



A Quantum Group Decision-Making Approach for Evaluating Decarbonization Strategies of Highway Infrastructure

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Abstract. In complex and uncertain group decision-making (GDM) environments, decision-makers' opinions are often mutually influential, while trust relationships typically exhibit multi-path structures and structural incompleteness. To address these challenges, this paper proposes a quantum GDM framework based on linguistic Z-numbers that explicitly incorporates quantum interference effects. Trust relationships among decision-makers are first aggregated within a quantum probability framework by jointly considering direct trust and trust transmission paths, thereby inferring missing pairwise trust information. Subsequently, an objective optimization model is developed to determine attribute weights by integrating attribute marginal contributions with closeness-based measures. Building on these results, a quantum-like Bayesian network is constructed to model interference effects among decision-makers, through which the quantum probabilities of alternatives are computed and used for ranking. Finally, the applicability of the proposed method is demonstrated through a case study on the decarbonization of highway infrastructure. Sensitivity, comparative, and simulation analyses further confirm the robustness of the proposed approach and its suitability for uncertain GDM scenarios.

Keywords: Multi-attribute group decision-making; Trust aggregation; Quantum probability theory; Social network; Shapley value.

1 Introduction

In the context of addressing climate change and advancing sustainable development, the decarbonization transformation of highway infrastructure has become a pressing issue in the global transportation sector^[1]. Due to the involvement of multiple technological pathways and stakeholders with diverse interests, the selection and evaluation of relevant strategies are marked by high levels of uncertainty and complexity. In recent years, group decision-making methods have been widely applied in such complex scenarios, as they can integrate opinions and knowledge from multiple sources^[2,3]. However, precise quantitative data is often difficult to obtain, making linguistic information

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Y. Xu et al. (eds.), *Proceedings of the 2026 5th International Conference on Engineering Management and Information Science (EMIS 2026)*, Advances in Computer Science Research 130,

https://doi.org/10.2991/978-94-6239-652-4_24

a key tool for handling uncertainty. Forms of linguistic information include binary linguistic sets^[4], hesitant fuzzy linguistic term sets^[5], and probabilistic linguistic term sets^[6], among others. Binary linguistic term sets are structurally simple but lack the ability to adequately capture evaluation uncertainty. Hesitant fuzzy linguistic term sets can reflect multiple possible assessment values; however, they do not explicitly model the reliability of the evaluations. Probabilistic linguistic term sets enhance expressive capability by introducing probability information, yet the assignment of probabilities is often subjective and it is difficult to clearly distinguish the evaluation results from their associated credibility. In complex group decision-making environments, these approaches still exhibit certain limitations in bridging qualitative judgments and quantitative analysis. To overcome the above limitations, this paper adopts linguistic Z-numbers^[7] as the fundamental information representation framework. By simultaneously modeling linguistic evaluations and their corresponding reliability levels, linguistic Z-numbers provide a unified representation that accounts for both assessment outcomes and credibility, thereby offering a more complete and stable basis for transforming qualitative information into quantitative modeling.

In group decision-making, trust relationships not only reflect the propensity of decision-makers to adopt information from one another but also largely determine the influence of their evaluation opinions in the group aggregation process^[8]. To date, various trust relationship aggregation methods have been proposed, such as the T-norm^[9], trust propagation models^[10], and adjacency matrix methods based on social network analysis^[11]. Beyond these classical approaches, several studies have attempted to incorporate behavioral and linguistic factors into trust modeling. Ji et al.^[12] constructed a social network group decision-making method centered on trust incentives, in which decision-makers' risk attitudes were integrated into the trust formation process and trust weight allocation, thereby enhancing cooperation and trust coordination at the group level. Wu et al.^[13] focused on the dissemination and aggregation of linguistic trust information in social networks and developed a feedback optimization mechanism driven by maximum self-esteem, achieving a balance between individual compromise behavior and group-level trust coordination. Although these methods have enriched the modeling of trust mechanisms in group decision-making to some extent, they are generally built upon simplified assumptions of path independence and do not fully account for the coupling relationships and interference effects among multiple trust propagation paths. As a result, the superposition, cancellation, and feedback phenomena that may arise during complex information transmission processes remain insufficiently characterized.

In GDM problems, the rational determination of attribute weights is essential for accurately reflecting the relative importance of evaluation criteria and improving the discriminative power of ranking results. To capture the significance of different attributes in the decision-making process, a variety of weighting approaches have been developed in the literature. Representative methods include the Analytic Hierarchy Process (AHP)^[14], which relies on pairwise comparisons and expert judgments, the entropy weight method^[15], which determines weights based on the dispersion of evaluation information, and combined weighting schemes^[16] that integrate subjective and objective

perspectives. These methods rely either on expert judgment or on the dispersion information of the evaluation matrix. To address the limitations of conventional attribute weight determination methods, Ma et al.^[17] introduced a risk perception mechanism under the framework of complex variable preference modeling, and reflected the differences in the importance of different attributes across dual dimensions of rational–emotional factors and heterogeneous risk attitudes through a standardized weight model. Although these methods enhance applicability to some extent, they often overlook the marginal contributions of attributes under different combinational structures and thus fail to fully capture the synergistic effects among attributes. The Shapley value from cooperative game theory can be employed to measure the average marginal contribution of an attribute across all possible subsets, thereby reflecting its structural importance within the decision system. Based on this idea, some studies directly normalize Shapley values for attribute weighting purposes^[18], while Liu and Zhu^[19] further integrated Shapley weights with the normal cloud model and cloud distance to characterize the importance of multi-level attributes in a multi-source information environment. However, since the Shapley value is essentially a relative contribution measure, its direct use or normalization in weighting schemes may distort proportional relationships and alter the ranking semantics of the original evaluation information.

Group interference effects refer to the evaluation bias and behavioral fluctuations caused by mutual influence among decision-makers during the group decision-making process^[20]. Traditional group decision-making models typically assume that decision-makers act independently, ignoring the interactive relationships among subjective opinions^[21]. In practice, however, individual decisions are often significantly influenced by others' views, attitudes, and emotions, exhibiting characteristics of non-independence and irrationality. Quantum probability theory introduces concepts such as interference terms and superposition states, enabling the modeling of interaction mechanisms among decision-makers and providing new tools for capturing group behavior in complex social contexts^[22,23]. For example, Xu et al.^[24] emphasized the need to fully consider the interference effects among multiple reference paths in trust network group decision-making to enhance the rationality of opinion revision and the coordination efficiency of group decisions. Xiao et al.^[25] introduced quantum interference effects to more accurately reflect the combined influence of multi-source trust information on group decision-making.

Based on the above analysis, existing studies still exhibit the following two research gaps:

(1) Existing weight allocation methods mainly rely on subjective judgment or statistical dispersion information, making it difficult to systematically reflect the structural importance of attributes under different combinational settings and their synergistic effects, which may compromise the rationality of weight determination.

(2) Most trust aggregation and group decision-making models are built upon assumptions of path independence or linear superposition, making it difficult to capture the interactions among multiple trust propagation paths as well as the interference behaviors among decision-makers, thereby limiting the ability to characterize complex information transmission processes and group behavioral characteristics.

In summary, this paper presents a quantum group decision-making approach that integrates linguistic Z -numbers with quantum interference mechanisms. Trust relationships among decision-makers are first inferred by aggregating direct trust and multi-path trust transmission within a quantum probability framework, enabling the completion of missing pairwise trust information. Subsequently, a characteristic function is formulated by incorporating attribute marginal contributions and closeness measures, and an optimization model is established to derive the optimal attribute weights for each decision-maker. Based on these results, a quantum-like Bayesian network is constructed to capture interference effects among decision-makers, through which the quantum probabilities of alternatives are computed and the final ranking outcomes are obtained.

2 Preliminaries

2.1 Linguistic Term Set

Let g be a given positive integer. A linguistic term set is defined as $L = \{l_t \mid t = 0, 1, \dots, 2g\}$, where l_t denotes the t -th linguistic term used to express a decision maker's qualitative assessment of an alternative with respect to a given attribute.

Definition 1^[7]: To enable quantitative processing of linguistic assessment information, a mapping is defined as :

$$F : l_t \rightarrow \theta = \theta_t, t = 1, 2, \dots, 2g. \quad (1)$$

where θ_t represents the semantic value associated with the linguistic term l_t , and θ_t is taken from a predefined semantic domain to characterize the semantic differences among linguistic terms.

2.2 Linguistic Z-Numbers

Definition 2^[7]: Let X be a finite universe of discourse. A linguistic Z -number is defined as:

$$Z = \left\{ \left(x, A_{\varphi(x)}, B_{\phi(x)} \right) \mid x \in X \right\}. \quad (2)$$

where $A_{\varphi(x)}$ is a linguistic variable representing the assessment value of element x , and $B_{\phi(x)}$ is a linguistic variable expressing the reliability or confidence degree associated with $A_{\varphi(x)}$. The functions $\varphi(\cdot)$ and $\phi(\cdot)$ denote the mappings from X to the corresponding linguistic term sets for evaluation and reliability, respectively.

In a linguistic Z -number, $A_{\phi(x)}$ characterizes the qualitative evaluation information, while $B_{\phi(x)}$ reflects the decision maker’s subjective confidence in that evaluation, thereby enabling a simultaneous representation of assessment information and its associated reliability.

2.3 Social Network

Definition 3^[24]: Let $D = \{d_1, d_2, \dots, d_s\}$ denote a finite set of decision makers, and let $E \subseteq D \times D$ represent the set of directed relationships among them. A social network is defined as a directed graph $G = (D, E)$, whose adjacency relationship can be expressed by a matrix $T = (t_{kh})_{s \times s}$, where

$$t_{kh} = \begin{cases} 1, & (d_k, d_h) \in E; \\ 0, & \text{Other}; \end{cases} \tag{3}$$

$$k = 1, 2, \dots, s, h = 1, 2, \dots, s, k \neq h.$$

where, $t_{kh} = 1$ indicates that there exists a direct relationship from decision maker d_k to decision maker d_h , while $t_{kh} = 0$ indicates that no direct relationship is observed between them. The matrix T thus provides a binary representation of the underlying social network structure.

2.4 Quantum Probability Theory

In quantum probability theory^[26], events are not evaluated solely through classical additive rules. Instead, the occurrence of an event is modeled as the superposition of multiple cognitive states, each contributing a probability amplitude rather than a deterministic probability value. This framework allows different states to interact with each other through phase information, thereby capturing contextual effects and mutual influences that cannot be adequately described by classical probability theory (see Figure 1).

Formally, let G and B denote two mutually exclusive cognitive states (or events), and let W be a target event. The probability of event W is defined by the squared modulus of the superposed probability amplitudes, i.e.,

$$P(W) = \left| \psi_G \psi_{W|G} + \psi_B \psi_{W|B} \right|^2. \tag{4}$$

where ψ_G and ψ_B are the probability amplitudes associated with states G and B , respectively, and $\psi_{W|G}$ and $\psi_{W|B}$ are the conditional probability amplitudes of event W given states G and B .

Definition 4^[24]. In a quantum Bayesian network, a decision outcome may be reached through multiple latent paths. Let the probability amplitudes of two representative paths be $\psi_G = \sqrt{P(G)}e^{i\theta_G}$ and $\psi_B = \sqrt{P(B)}e^{i\theta_B}$, where $P(\cdot)$ denotes the classical probability component and θ is the associated phase parameter. When the specific decision path is not observed, different paths remain in a superposition state, and their interaction gives rise to a quantum interference effect. The interference term between paths G and B is given by $I_{G,B} = 2\sqrt{P(G)P(B)} \cos(\theta_G - \theta_B)$, which characterizes the strength and direction of interaction between alternative decision paths. Positive and negative values correspond to constructive and destructive interference, respectively, while $\cos(\theta_G - \theta_B) = 0$ indicates the absence of interference.

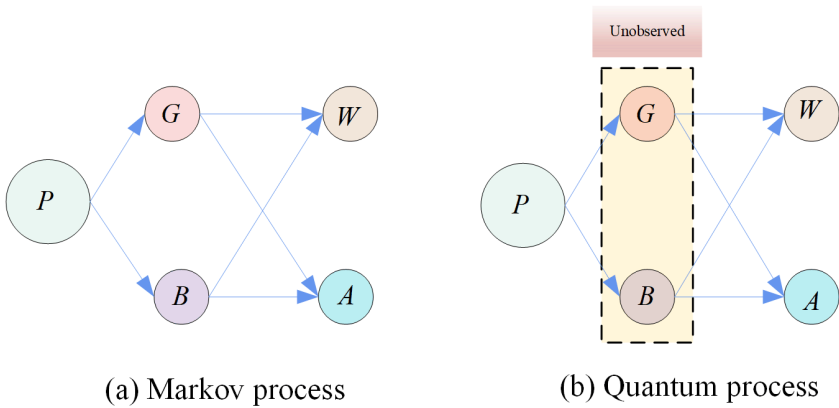


Fig. 1. Markov link path diagram.

3 A Multi-Attribute Quantum Group Decision-Making Model Considering Multi-Source Information Interference

In a GDM setting, the decision task aims to identify a consensus-oriented outcome by aggregating the assessments of multiple decision-makers over a set of alternatives under several evaluation attributes. Such a problem can be characterized by the following components.

Let $D = \{d_1, d_2, \dots, d_s\}$ denote a group of s decision-makers, where $s \geq 3$. Each decision-maker d_k is associated with a non-negative importance weight λ_k , and the collective weight vector is given by $\Lambda = \{\lambda_1, \lambda_2, \dots, \lambda_s\}$, satisfying

$$\sum_{k=1}^s \lambda_k = 1.$$

The decision space consists of a finite collection of candidate alternatives $A = \{a_1, a_2, \dots, a_m\}$ with $m \geq 2$, which represent feasible options to be evaluated and compared. The evaluation criteria are described by an attribute set $C = \{c_1, c_2, \dots, c_n\}$, where $n \geq 2$. For each decision-maker d_k , a personalized attribute weight vector $W^k = \{\omega_1^k, \omega_2^k, \dots, \omega_n^k\}$ is specified to reflect the relative importance of attributes, with $\omega_j^k \geq 0$ and $\sum_{j=1}^n \omega_j^k = 1$.

3.1 Collection and Processing of Decision Information

For each decision-maker d_k ($k = 1, 2, \dots, s$), the performance of alternative a_i ($i = 1, 2, \dots, m$) under attribute c_j ($j = 1, 2, \dots, n$) is assessed using linguistic information. To simultaneously capture both the evaluation result and its associated reliability, the assessment provided by d_k is expressed in the form of a linguistic Z -number, denoted by z_{ij}^k . Accordingly, the complete evaluation information of decision-maker d_k can be organized into a linguistic Z -number decision matrix:

$$Z^k = \begin{pmatrix} z_{11}^k & z_{12}^k & \cdots & z_{1n}^k \\ z_{21}^k & z_{22}^k & \cdots & z_{2n}^k \\ \vdots & \vdots & \ddots & \vdots \\ z_{m1}^k & z_{m2}^k & \cdots & z_{mn}^k \end{pmatrix}, k = 1, 2, \dots, s \tag{5}$$

To facilitate subsequent quantitative analysis, the original linguistic Z -number decision matrix Z^k is further converted into a crisp decision matrix $Z^{(k)} = (z_{ij}^k) m \times n$, by applying a predefined linguistic scale function together with the concept of relative closeness^[7]. This transformation enables the linguistic evaluation information to be consistently mapped into numerical values while preserving the underlying semantic ordering.

3.2 Computation of Decision-Maker Weights Considering Quantum Interference

3.2.1 Trust Relationship Aggregation Based on Quantum Interference.

In group decision-making contexts, social trust networks are widely employed to describe the relational structure among decision-makers, where nodes correspond to decision-makers and edges reflect trust relationships. In practice, however, trust information is often incomplete due to limited interaction records or restricted information

acquisition. As a consequence, trust networks frequently suffer from missing links and disconnected structures, especially in terms of indirect trust paths. Such structural incompleteness may interrupt trust propagation and weaken the coherence of group cognition, thereby impeding effective consensus formation.

To overcome this limitation, this paper adopts the framework of quantum probability theory and interprets trust propagation as a quantum-like state evolution process. Within this framework, different trust transmission paths are regarded as components of a composite quantum state, and their joint influence on indirect trust inference is captured through quantum interference. By representing the contribution of each path in terms of probability amplitudes, an indirect trust aggregation mechanism that explicitly accounts for multi-path interactions is established.

First, a fuzzy trust matrix is introduced to describe the trust relationships among decision-makers, extending conventional binary trust representations to continuous values. Specifically, the trust matrix is defined as $T = (t_{kh})_{s \times s}$, where $t_{kh} \in [0, 1]$ denotes the degree of trust from decision-maker d_k to decision-maker d_h , with $k, h = 1, 2, \dots, s$ and $k \neq h$. This formulation allows different intensities of trust to be consistently represented within the network.

Next, a quantum-like Bayesian network is constructed to model trust transmission. Suppose that there exist q feasible trust transmission paths in the network, denoted by E_r ($r = 1, 2, \dots, q$), where each E_r represents a distinct trust path between two decision-makers. For instance, E_1 may correspond to a direct path linking d_1 and d_2 . In the first layer of the network, each path is assigned a weight $p(E_r) = w_r$, reflecting its relative importance. The second layer characterizes the normalized indirect trust contribution of each path, given by:

$$p(T_r | E_r) = \frac{t_r}{\sum_{r=1}^q t_r}, r = 1, 2, \dots, q \tag{6}$$

where T_r denotes the r -th indirect trust path and t_r represents the aggregated trust degree associated with that path.

Finally, the overall indirect trust degree is obtained by superposing the probability amplitudes corresponding to all trust transmission paths, as follows:

$$\begin{aligned}
 t_{kh} = & \sum_{r=1}^q p(E_r) p(T_r | E_r) \\
 & + 2 \sum_{r=1}^{y-1} \sum_{r'=r+1}^y \sqrt{p(E_r) p(T_r | E_r) p(E_{r'}) p(T_{r'} | E_{r'})} \cos \zeta_{rr'}, \tag{7} \\
 & k = 1, 2, \dots, s, h = 1, 2, \dots, s
 \end{aligned}$$

3.2.2 Computation of Decision-Maker Weights Based on Trust Relationships.

Based on the aggregated trust structure among decision-makers, the relative importance of each decision-maker is further quantified from a network perspective. Specifically, degree centrality is employed to measure the structural influence of decision-makers within the trust network, and the corresponding decision-maker weights are determined as follows:

$$\lambda_k = \frac{t_{kh}}{\sum_{h=1, h \neq k}^s t_{kh}}, k = 1, 2, \dots, s \tag{8}$$

3.3 Computation of Attribute Weights Considering Marginal Contribution

To quantitatively characterize the average marginal contribution of attributes under different combination structures, this paper adopts the Shapley value from cooperative game theory as a contribution measurement tool. The Shapley value evaluates how an attribute affects the overall evaluation outcome by examining the incremental change it brings when joining different attribute coalitions, thereby capturing both marginal effects and potential interaction-induced synergies among attributes.

Let the attribute set be $C = \{c_1, c_2, \dots, c_n\}$, and let any subset $S \subseteq C$ represent an attribute coalition. Denote by $v^k(S)$ the characteristic function value associated with coalition S from the perspective of decision-maker d_k . Then, the Shapley value of attribute c_j for decision-maker d_k is defined as:

$$\phi_j^k = \sum_{S \subseteq C \setminus \{c_j\}} \frac{|S|!(n-|S|-1)!}{n!} [v^k(S \cup \{c_j\}) - v^k(S)] \tag{9}$$

In this formulation, $v^k(S \cup \{c_j\}) - v^k(S)$ represents the marginal contribution of attribute c_j to coalition S , while the coefficient $\frac{|S|!(n-|S|-1)!}{n!}$ corresponds to the probability that coalition S appears immediately before c_j under a random permutation of all attributes. Consequently, ϕ_j^k can be interpreted as the average marginal impact of attribute c_j on the evaluation results across all possible coalition structures.

It should be noted that the Shapley value is essentially a relative contribution indicator derived from the characteristic function. If it is directly normalized to compute attribute weights^[27], the proportional relationships embedded in the original evaluation information and the consistency of ranking semantics may be distorted. Therefore, in

this paper, the Shapley value is employed as a theoretical foundation to reflect the average marginal effect of each attribute. By further incorporating the concept of relative closeness to construct the Shapley characteristic function, an objective optimization model is subsequently developed to determine the optimal attribute weights for each decision-maker, as follows:

$$\begin{aligned}
 M_1 : & \text{Min} \left(\sum_{j=1}^n |\omega_j^k - \phi_j^k|^p \right)^{1/p} \\
 & \left. \begin{aligned}
 & \sum_{j=1}^n \omega_j^k = 1 \\
 & \omega_j^k \geq \omega_{j'}, \text{ if } \phi_j^k \geq \phi_{j'}^k \\
 & \phi_j^k = \sum_{S \subseteq C \setminus \{c_j\}} \frac{|S|!(n-|S|-1)!}{n!} [v^k(S \cup \{c_j\}) - v^k(S)] \\
 & v^k(S) = \frac{1}{m} \sum_{i=1}^m C_i^k \\
 & C_i^k = \frac{\sqrt{\sum_{\{c_j\} \in S} (r_{ij}^k - r_j^-)^2}}{\sqrt{\sum_{\{c_j\} \in S} (r_{ij}^k - r_j^+)^2} + \sqrt{\sum_{\{c_j\} \in S} (r_{ij}^k - r_j^-)^2}} \\
 & r_{ij}^k = \omega_j^k z_{ij}^k \\
 & \omega_j^k \geq 0, p \geq 1 \\
 & i = 1, 2, \dots, m, j = 1, 2, \dots, n, k = 1, 2, \dots, s
 \end{aligned} \right\} \text{s.t.} \tag{10}
 \end{aligned}$$

For generality, the value of p is set to 2.

The objective function in model (9) aims to minimize the deviation between the attribute weights and their corresponding Shapley values. Constraint 1 ensures normalization, i.e., the sum of all attribute weights equals 1. Constraint 2 enforces ranking consistency, meaning that if the Shapley value of one attribute is greater than or equal to another, its corresponding weight should not be smaller, thus maintaining semantic consistency at the ranking level. Constraints 3–4 define the Shapley values, which are computed by enumerating all possible subsets and calculating their marginal utility gains based on relative closeness. Constraints 5–6 define the computation of closeness, which is based on the Euclidean distance from the positive and negative ideal solutions within a given attribute subset, reflecting the impact of attribute combinations on ranking differentiation. Constraints 7–8 specify the feasible ranges of relevant parameters.

Theorem 1. Model (10) has an optimal solution.

Proof. The objective function of this model is a continuous convex function, and strictly differentiable when $p > 1$, satisfying the continuity conditions required in optimization theory. Meanwhile, the feasible region is defined by $\sum_{j=1}^n \omega_j^k = 1$ and $\omega_j^k \geq 0$, forming a closed convex set bounded within a closed and finite domain.

According to the Weierstrass extreme value theorem, any continuous function on a non-empty compact set must attain a minimum. Therefore, the objective function of this model necessarily has an optimal solution over the given feasible domain.

Theorem 2. The time complexity of model (10) is $\mathcal{O}(mn^2 2^n + n^3)$.

Proof. The computational complexity of this model mainly arises from two parts: Shapley value computation and optimization solving. Firstly, in the computation of the Shapley value, each attribute must enumerate 2^{n-1} subsets, and for each subset, the utility is calculated based on relative closeness. Each closeness calculation involves m alternatives and up to n attributes, leading to a complexity of $\mathcal{O}(mn2^n)$ per attribute. Thus, the total complexity for all attributes is $\mathcal{O}(mn^2 2^n)$.

Secondly, the optimization problem is convex. For n attributes, the number of variables is n , and ranking consistency may introduce up to $\mathcal{O}(n^2)$ inequality constraints. The problem is solved using the interior point method, with a computational complexity of up to $\mathcal{O}(n^3)$.

In summary, the total time complexity of the model is $\mathcal{O}(mn^2 2^n + n^3)$.

3.4 Ranking of Alternative Solutions Based on Quantum Probability

1) The weights assigned to decision-makers are incorporated as the prior probabilities in the first layer of the Bayesian network, defined as follows:

$$p(d_k) = \lambda_k, k = 1, 2, \dots, s \tag{11}$$

2) The conditional probability of an alternative, given the corresponding decision-making state, is determined as follows:

$$p(a_i | d_k) = \frac{\sum_{j=1}^n \omega_j z_{ij}^k}{\sum_{i=1}^m \sum_{j=1}^n \omega_j z_{ij}^k}, i = 1, 2, \dots, m, k = 1, 2, \dots, s \tag{12}$$

3)The quantum probabilities of the alternatives are calculated as follows:

$$\begin{aligned} p(a_i) &= \sigma \left| \sum_{k=1}^s \varphi_{d_k} e^{i\theta_{d_k}} \varphi_{a_i|d_k} e^{i\theta_{a_i|d_k}} \right|^2 \\ &= \sigma \left[\sum_{k=1}^s \left| \varphi_{d_k} \varphi_{a_i|d_k} \right|^2 + 2 \sum_{k=1}^{s-1} \sum_{k=k+1}^s \varphi_{d_k} \varphi_{a_i|d_k} \varphi_{d_k} \varphi_{a_i|d_k} \cos(\theta_{ik} - \theta_{i\hat{k}}) \right], \tag{13} \\ &i = 1, 2, \dots, m \end{aligned}$$

where,

$$\sigma = \frac{1}{\sum_{i=1}^m \left| \sum_{k=1}^s |q'_{ik}|^2 + 2 \sum_{k=1}^{s-1} \sum_{\hat{k}=k+1}^s q'_{ik} q'_{i\hat{k}} \cos(\theta_{ik} - \theta_{i\hat{k}}) \right|^2} \tag{14}$$

3.5 Decision Steps

This paper develops a multi-attribute quantum group decision-making approach that integrates trust network information and quantum interference effects. The overall decision-making procedure can be summarized as follows:

Step 1: The evaluation matrices provided by all decision-makers are collected, where the assessments are expressed in the form of linguistic Z-numbers. These linguistic evaluations are subsequently transformed into crisp numerical matrices by employing a linguistic scale function together with the concept of relative closeness.

Step 2: Aggregation of trust relationships considering quantum interference. Obtain the indirect trust relationships among decision-makers using Eqs. (6)–(7), and calculate the weights of decision-makers using Eq. (8).

Step 3: Solve model (10) to obtain the optimal attribute weights.

Step 4: Construct a quantum-like Bayesian network using Eqs. (11)–(12). Based on this, compute the quantum probabilities of each alternative using Eqs. (13)–(14).

Step 5: Finally, the set of alternatives is ranked in descending order according to their corresponding quantum probability values, yielding the final decision outcome.

4 Case Analysis

4.1 Background

Under the continuous advancement of the “dual-carbon” strategy, highway infrastructure, as a sector characterized by high energy consumption and high carbon emissions, has attracted increasing policy attention regarding carbon emission control during the construction stage. The improvement of standard systems related to carbon emission accounting and management in highway construction has become an important institutional foundation for promoting green and low-carbon transformation, thereby creating practical demands for the systematic evaluation of different decarbonization strategies. In the Xinjiang region, the standardization of carbon emission accounting and management during highway construction has entered an implementation-oriented stage. The Calculation Method for Carbon Emissions during Highway Construction (XJ23-106) has been officially included in the regional local standard development and revision plan

(https://jtyst.xinjiang.gov.cn/hd/yjzj_result?id=ec89da740a484abc9510c862adfb5b43&site=6500000043&url=/xjjtysj/zjxqy/new_hd_myzj_details.shtml). The advancement of this standard indicates that, in practical decision-making processes, it is necessary to conduct systematic comparisons and

scientific selections among multiple potential decarbonization pathways in order to support carbon emission management and control objectives during highway construction.

In the context of decarbonization decision-making for highway construction, a variety of technological and managerial pathways are usually available, including material substitution, optimization of construction methods, electrification of construction equipment, and coordination of operation and management practices. These decarbonization strategies exhibit significant differences in terms of cost input, emission reduction potential, technological maturity, implementation period, and public acceptance. Moreover, relevant evaluation information often relies on expert experience and subjective judgment, which is difficult to characterize using a single deterministic indicator. Meanwhile, the decision-making process typically involves multiple stakeholders, such as government authorities, construction enterprises, research institutions, and public groups, whose cognitive perspectives and concerns differ substantially. As a result, this decision context constitutes a typical multi-attribute group decision-making problem.

Based on the above background, five candidate decarbonization strategy alternatives a_i , ($i = 1, 2, \dots, 5$) are considered, including: (1) adoption of recycled or green construction materials; (2) promotion of prefabricated low-carbon construction technologies; (3) application of electrified construction equipment; (4) deployment of intelligent energy efficiency control systems; and (5) implementation of green supply chain coordination mechanisms. To comprehensively characterize the overall performance of these alternatives, five evaluation attributes c_j ($j = 1, 2, \dots, 5$) are introduced, namely initial construction cost, carbon emission reduction potential, technological maturity, implementation period, and public acceptance.

In addition, four decision-makers d_k ($k = 1, 2, \dots, 4$) are involved, representing government authorities, construction enterprises, environmental research institutions, and public groups, respectively. Each decision-maker provides comprehensive evaluations of the decarbonization alternatives based on their professional background and cognitive perspective. Considering the uncertainty and subjectivity inherent in the evaluation information, linguistic Z-numbers are employed as the information representation form to simultaneously capture evaluation outcomes and their associated credibility, thereby providing a foundation for subsequent multi-attribute group decision-making analysis.

4.2 Decision Steps

Step 1: The two linguistic term sets used in this paper are as follows:

$$A = \{A_0: \text{Very Poor}, A_1: \text{Poor}, A_2: \text{Average}, A_3: \text{Good}, A_4: \text{Very Good}\}$$

$$B = \left\{ \begin{array}{l} B_0: \text{Completely Impossible}, B_1: \text{Extremely Unlikely}, B_2: \text{Very Unlikely}, B_3: \text{Unlikely}, \\ B_4: \text{Moderately Likely}, B_5: \text{Somewhat Likely}, B_6: \text{Very Likely}, B_7: \text{Highly Likely}, \\ B_8: \text{Completely Likely} \end{array} \right\}$$

On this basis, the evaluation information matrices provided by the decision-makers, represented using linguistic Z-numbers, are collected as follows:

$$Z^1 = \begin{bmatrix} (A_4, B_3) & (A_1, B_5) & (A_3, B_6) & (A_3, B_2) & (A_4, B_1) \\ (A_2, B_6) & (A_3, B_5) & (A_3, B_6) & (A_1, B_6) & (A_2, B_8) \\ (A_3, B_7) & (A_2, B_6) & (A_1, B_4) & (A_2, B_4) & (A_1, B_4) \\ (A_2, B_5) & (A_1, B_2) & (A_4, B_1) & (A_4, B_7) & (A_3, B_6) \\ (A_1, B_1) & (A_3, B_4) & (A_2, B_8) & (A_2, B_5) & (A_3, B_1) \end{bmatrix},$$

$$Z^2 = \begin{bmatrix} (A_2, B_3) & (A_1, B_6) & (A_3, B_1) & (A_1, B_1) & (A_1, B_3) \\ (A_1, B_7) & (A_3, B_2) & (A_2, B_7) & (A_2, B_7) & (A_4, B_6) \\ (A_2, B_2) & (A_3, B_5) & (A_1, B_6) & (A_4, B_8) & (A_4, B_4) \\ (A_4, B_1) & (A_4, B_8) & (A_4, B_5) & (A_2, B_6) & (A_4, B_6) \\ (A_2, B_3) & (A_3, B_6) & (A_1, B_4) & (A_3, B_2) & (A_3, B_7) \end{bmatrix},$$

$$Z^3 = \begin{bmatrix} (A_2, B_6) & (A_0, B_7) & (A_4, B_8) & (A_2, B_2) & (A_2, B_7) \\ (A_3, B_5) & (A_2, B_6) & (A_2, B_6) & (A_3, B_7) & (A_4, B_7) \\ (A_4, B_7) & (A_1, B_5) & (A_1, B_1) & (A_4, B_6) & (A_1, B_5) \\ (A_2, B_8) & (A_4, B_7) & (A_3, B_7) & (A_1, B_3) & (A_3, B_8) \\ (A_1, B_5) & (A_1, B_8) & (A_3, B_3) & (A_1, B_6) & (A_4, B_5) \end{bmatrix},$$

$$Z^4 = \begin{bmatrix} (A_4, B_8) & (A_2, B_6) & (A_2, B_7) & (A_2, B_3) & (A_4, B_8) \\ (A_2, B_6) & (A_1, B_7) & (A_2, B_7) & (A_3, B_5) & (A_1, B_5) \\ (A_1, B_5) & (A_0, B_8) & (A_1, B_2) & (A_4, B_8) & (A_1, B_5) \\ (A_2, B_8) & (A_2, B_5) & (A_4, B_5) & (A_1, B_1) & (A_2, B_6) \\ (A_4, B_7) & (A_4, B_6) & (A_3, B_2) & (A_1, B_7) & (A_3, B_7) \end{bmatrix}.$$

Step 2: Collect the direct trust relationships among the decision-makers, as shown in Figure 2.

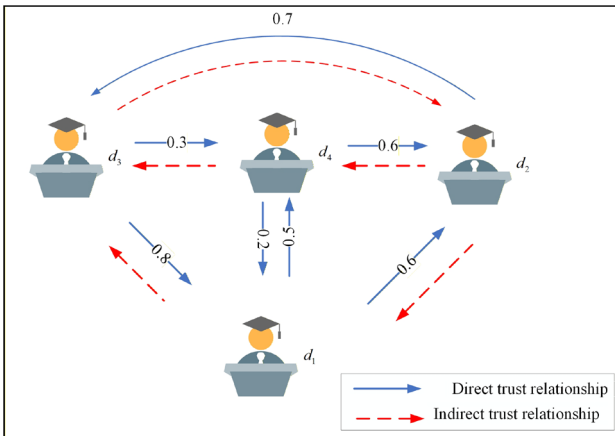


Fig. 2. The trust relationship among decision-makers.

From Figure 2, it can be observed that there are three indirect trust paths from decision-maker d_3 to decision-maker d_2 , with assumed trust path weights of 0.3, 0.3, and 0.4. For all other pairs of decision-makers, there are two indirect trust paths, with assumed path weights of 0.4 and 0.6. On this basis, the indirect trust relationships among decision-makers are aggregated using Eqs. (6)–(7), where the phase angle is set to 45° . The complete trust matrix is as follows:

$$T = \begin{bmatrix} \times & 0.3 & 0.7759 & 0.5 \\ 0.7619 & \times & 0.7 & 0.5880 \\ 0.8 & 0.6258 & \times & 0.3 \\ 0.2 & 0.6 & 0.6158 & \times \end{bmatrix}.$$

Finally, the weights of the decision-makers are calculated according to Eq. (8), as follows:

$$\Lambda = \{0.2329, 0.3029, 0.2550, 0.2092\}.$$

Step 3: Solve model (10) to obtain the optimal attribute weights for each decision-maker, as shown below:

$$\begin{aligned} W^1 &= \{0.2597, 0.2794, 0.1581, 0.1658, 0.1370\}, \\ W^2 &= \{0.0997, 0.2252, 0.1631, 0.2057, 0.3062\}, \\ W^3 &= \{0.2101, 0.1782, 0.2331, 0.1439, 0.2347\}, \\ W^4 &= \{0.2296, 0.2159, 0.2178, 0.1751, 0.1616\}. \end{aligned}$$

Step 4: Construct the quantum-like Bayesian framework. Let the decision-maker weights constitute the first layer of the quantum-like Bayesian network. The conditional probabilities are then calculated using Eq. (12) as follows:

$$\begin{aligned} p(a_1 | d_1) &= 0.1912, p(a_2 | d_1) = 0.2562, p(a_3 | d_1) = 0.2170, \\ p(a_4 | d_1) &= 0.1881, p(a_5 | d_1) = 0.1474, p(a_1 | d_2) = 0.1111, \\ p(a_2 | d_2) &= 0.2230, p(a_3 | d_2) = 0.2218, p(a_4 | d_2) = 0.2449, \\ p(a_5 | d_2) &= 0.1991, p(a_1 | d_3) = 0.1923, p(a_2 | d_3) = 0.2332, \\ p(a_3 | d_3) &= 0.1697, p(a_4 | d_3) = 0.2152, p(a_5 | d_3) = 0.1896, \\ p(a_1 | d_4) &= 0.2181, p(a_2 | d_4) = 0.2175, p(a_3 | d_4) = 0.1527, \\ p(a_4 | d_4) &= 0.1986, p(a_5 | d_4) = 0.2310. \end{aligned}$$

On this basis, the interference angles are measured using a similarity-based heuristic algorithm^[21]. Then, the quantum probabilities of the alternatives are calculated using Eq. (13) as follows:

$$p(a_1) = 0.1592, p(a_2) = 0.2111, p(a_3) = 0.1645,$$

$$p(a_4) = 0.2313, p(a_5) = 0.2339.$$

Finally, the alternatives are ranked based on their quantum probabilities, and the ranking result is: $a_5 \succ a_4 \succ a_2 \succ a_3 \succ a_1$.

4.3 Sensitivity Analysis

In the decision-maker weight calculation method proposed in this paper, the value of the phase angle directly affects the characterization of trust relationships. To further conduct sensitivity analysis, five typical phase angle values (0° , 45° , 90° , 135° , and 180°) are selected. The changes in decision-maker weights under different phase angle settings are systematically observed, as shown in Figure 3.

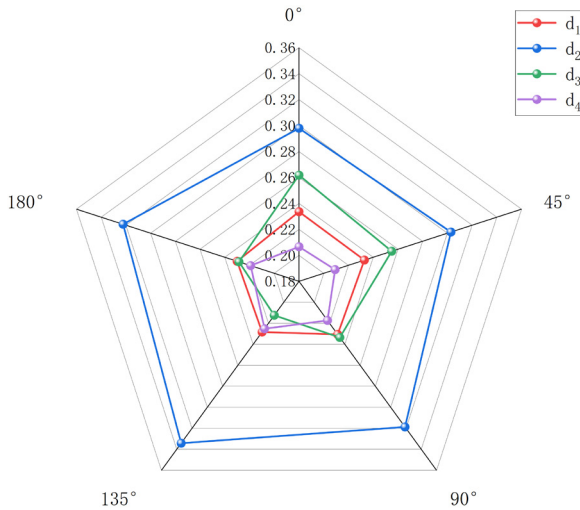


Fig. 3. Influence of phase angle variation on decision-maker weights.

As shown in Figure 3, the weights of the decision-makers exhibit noticeable variation trends under different phase angle settings, indicating that the phase angle, as a key parameter reflecting trust interference, has a significant impact on weight allocation. For example, the weight of decision-maker d_2 fluctuates the most, with its maximum value appearing at a phase angle of 135° and the minimum near 0° , showing a wide distribution range. This indicates that d_2 is more sensitive to changes in the trust

structure. In contrast, the weight of decision-maker d_4 shows relatively small variation across different phase angles and remains within a relatively stable range, suggesting that d_4 maintains a robust position under different trust interference scenarios. Moreover, when the phase angle is set to 135° and 180° , the weights of all decision-makers tend to converge, implying that the model's ability to distinguish individual differences becomes relatively weaker under these settings.

Therefore, the sensitivity analysis results demonstrate that the proposed method can effectively capture the influence of trust relationship variations on weight allocation. It also exhibits strong dynamic adaptability, making it suitable for complex, variable, and interference-prone multi-attribute group decision-making scenarios.

4.4 Comparative Analysis

To evaluate the performance of the proposed approach, comparative experiments were carried out against several representative decision-making methods from the existing literature. These benchmark models are either directly applicable to the case problem considered in this study or can be appropriately adjusted to ensure methodological consistency. An overview of the selected comparison methods is provided in Table 1.

Table 1. Ranking comparison results of different methods

Method	Ranking indicator	Ranking results
TOPSIS ^[28]	$C_1^* = 0.4159, C_2^* = 0.8868, C_3^* = 0.4842,$ $C_4^* = 0.6216, C_5^* = 0.4189.$	$a_2 \succ a_4 \succ a_3 \succ a_5 \succ a_1$
TODIM ^[29]	$\phi(a_1) = -3.5581, \phi(a_2) = 1.4644, \phi(a_3) = -2.4484,$ $\phi(a_4) = -0.0631, \phi(a_5) = -2.0052.$	$a_2 \succ a_4 \succ a_5 \succ a_3 \succ a_1$
Quantum model 1 ^[30]	$p(a_1) = 0.1582, p(a_2) = 0.2311, p(a_3) = 0.1637,$ $p(a_4) = 0.3092, p(a_5) = 0.1378.$	$a_4 \succ a_2 \succ a_3 \succ a_1 \succ a_5$
Quantum model 2 ^[26]	$p(a_1) = 0.1738, p(a_2) = 0.2334, p(a_3) = 0.1915,$ $p(a_4) = 0.2136, p(a_5) = 0.1876.$	$a_2 \succ a_4 \succ a_3 \succ a_5 \succ a_1$
The proposed method in Bayesian network	$p(a_1) = 0.1729, p(a_2) = 0.2322, p(a_3) = 0.1929,$ $p(a_4) = 0.2144, p(a_5) = 0.1876.$	$a_2 \succ a_4 \succ a_3 \succ a_5 \succ a_1$
The proposed method	$p(a_1) = 0.1592, p(a_2) = 0.2111, p(a_3) = 0.1645,$ $p(a_4) = 0.2313, p(a_5) = 0.2339.$	$a_5 \succ a_4 \succ a_2 \succ a_3 \succ a_1$

In comparison with classical multi-attribute decision-making models, both the TOPSIS^[28] and TODIM^[29] methods identify a_2 as the optimal alternative, and their overall ranking results differ significantly from those obtained by the proposed method. Specifically, TOPSIS^[28] evaluates alternatives based on the relative closeness to ideal

solutions, while TODIM^[29] incorporates prospect theory to model gains and losses perception. However, both methods fail to consider cognitive biases or information interference that may arise during the evaluation process. When the proposed model is degenerated into a classical Bayesian framework, the resulting ranking remains generally consistent with traditional methods, which validates the effectiveness of the proposed approach.

In terms of comparison with quantum models, Quantum Model 1^[30] primarily focuses on the measurement of interference angles and performs alternative ranking directly based on quantum probabilities. It overlooks the attribute weight structure and thus cannot reflect the relative contribution of different attributes to the final ranking. Although Quantum Model 2^[26] determines attribute weights using the entropy weight method, it fails to consider the marginal effects and interaction structures of attributes as expressed in group ranking processes. As a result, its ranking outcome is constrained by the limitations of a static weight configuration.

In summary, the proposed method demonstrates significant advantages in uncertain multi-attribute group decision-making, primarily in two aspects: Firstly, it introduces a quantum interference modeling mechanism that fully accounts for the mutual interference effects among decision-makers caused by cognitive interdependence, effectively capturing the nonlinear cognitive structure within the group. Secondly, it establishes objective computation mechanisms for both decision-maker weights and attribute weights. The decision-maker weights are dynamically generated based on the trust network, while the attribute weights are determined through an objective optimization model. This ensures that attribute weights reflect marginal contributions within combinatorial structures while maintaining overall ranking consistency. Therefore, the proposed method is better suited to complex and uncertain decision-making environments, offering more interpretable and discriminative decision support under multi-source information interference and fluctuating subjective preferences.

4.5 Simulation Analysis

To verify the effectiveness and robustness of the proposed attribute weight model, a simulation analysis is conducted. Different matrix dimensions are considered to simulate various decision-making scenarios, and the elements of each evaluation matrix are assumed to be randomly generated data within the range $[0, 1]$. To avoid randomness in individual cases, the model is independently executed 100 times, and the distribution of attribute weights is observed, as shown in Figure 4.

As shown in Figure 4, the attribute weight optimization model proposed in this paper exhibits a clearly structured and distinguishable distribution of attribute weights across 100 runs under decision matrices of different dimensions. Specifically, when the matrix dimension is relatively low, the distribution of attribute weights shows strong concentration and differentiation, effectively identifying key attributes. As the matrix dimension increases, although the overall weight values tend to converge, significant differences among attributes remain, indicating that the model can still effectively characterize attribute importance in more complex structures.

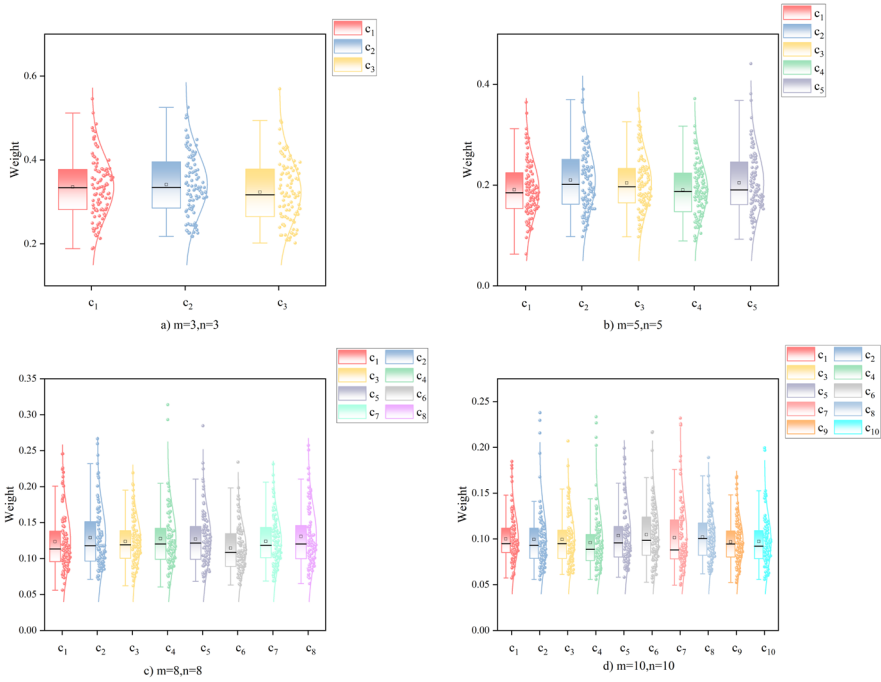


Fig. 4. Distribution of attributes' weights under decision matrices of different dimensions.

Therefore, the proposed attributes' weights optimization model not only maintains high performance stability under evaluation environments of varying scales and complexity, but also responds sensitively to changes in attribute structures and evaluation matrices. This demonstrates its effectiveness in addressing highly uncertain and structurally complex multi-attribute group decision-making problems.

4.6 Discussion on Practical Applicability and Advantages

In practical highway infrastructure decarbonization planning, decision-makers are often required to conduct comprehensive evaluations among multiple alternatives, such as different material options, construction technology pathways, or operation and maintenance strategies. Such decisions are typically confronted with insufficient quantitative data, expert judgments mainly expressed in linguistic terms, and significant cognitive heterogeneity among multiple participating stakeholders. Conventional multi-attribute decision-making methods usually rely on fixed weights and deterministic evaluation values to rank alternatives, which exhibit limitations in handling uncertain linguistic information, heterogeneous stakeholder influence, and interaction effects among evaluation attributes.

Under the above decision scenarios, the proposed decision framework mainly exhibits the following advantages in practical applications:

(1) The framework allows decision information to be expressed in linguistic form while simultaneously reflecting the credibility of judgments, thereby enabling the integration of multi-source subjective information without forcing precise quantification. This approach helps reduce bias risks caused by information simplification or over-deterministic processing, and is more consistent with the realistic characteristics of highway decarbonization decisions, where expert judgments are often inconsistent and information quality varies.

(2) By characterizing collaborative relationships among multiple stakeholders, the influence of decision-makers is no longer assumed to be static or equal, but can be dynamically adjusted in response to changes in collaboration structures and decision behaviors. This feature helps reflect the objective reality that stakeholder roles and influences vary across decision contexts, thereby improving the adaptability of group decision-making in complex collaborative environments.

(3) At the attribute evaluation level, conventional methods usually assume that evaluation attributes are mutually independent and that their weights remain fixed once determined, making it difficult to reflect interaction effects when attributes are used in combination. In contrast, the proposed framework focuses on changes in the roles of attributes under different combination structures, which helps identify critical attribute combinations that have a significant impact on ranking results, thereby enhancing the interpretability consistency and stability of evaluation outcomes across different decision contexts. This advantage is particularly important in decarbonization decisions, where attributes such as cost, technological maturity, emission reduction potential, and social impacts are often characterized by complex interdependencies.

Overall, compared with conventional multi-attribute decision-making methods, the proposed decision framework demonstrates stronger adaptability and interpretability in handling uncertain linguistic information, multi-stakeholder collaboration, and attribute synergy effects. Therefore, it is more suitable for real-world decision problems such as highway infrastructure decarbonization, which are characterized by high uncertainty, multiple stakeholders, and complex attribute structures, and can provide useful references for method selection and process design in practical planning and evaluation activities.

5 Policy and Practical Implications

In decision-making for the decarbonization of highway infrastructure, the decision environment is typically characterized by the involvement of multiple stakeholders, incomplete information, and significant cognitive heterogeneity. Focusing on key issues such as uncertain information expression, multi-stakeholder collaboration, attribute combination structures, and group cognitive interactions, this paper develops a systematic decision analysis framework and proposes the following three policy recommendations with practical significance. These recommendations are particularly relevant to key stakeholders in the highway infrastructure sector, including regulatory authorities

responsible for policy formulation, project owners and operators overseeing implementation, and technical support or evaluation institutions involved in assessment and coordination, as follows:

(1) Establish a decision information management mechanism that jointly considers judgment content and judgment credibility. In the evaluation and deliberation of highway infrastructure decarbonization strategies, different stakeholders often provide judgments based on their experience, positions, and professional backgrounds, and the reliability of these judgments may vary considerably. It is therefore recommended that practical decision-making processes not only record stakeholders' evaluations of alternatives and criteria, but also explicitly annotate and manage the credibility of these judgments, so as to avoid treating opinions with different confidence levels equally. Such a mechanism can enhance the transparency of multi-source qualitative information in integrated analysis and collective discussions, thereby reducing decision bias caused by insufficient information expression or misinterpretation.

(2) Construct a dynamic trust adjustment mechanism suitable for multi-stakeholder collaboration. Highway infrastructure decarbonization decisions usually involve collaboration among government authorities, construction and operation entities, and technical support institutions, and trust relationships among these stakeholders are often neither fully explicit nor permanently stable. It is suggested to introduce a dynamic trust assessment and updating mechanism in collaborative decision-making processes, which comprehensively considers historical collaboration performance, information-sharing quality, and current decision behavior, and periodically revises trust relationships among stakeholders. Based on the updated trust levels, the influence of each stakeholder in discussions, evaluations, or negotiations can be adjusted accordingly. This approach helps reduce efficiency losses caused by trust imbalance or information asymmetry, and enhances the stability of group decision-making.

(3) Implement a comprehensive evaluation and weight coordination mechanism that accounts for indicator synergy effects. In the evaluation of decarbonization strategies, different indicators are often interrelated, and their importance depends not only on individual performance but also on the underlying combination structure. It is therefore recommended to avoid fixing indicator weights as a one-time setting in practical decision-making. Instead, through multiple rounds of discussion and structured analysis, key indicator combinations and their synergy effects should be identified, and weight configurations should be coordinated and adjusted accordingly. Meanwhile, consistency constraints or review procedures should be established to prevent inconsistencies in evaluation logic caused by weight adjustments, thereby improving the interpretability and acceptability of decision outcomes.

6 Conclusion

To cope with incomplete trust structures and interaction effects among decision-makers in complex and uncertain decision environments, this paper develops a multi-attribute quantum group decision-making framework grounded in a trust network. The evalua-

tion information provided by individual decision-makers is first modeled using linguistic Z-numbers to jointly capture assessment values and their associated reliability. Subsequently, by incorporating superposition and interference mechanisms among multiple trust transmission paths, the trust network is inferred and completed within a quantum probability framework, from which decision-maker weights are derived via degree centrality. Meanwhile, an optimization model based on the Shapley value is formulated to obtain attribute weights, explicitly accounting for inter-attribute synergy and the consistency of ranking outcomes. Finally, quantum probabilities of alternatives are computed by considering interference effects among decision-makers, leading to the final decision results.

Although the proposed method demonstrates certain innovation and effectiveness, it still has the following limitations: 1) The computational complexity of the Shapley value increases significantly with the number of attributes, limiting the model's applicability in high-dimensional attribute spaces; 2) The current approach for measuring interference angles mainly relies on similarity-based heuristic algorithms, lacking a precise mapping to cognitive data.

To address these issues, future research can be carried out in two directions: 1) Design a Monte Carlo approximation algorithm for the Shapley value to reduce computational complexity and enhance the model's scalability; 2) Develop data-driven methods for measuring interference angles, integrating behavioral data to achieve adaptive identification of interference parameters, thereby improving the model's cognitive interpretability and predictive accuracy.

Acknowledgment

This work is funded by the Science and Technology Projects of the Transportation Industry of Xinjiang Uygur Autonomous Region: Research on Strategic Goals and Pathways for Carbon Peak and Carbon Neutrality in Highway Transportation (2021-ZD-001), the Science and Technology R&D Project of Xinjiang Transportation Investment (Group) Co., Ltd.: Research on Carbon Emission Monitoring and Control Technology during Highway Construction Period (2022-KT-15), and the 2023 Science and Technology Innovation Talent Team Project of Xinjiang Transportation Investment (Group) Co., Ltd.: Research on Carbon Emission Accounting Methods and Emission Reduction Paths for the Entire Lifecycle of Highway Infrastructure (2023-KT-22).

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