



Pricing Differentiated Expressway Tolls for Freight Trucks Balancing Equity and Efficiency

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Abstract. To address the challenge of balancing efficiency improvement and equity assurance in differentiated expressway tolls for freight trucks. This paper develops a bi-level programming model embedding group equity and spatial-temporal indicators into the objective function and constraints of expressway operator. The equity demands transform post-evaluation into upfront optimization design. The upper-level guides operator decisions through setting fairness penalty costs and rate difference constraints, while the lower-level employs elastic-demand stochastic user equilibrium to capture truckers' route choice to rate variations. The improved particle swarm optimization and Frank-Wolfe algorithm solve this model. Using the Phnom Penh–Sihanoukville–Kampot freight corridor as the study context, a comparative analysis is conducted of systematic impacts under equity-oriented, efficiency-oriented, and neutral policy configurations. The results show that the neutral scenario achieves optimal performance: revenue increases by 13.5%, user costs decrease by 4.3%, group and spatiotemporal equity metrics improve by 43.4% and 40.4%, respectively, and expressway freight-truck volumes rise by 12.1%, with heavy vehicles exhibiting a discount elasticity of 16.8% and off-peak guidance effects of 15.3%. For policymakers and operators, this implies a policy-ready starting point: set moderate peak surcharges for heavy trucks and small off-peak discounts for light trucks, implement simple caps on inter-period rate changes and on cross-group rate differentials, and maintain a revenue floor—together delivering equity gains without undermining financial viability and with clear indicators to monitor in operation. This study provides quantitative decision tools and empirical evidence for developing countries to design differentiated tolling policies balancing economic viability and social acceptability.

Keywords: Expressway; Differential toll; Equity; Two-layer optimization model; Optimization algorithm

1 Introduction

Differentiated highway tolling has become an important policy tool in many countries for optimizing freight logistics costs and improving road network efficiency^[1]. Differentiating tolls by vehicle type, time window, and road enables operators to shift freight traffic temporally and spatially, thereby relieving peak-hour congestion and increasing off-peak road use. Existing differentiated pricing frameworks often prioritize economic efficiency at the expense of careful consideration of equity implications. Such single-objective optimization can saddle freight-truck groups with disproportionate cost burdens, provoking public dissatisfaction and policy pushback^[2].

The existing literature largely concentrates on pricing-model design for differentiated tolling, generally employing a bilevel framework to capture the operator–user game. The Upper-level models typically aim to maximize corporate profits^[3]. The lower-level model describes freight drivers' route selection through the user equilibrium theory^[4]. Although these studies have made significant progress, the fairness assessment of charging schemes still remains at the post-event analysis stage^[5], which may make the optimal pricing scheme difficult to promote during the implementation process due to fairness deficiencies^[6]. In addition, much of the literature treats equity in a single dimension, emphasizing differences in burdens among vehicle types^[7] but neglecting road and time service variations, thereby failing to capture the full fairness implications of differentiated pricing^[8].

To fill the gap, this study proposes a dual-dimensional evaluation framework encompassing both group equity and spatiotemporal equity, providing a practical quantitative tool for assessing the equity of differentiated tolling. Secondly, embedding equity constraints directly into the bilevel program model enables joint modeling of efficiency optimization and equity assurance, rather than the traditional post-event evaluation mode. To solve the two-layer model with complex equity constraints, this paper designs an improved particle swarm optimization algorithm, introducing adaptive inertia weights and constraint repair mechanisms to enhance the convergence speed and the feasibility of the solution. Finally, through comparative analysis of different policy scenarios, it provides decision support for the design of differentiated charging schemes under different development stages and policy orientations.

2 Problem Formulation

2.1 Problem description

Highway operators are responsible for toll collection, operation, maintenance, mainly focusing on achieving maximum benefits at the lowest operating cost. Freight-truck drivers choose their preferred routes in response to the base toll rate and available discounts, minimizing user cost (UC). Policy-making departments need to balance

efficiency and fairness and take social benefits into account. The three-player game is modeled via a bilevel formulation, with the operator and government at the upper level and freight-truck drivers at the lower level. The problem is formulated based on the following assumption.

- (1) Driver route selection is only related to the utility of cost and time cost
- (2) Once the government sets the price bounds and the allowable range of rate adjustments, these parameters are held fixed over the study period.

2.2 Upper-level model

The upper-level model takes into account both efficiency and fairness, and the objective function consists of three parts. The operator revenue (OR) comes from the tolls paid by freight drivers. Road construction, environmental mitigation, and maintenance are treated as necessary operating costs of the operator (OC). The fairness penalty term represents the soft constraints imposed by the government on enterprises through policy guidance. The fairness penalty term represents the soft constraints imposed by the government on operator through policy guidance. The model is as follows.

$$\max_y F(\mathbf{y}) = \sum_{a \in A} \sum_{i \in I} \sum_{g \in G} y_{ai}^g \cdot x_{ai}^g \cdot L_a - \sum_{a \in A} \sum_{i \in I} C_a^{op} - \lambda \cdot [\omega_1 \cdot GE(\mathbf{y}) + \omega_2 \cdot TE(\mathbf{y})] \quad (1)$$

s.t.

$$GE(\mathbf{y}) = \frac{1}{|A| \cdot |I|} \sum_{a \in A} \sum_{i \in I} \sqrt{\frac{1}{|G|} \sum_{g \in G} \left(\frac{y_{ai}^g \cdot \xi^g}{y_{ai}} - 1 \right)^2} \quad (2)$$

$$\bar{y}_{ai} = \frac{1}{|G|} \sum_{g \in G} y_{ai}^g \cdot \xi^g, \forall a \in A, i \in I \quad (3)$$

$$TE(\mathbf{y}) = \frac{1}{|G|} \sum_{g \in G} \frac{\max_{a,i} y_{ai}^g - \min_{a,i} y_{ai}^g}{\bar{y}^g}, \forall a \in A, i \in I \quad (4)$$

$$\bar{y}^g = \frac{1}{|A| \cdot |I|} \sum_{a \in A} \sum_{i \in I} y_{ai}^g, \forall g \in G \quad (5)$$

$$y_{aig}^{\min} \leq y_{ai}^g \leq y_{aig}^{\max}, \forall a \in A, i \in I, g \in G \quad (6)$$

$$\sum_{a \in A} \sum_{i \in I} \sum_{g \in G} y_{ai}^g \cdot x_{ai}^g \cdot L_a - \sum_{a \in A} \sum_{i \in I} C_a^{op} \geq R_{\min} \quad (7)$$

$$\sum_{g \in G} \beta_g \cdot x_{ai}^g \leq C_a, \forall a \in A, i \in I \quad (8)$$

$$\frac{\max_{g \in G} (y_{ai}^g \cdot \xi^g)}{\min_{g \in G} (y_{ai}^g \cdot \xi^g)} \leq \rho_1, \forall a \in A, i \in I \quad (9)$$

$$\frac{\max_{a \in A, i \in I} y_{ai}^g}{\min_{a \in A, i \in I} y_{ai}^g} \leq \rho_2, \forall g \in G \quad (10)$$

$$y_{ai}^g \geq y_{ai}^{g-1}, \forall a \in A, i \in I, g \in G \setminus \{1\} \quad (11)$$

Where A denotes the set of roads $a \in A$. $|A|$ is the total number of roads. I denotes a set of time periods $i \in I$. $|I|$ is the total number of time periods. G denotes the set of vehicle models $g \in G$. $|G|$ is the total number of vehicle models. W is the set of OD pairs $w \in W$. y_{ai}^g denotes the decision variable representing the differentiated toll rate for vehicle type g on road a during period i . x_{ai}^g is a state variable, representing the traffic flow of vehicle type g on road a during period i . C_a^{op} represents the operating cost of road a . $GE(y)$ is the measure of group equity. $TE(y)$ is the measure of spatial-temporal equity. ω_1 and ω_2 are the weight coefficients. L_a represents the length of road a . ξ^g represents the extreme differential coefficient for vehicle type g relative to the base type. $\bar{y}_{ai}$ denotes the standardized average toll rate on road a during period i . $\bar{y}^g$ denotes the average toll rate of vehicle type g . $y_{aig}^{\min}$ and $y_{aig}^{\max}$ denote the minimum and maximum toll rates for vehicle type g on road a during period i . $R_{\min}$ represents the financial lower limit target of the operator. C_a represents the traffic capacity of road a . β_g is the PCU conversion factor for vehicle type g . ρ_1 and ρ_2 denote the upper bound on the inter vehicle type toll-rate range ratio (GV) and the upper bound on the toll-rate range across road/time periods (CV) respectively. The road traffic flow x_{ai}^g in Eq. (1) is obtained from the lower-level model.

2.3 Lower-level model

The lower-level model is the stochastic user equilibrium model (SUE) of elastic demand. the objective function consists of three parts. The first term is used to capture route-choice randomness, the second to integrate road impedance as the system's generalized cost, and the third to represent demand response to tolls and service quality.

$$\begin{aligned} \min_{x,q} Z = & \frac{1}{\theta} \left[\sum_{w \in W} \sum_{k \in K_w} \sum_{g \in G} \beta_g x_{kg}^w (\ln x_{kg}^w - 1) - \sum_{w \in W} \sum_{g \in G} \beta_g q_g^w (\ln q_g^w - 1) \right] \\ & + \sum_{a \in A} \sum_{i \in I} \int_0^{x_{ai}} t_{ai}(u) du - \sum_{w \in W} \sum_{g \in G} \beta_g \int_0^{q_g^w} [D_g^w(u)]^{-1} du \end{aligned} \quad (12)$$

s.t.

$$\sum_{k \in K_w} x_{kg}^w = q_g^w, \forall w \in W, g \in G \quad (13)$$

$$x_{ai}^g = \sum_{w \in W} \sum_{k \in K_w} x_{kg}^w \delta_{ak}^w, \forall a \in A, i \in I, g \in G \quad (14)$$

$$q_g^w \leq q_g^{w, \max}, \forall w \in W, g \in G \quad (15)$$

$$t_{ai}^g(x_{ai}^g) = \frac{\eta_1 \cdot L_a}{v_a^0} \left[1 + \alpha \left(\frac{x_{ai}^g}{C_a} \right)^\beta \right] + \frac{\eta_2 (y_{ai}^g \cdot L_a + f_g \cdot L_a)}{\phi_g}, \forall a \in A, i \in I, g \in G \quad (16)$$

$$x_{ai} = \sum_{g \in G} \beta_g x_{ai}^g, \forall a \in A, i \in I \quad (17)$$

$$x_{kig}^w = \frac{q_g^w \cdot \exp \left(-\theta \cdot \sum_{a \in k} t_{ai}^g(x_{ai}^g) \right)}{\sum_{k \in K_w} \exp \left(-\theta \cdot \sum_{a \in k} t_{ai}^g(x_{ai}^g) \right)}, \forall k \in K_w, i \in I, g \in G, w \in W \quad (18)$$

$$q_g^w = q_g^{w, \max} \cdot \exp \left(\mu_g^w \cdot \frac{1}{\theta} \ln \sum_{k \in K_w} \exp \left(-\theta \sum_{a \in k} t_{ai}^g(x_{ai}^g) \right) \right), \forall g \in G, w \in W \quad (19)$$

$$\left[D_g^w \right]^{-1} (q_g^w) = -\frac{1}{\mu_g^w} \ln \frac{q_g^w}{q_g^{w, \max}}, \forall g \in G, w \in W \quad (20)$$

$$D(\mathbf{y}) = \sum_{g \in G} \sum_{a \in A} \sum_{i \in I} x_{ai}^g \cdot \left[t_{ai}^g(x_{ai}^g) - \frac{L_a}{v_a^0} \right] \quad (21)$$

$$x_{kg}^w \geq 0, q_g^w \geq 0, \forall w \in W, k \in K_w, g \in G \quad (22)$$

Where K_w denotes the set of paths for OD pair w . x_{kg}^w is a state variable denoting the flow of vehicle type g on path k for OD pair w . q_g^w is a state variable denoting the actual travel demand of vehicle type g for OD pair w . δ_{ak}^w is a 0-1 variable when path k of contains road a , $\delta_{ak}^w=1$, otherwise, $\delta_{ak}^w=0$. $t_{ai}^g(x_{ai}^g)$ denotes the generalized impedance for vehicle type g on road a during period i , as determined by x_{ai}^g . α and β are parameters of the BPR function, $\alpha=1.5$, $\beta=4$. x_{ai} is the flow of vehicle type g on road a . v_a^0 represents the free-flow velocity of road a . f_g represents the fuel cost of type g . ϕ_g represents the time value coefficient of vehicle type g . η_1 and

η_2 are the weight coefficients. θ is the perceptual parameter of the Logit model. $q_g^{w,\max}$ denotes the max travel demand of vehicle type g for OD pair w . x_{kig}^w denotes the flow of vehicle type g on path k for OD pair w during period i . μ_g^w represents the demand price elasticity coefficient of vehicle type g for OD pair w . D_g^w is travel demand function.

3 Model Solution

The bilevel programming model constructed in this paper has non-convex and nonlinear characteristics. Traditional gradient-based algorithms have difficulty effectively handling equity constraints. An improved particle swarm method^[9] (IPSO) is adopted for the upper-level toll optimization, and a Frank–Wolfe algorithm^[10] (FW) is embedded to solve the lower-level SUE model. The lower-level model returns the flow x_{ai}^g and demand q_g^w , which are then used to evaluate the upper-level objective function. In view of the characteristics of equity constraints, the following improvements have been made to the PSO.

(1) This study employs a nonlinear decreasing adaptive inertia weight $\omega(t) = \omega_{\max} - (\omega_{\max} - \omega_{\min}) \cdot (t / T_{\max})^2$, where t represents the current iteration and $T_{\max}$ represents the max number of iterations. The strategy employs a quadratic function to affect a smooth transition from global exploration to local exploitation.

(2) A two-stage equity constraint repair mechanism is designed. When particles violate Eq. (9), the rate is adjusted through scaling $y_{ai}^{g'} = \bar{y}_{ai} + (y_{ai}^g - \bar{y}_{ai}) \cdot \min(1, \rho_1 r_{\min} / r_{\max})$. $\bar{y}_{ai}$ represents the standard average toll rate. $r_{\max}$ and $r_{\min}$ represent the maximum and minimum standard toll rates respectively. When Eq. (10) is violated, apply spatial smoothing $y_{ai}^{g'} = (1 - \gamma)y_{ai}^g + \gamma\bar{y}_g$. The smoothing coefficient $\gamma \in [0, 1]$ is determined via bisection to satisfy the constraints.

(3) Every N_{local} iterations, a simulated-annealing local search is applied to the global-best particle. A subset of variables is randomly perturbed and worsening moves are accepted with probability $\exp(\Delta F / T)$ to prevent premature convergence. Fitness function is $F(y) = F(y) - M(t) \sum_j [\max(0, g_j(y))]^2$. The dynamic factor increases with each iteration. $g_j(y)$ represent Eq. (7) and Eq. (8) $j = 1, 2$. The particle update rules are fully demonstrated in studies of Malik⁹.

The steps of the double-layer optimization algorithm are as follows.

(1) Initialize the particle swarm $\{y_1, \dots, y_N\}$. Uniformly sample within the range of $[y_{\min}, y_{\max}]$, speed $\{v_1, \dots, v_N\} = 0$.

(2) For each particle y_i , the FW algorithm is called to solve the lower-level model to obtain (x_i, q_i) , calculate the fitness function $F(y)$, and update the individual optimum p_i .

(3) Update the global optimal $p_g = \arg \max \{F(p_i)\}$.

(4) Compute the adaptive inertia weight $\omega(t)$, update all particles' velocities and positions, and apply a repair operator to enforce the equity constraints.

(5) Perform a local search on the global optimal particle p_g every N_{local} iterations.

(6) Evaluate convergence: if the change in the global-best fitness over 20 consecutive iterations is less than 10^{-3} , or if the maximum iteration count T_{max} is reached, terminate and return $y^* = p_g$; else, return to Step (2).

4 Case Study

4.1 Basic data

Phnom Penh, Sihanoukville, and Kampot in Cambodia are taken as the nodes defining the study area. Table 1 shows the basic information of three groups of parallel expressways and free roads among the nodes. Table 2 shows the type of vehicle and characteristic coefficients. Based on two-axle vehicles, $y_0=0.5$, only tolls are charged on expressways. Based on freight characteristics, the 24-hour period is divided into peak (7:00-10:00, 16:00-20:00), off-peak (10:00-16:00, 20:00-22:00) and night periods (22:00-7:00). Phnom Penh – Sihanoukville: $q_g^{w,max} = \{120, 80, 50, 30\}$ veh/h, $g = 1, 2, 3, 4$. Sihanoukville – Kampot: $q_g^{w,max} = \{45, 30, 20, 10\}$ veh/h, $g = 1, 2, 3, 4$. Phnom Penh – Kampot: $q_g^{w,max} = \{60, 40, 25, 15\}$ veh/h, $g = 1, 2, 3, 4$. $\theta = 1.5$, $\eta_1 = 0.4$, $\eta_2 = 0.6$. Define three policy scenarios, scenario 1 is equity-first approach (S1), $\lambda = 0.8$, $\rho_1 = 1.5$, $\rho_2 = 1.8$, scenario 2 is balanced approach (S2), $\lambda = 0.5$, $\rho_1 = 2.1$, $\rho_2 = 2.3$, scenario 3 is efficiency-first approach (S3), $\lambda = 0.2$, $\rho_1 = 2.8$, $\rho_2 = 3.2$. The range of toll rate is $[y_{min}, y_{max}] = [0.3y_0, 1.2y_0]$. R_{min} represents 85% of the current income. Particle swarm size $N=100$. Maximum number of iterations $T_{max}=200$. Inertia weighting range $[\omega_{min}, \omega_{max}] = [0.4, 0.9]$, $c_1 = c_2 = 2.0$, $N_{local} = 35$, $M_0 = 10^5$. The model is solved using MATLAB R2023a.

Table 1. Basic parameters of the road network.

a	OD	type	L_a (km)	v_a^0 (km/h)	C_a (pcu/h)	C_a^{op} (CNY/h)
E1	Phnom Penh – Si-	Expressway	205	120	1800	1500
F1	hanoukville	Freeway	238	80	1100	400
E2	Sihanoukville –	Expressway	98	100	1350	900
F2	Kampot	Freeway	132	80	850	300

E3	Phnom Penh –	Expressway	148	100	1600	1200
F3	Kampot	Freeway	165	80	1000	350

Table 2. Vehicle type and characteristic coefficients.

g	axes	β_g	ξ^g	ϕ_g	f_g	μ_g^w
1	2	1.5	1	35	0.8	0.05
2	3	2	1.5	50	1.2	0.06
3	4	2.5	2	65	1.6	0.07
4	≥ 5	3.2	2.8	85	2.1	0.09

4.2 Result analysis

The two-layer model constructed in this paper is solved through the designed algorithm. The result in Figure 1 shows that as the weight on equity constraints in pricing increases, the per-kilometer rate differential across vehicle types (GV) narrows, while the road network total delay rises modestly, revealing equity–efficiency trade off. Scenario 3 exhibits the largest group disparity and the lowest delay. Scenario 1 yields the smallest disparity with slightly higher delay, while Scenario 2 falls in between and offers a more balanced trade-off between improvement and cost. From the perspective of time periods, on expressways, the rate of Scenario 1 is smoother among peak, off-peak and night periods, with the overall coefficient of variation (CV) being the lowest. It indicates that the equity of time periods has been effectively enhanced. Under Scenario 3, rates and costs fluctuate substantially during peak hours, yielding clear differences across time periods. Scenario 2 maintains operational stability while reducing the dispersion between peak, off-peak and night periods.

To sum up with, raising the weight on equity does not lead to uncontrolled efficiency losses. The greatest payoff arises from structural rebalancing at heavy-truck peaks and light-truck off-peaks, yielding concurrent improvements in group and time period equity. Under a given demand elasticity, choosing a balanced solution can achieve a more robust trade-off between equity improvement and delay time growth.

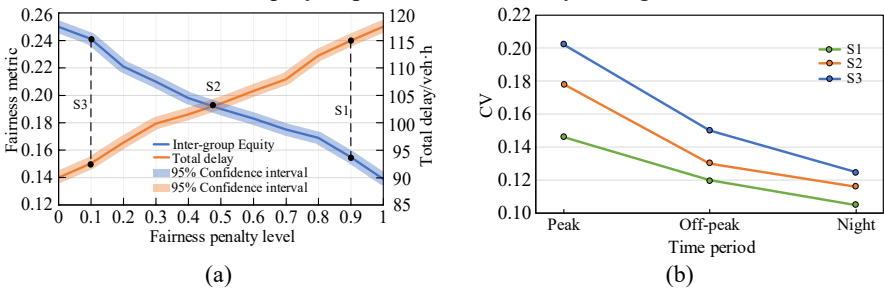


Fig. 1. Comparison of equity indicators in various scenarios.

Furthermore, calculate the key indicators in different scenarios. As shown in Table 3, Scenario 2 achieves a 13.5% increase in operator revenue (OR) alongside a 4.3% reduction in user costs (UC), demonstrating a significant improvement in equity metrics. The expressway share ratio (SR) reaches its maximum value of 71.4% across all

scenarios, optimizing the balance between equity and efficiency while enhancing the utilization rate of the road network. Although Scenario 1 offers the best equity, the increase in operator revenue is limited. Scene S3 has the highest operator returns, the improvement in equity is not significant and the reduction in user costs is the smallest.

Table 3. Comparison of the key indicators in different key indicators in different scenarios.

Scenarios	OR (CNY/d)	UC (CNY/km)	GE	TE	Equity Loss (%)	SR (%)	Total delay (veh/h)
Base	45.22	0.921	0.382	0.421	0.403	0.623	102.2
S1	46.83	0.864	0.144	0.168	0.155	0.687	92.34
S2	51.39	0.882	0.223	0.251	0.235	0.714	104.2
S3	55.71	0.918	0.356	0.398	0.377	0.659	112.7

This study specifically analyzes the increase in traffic volume by vehicle type on the expressway in Scenario 2. As shown in Figure 2, the increase in traffic is positively correlated with the type of vehicle. Across all expressways, vehicles with five or more axles register increases exceeding 15%, whereas two-axle freight trucks show increases below 10%. It indicates that heavy-duty trucks are more sensitive to toll rate discounts. Comparing each expressway show that the strongest policy impact on expressway E1, where the average increase reaches 13.2%, exceeding that of other segments and implying greater responsiveness among long-distance freight.

Figure 3 reports are duction in solution oscillation after 60 iterations and convergence by iteration 80. The total computation time is 93.12 s, underscoring the algorithm's stability and convergent behavior.

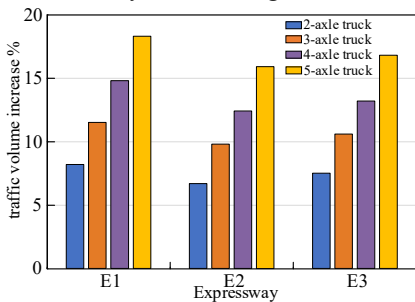


Fig. 2. Truck traffic volume Changes.

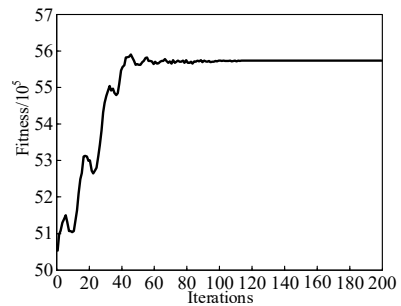


Fig. 3. Upper-level model results iteration.

5 Conclusion

To address the equity–efficiency coordination problem in differentiated expressway tolling for freight trucks, a bilevel model is developed that embeds dual constraints on group equity and spatial-temporal equity. The upper-level model introduces equity constraints, while the lower-level model adopts elastic demand stochastic user equilibrium to characterize the path selection behavior of freight drivers. The model was solved by IPSO algorithm and verified with real cases. The research results show that the balanced

scheme (S2) performs best in the three policy scenarios. operator revenue increased by 13.5%, the average users cost decreased by 4.3%, group equity and spatial-temporal equity improved by 43.4% and 40.4% respectively, and the volume of truck on expressways increased by 12.1%. Heavy-duty vehicle type responded more significantly to rate discounts, with the traffic volume of 5-axle and above trucks increasing by 16.8%, while that of 2-axle trucks was only 7.5%. The increase in traffic during off-peak hours (15.3%) was significantly higher than that during peak hours (8.9%), verifying the spatial-temporal guidance effect of differentiated tolling. The equity–efficiency trade-off curve indicates that overemphasizing equity sacrifices economic efficiency, whereas neglecting equity amplifies social discontent. The balanced scheme operates at the intersection point of the trade-off curve, achieving coordinated multi-objective optimization and providing a theoretical basis for differentiated toll policy design.

This study is limited by static demand elasticity and an SUE calibrated to a single corridor/period, aggregate equity metrics. Future work should validate the model across multiple corridors and seasons with operational data, incorporate stochastic elements and distributional analyses by origin–destination and shipper segments, and extend to dynamic time-of-day pricing with explicit enforcement and evasion considerations.

Acknowledgements

This research was supported by China Communications Construction Company.

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