

(b) Woody lateral roots



(c) Woody main roots

Fig. 7. Root Axial Force Diagrams Under Typical Condition (100mm/day * 1day)

The root axial force diagrams under natural conditions and the 20mm/day * 5days condition show patterns similar to Fig. 7 and are not detailed here. From Fig. 7, it can be observed:

(1) The maximum tensile force in herbaceous roots occurs at the shear exit of the landslide body, with a maximum tension of 17.7 N. The maximum compressive force in herbaceous roots occurs at the rear edge of the landslide, with a maximum pressure of 9.1 N. Herbaceous roots in the middle and lower parts mainly experience tension, while those in the middle and upper parts experience more compression. Even at the same location, roots in different directions experience different stress states. Generally, roots aligned with the sliding direction experience more compressive stress. Roots oriented opposite to the sliding direction experience more tensile stress. For roots perpendicular to the slope surface, the lower part of the slope body experiences tensile stress, and the upper part experiences compressive stress.

(2) The maximum tension and compression in woody lateral roots both appear in the roots in the lower part of the slope body. The maximum tension is 529.2 N, and the maximum compression is 910.3 N. Lateral roots aligned with the sliding direction experience more compressive stress, while those oriented opposite to the sliding direction experience more tensile stress.

(3) The maximum tensile stress in the woody main root appears in the lower part of the slope body, reaching 6140.6 N. The maximum compressive stress appears in the middle and upper part of the slope body, at 2814.0 N. The main root is perpendicular to the slope surface. The lower part of the slope body experiences tensile stress, and the upper part experiences compressive stress.

The fundamental reason for these phenomena lies in the movement mode of the landslide body. The landslide is not a rigid monolithic slide but a plastic body undergoing shear deformation. Its movement mode can be simplified as a composite motion combining rotation and translation. The middle and lower parts of the landslide (especially near the shear exit) are the areas where the landslide body experiences the largest displacement. The forward movement of the soil exerts a forward "dragging" force on the roots, causing them to experience tensile stress. The soil at the rear edge of the landslide undergoes significant downward displacement, transmitted from the deep layers to the surface. Therefore, the downward movement of the surface soil compresses the roots, causing them to experience compressive stress. When a root is aligned with the sliding direction, landslide movement pushes the soil along the root's axis, so it is compressed. When a root is opposite to the sliding direction, landslide movement pulls the soil along the root's axis, so it is tensioned.

5 Conclusion

The conclusions of this paper are as follows:

(1) The fractal dimension of plant roots effectively quantifies structural complexity and correlates positively with the shear strength of root–soil composites. Fractal-based beam-network models therefore offer an accurate way to represent intricate root architecture in numerical simulations.

(2) In most bio-engineered slopes the root zone does not penetrate to the potential slip surface, so vegetation increases the factor of safety only marginally and cannot arrest deep-seated failure. It does, however, reduce surficial displacement and mitigates

rainfall-induced deterioration. Dense, shallow herbaceous roots suppress surface erosion, while woody taproots anchor deeper layers and lateral roots provide horizontal shear resistance; their combined action improves overall slope stability. Roots experience both tension and compression depending on their position and growth direction within the slope.

(3) Eco-slope design should favour native species with well-developed taproots, extensive lateral spread and high root-fractal dimensions. Planting location and orientation must be optimised for the anticipated failure mechanism to maximise tensile/compressive root reinforcement. Because purely biological measures have limitations, they should be integrated with engineering stabilisation to create a "bio-engineering" composite system capable of resisting instability under complex geology and extreme climate.

Acknowledgements

This work was supported by the Open Research Fund of Hunan Provincial Key Laboratory of Key Technology on Hydropower Development (Grant No. PKLHD202102).

Declaration of AI-assisted Technologies

At certain stages of this study, we used artificial intelligence (AI)-assisted technologies. This mainly included polishing the language expressions and checking the grammar.

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