Limitations of CAPM and Empirical Analysis based on the Fama-French Three-Factor Model

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Abstract. This paper critically evaluates the inherent limitations of the Capital Asset Pricing Model (CAPM) and explores the improvements introduced by the Fama-French three-factor model. As a pioneering achievement in asset pricing theory, CAPM established a linear relationship between systematic risk and expected return. However, its explanatory power is constrained by several restrictive assumptions, including perfectly efficient markets, rational and risk-averse investors, and homogeneous expectations. Furthermore, the model relies solely on a single market beta coefficient. It overlooks the impact of other dimensions of systemic risk and thereby limits its empirical validity. An applied case study of Apple Inc. demonstrates that CAPM fails to capture firm-specific performance drivers such as technological innovation and investor sentiment, resulting in significant deviations between predicted and actual returns. The Fama-French model partially addresses these shortcomings by incorporating size (SMB) and value (HML) factors, providing an improved framework for explaining cross-sectional return differences. Nevertheless, this model retains flaws: its factors exhibit unstable correlations across markets and time periods, and it overlooks anomalies like momentum, profitability, and investment activity. These findings underscore that asset pricing models should be viewed as dynamically evolving tools rather than fixed formulas. Despite the significance of the three-factor model, it ultimately fails to resolve the complexity inherent in estimating expected returns.

Keywords: Fama-French Three-Factor Model, Capital Asset Pricing Model (CAPM), Size Effect (SMB).

1 Introduction

CAPM model was first developed by Willian Sharpe, Jack Treynor and other finance scholars in the 1960s [1]. It builds on the modern portfolio theory to further determine whether an asset is being valued fairly, helping investors make financial decisions by quantifying risk using returns. Based on the trade-off of risk and returns, CAPM model provides calculations for asset return given different market and risk-free rates, regardless of its simplified assumptions [2]. Inevitably, it became a crucial asset pricing

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model and was widely used by investors. However, CAPM were also facing significant drawbacks due to its narrow assumptions, which make it hard to adapt to real-world applications.

There are multiple assumptions of CAPM that lead to its limitations. It firstly assumes all investors are rational and risk-averse, which, in fact, many investors are irrational due to emotional and cognitive bias. Also, investors are assumed to have homogeneous expectations towards asset returns. It then assumes a linear relationship between risks and asset returns as well.

As a single-factor model, there are common drawbacks. It bears all the variation of the independent variable to only one single coefficient beta, which, here, means making the market portfolio to explain all the variation in asset returns. Since market anomalies are unavoidable, a single factor cannot include all the variations in the market, causing invalidity of the model empirically.

This study will provide an in-depth analysis of the limitations of CAPM through an empirical case study and document study. By using real-life examples to address weaknesses in specific circumstances, and provide mitigation methods towards those weaknesses, like using the Fama-French 3-factor Model.

2 Limitation of CAPM

2.1 CAPM Model

$$ER_{it} = R_{ft} + \beta_i\left(R_{Mt} - R_{ft}\right) \tag{1}$$

$$\beta = \frac{Cov(R_i,R_m)}{Var(R_m)} \tag{2}$$

Based on the CAPM model function above, ER_{it} is the expected returns of the asset R_{ft} represent the risk-free rate of return and R_{Mt} is the market portfolio return at time t. Therefore, $R_{Mt} - R_{ft}$ indicates the market risk premium [3]. β_i is calculated using the covariance of the given stock or portfolio returns and the market returns, divided by the variance of the market returns. It measures how the volatility of asset returns moves with the fluctuation in the market.

2.2 Theoretical Limitation

The main weaknesses of CAPM model lie in its oversimplified assumptions and overreliance on beta. The model assumes a perfectly frictionless market, meaning no taxes or transaction costs exist. It also assumes a single-period horizon and trades at the risk-free rate. Additionally, all investors have homogeneous expectations towards market risk and return. As a single-factor model, beta also plays a significant role in the function. Beta measures the systematic risks, given the assumption that the investor’s portfolio is fully diversified, which means the model will only care about the market risks. Thus, for a higher beta, the returns should compensate only for the systematic risks instead of the diversifiable non-systematic risks. Regarding a pool of assumptions, it sets a high bar for return calculation. If any of them fail, it will lead to problematic estimation for the risk-return trade-off. In short, the validity of beta only exists when those assumption stands within CAPM model.

2.3 Academic Critiques of CAPM

Over the period people adopted CAPM model, there have already been academic critiques about the validity of CAPM model. Roll's critiques about CAPM argue that the true market portfolio could not be observed, so the model is not testable unless all forms of wealth are included, like bonds, equities, real estate, etc. Then, when using a proxy for the market, the proxy should be truly efficient, but in fact, the real-world market is hardly efficient [4]. Thus, using an inefficient proxy may lead to incorrect testing for CAPM models. Other scholars also point out that, due to the restriction on the single-period horizon, it is impossible to capture the multi-period horizon for investors [5]. Furthermore, as beta is a measure for systematic risks, there are also criticisms about not capturing the upside or downside risks – investors tend to care more about downside volatilities of assets [6]. In summary, the academic consensus is that CAPM's beta-risk story relies on its restrictive assumptions: any deviation from those assumptions (incomplete markets, investor irrationality, multiple risk factors) undermines beta's theoretical sufficiency as the sole measure of risk.

2.4 Empirical Case Study of CAPM Limitations

The empirical validity of the Capital Asset Pricing Model (CAPM) can be evaluated by applying it to Apple Inc. (AAPL), one of the most widely traded equities in global markets. Using five years of daily trading data for Apple and the S&P 500 index, combined with the most recent 90-day U.S. Treasury bill yield as the proxy for the risk-free rate.

Daily returns were calculated for both Apple and the S&P 500. Apple's returns were measured as the percentage change from daily opening to closing prices, while the S&P 500 returns were computed on a close-to-close basis. Aligning these datasets ensured consistency in the estimation of systematic risk. The risk-free rate was set at 5.44 percent, corresponding to the most recent three-month Treasury bill yield, reflecting current market conditions.

The estimation of Apple's beta involved regressing Apple's daily returns on the returns of the S&P 500. Formally, beta is defined as the covariance of Apple's returns with those of the market, divided by the variance of the market's returns. The resulting coefficient, approximately 1.21, indicates that Apple exhibits about 21 percent more volatility than the market in terms of systematic risk. In other words, Apple's stock tends to amplify market movements, rising more in bull markets and falling more in bear markets, consistent with the interpretation of a high-beta security.

The average daily return of the S&P 500 during this period was approximately 0.035 percent, which annualizes to an expected market return of about 9.3 percent. Applying the CAPM formula, Apple's expected return is calculated as:

$$ER_{AAPL} = R_{ft} + \beta_i\left(R_{Mt} - R_{ft}\right) \quad (3)$$

$$ER_{AAPL} = 5.44\% + 1.21 \times (9.3\% - 5.44\%) \approx 10.1\% \quad (4)$$

The CAPM therefore predicts that, given its level of systematic risk, Apple's expected annual return should be approximately 10.1 percent. This discrepancy reflects the inability of a single-factor framework to fully capture the drivers of Apple's performance. Company-specific factors such as technological innovation, brand strength, market share expansion, and investor sentiment have contributed substantially to Apple's excess returns. These elements are treated as diversifiable risk under CAPM and thus not rewarded, yet empirically they have played a crucial role in shaping shareholder value.

3 Construction of the Fama-French Three-Factor Model

3.1 From Single Factor to Multi-Factor Model

Beta alone cannot explain all the factors that influenced cross-sectional differences of asset returns, found by Fama and French, who used to support CAPM model, now fighting against it [7]. They came up with an adjusted model including more factors that address other market anomalies, for instance, the size effect and the value effect of stocks are the two most significant factors other than market risks [8]. Technically, when adding more factors to the regression, the regression results tend to be more representative of the actual value [9]. This adjustment made the single-factor model into a multifactor model, enhancing the explainability of the model in estimating asset returns.

3.2 Construction of the Three-Factor Model and Additional Factors

$$R_{it} - R_{ft} = \alpha_{it} + \beta_1\left(R_{Mt} - R_{ft}\right) + \beta_2 SMB_t + \beta_3 HML_t + \epsilon_{it} \quad (5)$$

Fama-French 3-factor model is expressed as the function above, where $\beta_{1,2,3}$ represents three coefficients for three factors, which indicate the correlation of the change in the factor and the asset returns. R_{it} is the return of a stock or portfolio at time t, R_{ft} represents the risk-free return at time t and R_{Mt} is the market portfolio returns at time t. α_{it} means the excess return of the asset at time t [10]. Two additional factors are SMB (small minus big) – the size premium and HML (high minus low) – the value premium.

SMB factor measures the market size of the stock in the market. The model assumes that stocks with a smaller market cap tend to perform better in excess returns, whereas large market cap stocks will have relatively lower excess returns. In a practical perspective, the SMB monthly factor indicates the differences of average returns between the smallest 30% of stocks and the largest 30% of stocks in the market, shown in the formula below [11]:

$$MB = \frac{}{3}(Small\,Value + Small\,Neutral + Small\,Growth\,)$$
$$-\frac{1}{3}(Big\,Value + Big\,Neutral + Big\,Growth) \quad (6)$$

When SMB factor is positive, small-cap stocks' performance beats large-cap stocks' performance, measured by excess returns "alpha". When SMB factor is negative, the results are reversed. One example of SMB would be the SPAC boom period during 2020 and 2021 at which time the Russell 1000 (commonly recognized as the small-cap firm index) outperformed the S&5 500 (recognized as the large-cap firm index) by approximately 9.8% [12].

HML factor, similarly, represents the differences of excess returns in companies with high book-to-market ratio and those with low book-to-market ratio. Book-to-market ratio (B/M ratio) explains the growth potential of the stock. High B/M ratio represents value stocks, where they tend to pay more dividends instead of reinvesting for stock growth. However, with a low B/M ratio, it usually refers to a growth stock, where the company decides to reinvest all the potential dividends for long-term growth instead of paying out to shareholders. In this case, investors are willing to pay a premium price as they are anticipating the future returns generated by the stock. Thus, with a positive HML, value stocks will outperform growth stocks, but with a negative HML, growth stocks show a better performance. Generally, 50% of stocks with the highest B/M and the lowest B/M are used for HML calculation, given the calculation formula below [11]:

$$HML = \frac{1}{2}(Small\,Value + Big\,Value\,) - \frac{1}{2}(Small\,Growth + Big\,Growth) \quad (7)$$

By adding two factors to the original CAPM model, theoretically, it will provide a better explanation for the excess return of the asset. With multiple factors, more market anomalies would be captured.

3.3 Empirical Analysis

To better justify the theoretical conclusion of the Fama-French 3-factor model would be a better explanatory model than CAPM in terms of the accuracy of measuring asset returns, the same case study is used as the CAPM model but adding SMB and HML factors to the calculation.

Table 1. Fama-French Factors 2020-2024 [13]

Year	SMB (%)	HML (%)	Market Returns (%)
2020	13.48	-46.94	71.48
2021	-3.78	25.61	34.56
2022	-7.01	25.68	-25.67
2023	-3.54	-13.98	33.43
2024	-11.34	-8.71	35.80
Averages	-2.438	-3.668	29.92

The data above in Table 1 is the Fama-French 3 factors in the US security market from the CRSP database. As shown, the value of SMB and HML factors is -2.438% and -3.668%, which are both negative. It is because the general moving trend would be that the more the stock is a large cap, the more negative the SMB factor will be. Similarly, the more the stock has high growth, the more negative the HML factor will be. Therefore, a stock like Apple, a mega-cap and a growth stock, will have negative values for both SMB and HML factors.

In addition, the average value of SMB and HML will be used in the formula to neutralize the changes over the four years. After gathering annual return data from Yahoo Finance, the market return in percentage is calculated for measuring the relationship between each Fama-French factor and the market returns, which helps calculate the coefficient (beta) here in the formula, as shown in Table 2.

Table 2. Covariance Matrix for SMB, HML and Market Returns

	SMB	*HML*	*Market Returns*
SMB	89.14		
HML	-201.51	930.68	
Market Returns	213.32	-839.96	1221.48

Moreover, it is also necessary to determine the sensitivity of changes in SMB and HML factors to changes of the market returns, which is the beta value for each factor. Like the beta calculation in CAPM model, the covariance between market returns and SMB or HML is used, as well as the variance of SMB and HML The formula is shown below:

$$\beta_{SMB} = \frac{Cov(R_m,SMB)}{Var(SMB)} \tag{8}$$

$$\beta_{HML} = \frac{Cov(R_m,HML)}{Var(HML)} \tag{9}$$

Therefore, based on the covariance table provided above, β_{SMB} and β_{HML} would be 2.39 and -0.9 respectively:

$$\beta_{SMB} = \frac{Cov(R_m,SMB)}{Var(SMB)} = \frac{213.32}{89.14} \approx 2.39 \tag{10}$$

$$\beta_{HML} = \frac{Cov(R_m,HML)}{Var(HML)} = \frac{-839.96}{930.68} \approx -0.9 \tag{11}$$

The expected return for Apple would be calculated as:

$$R_{it} - R_{ft} = \alpha_{it} + \beta_1\left(R_{Mt} - R_{ft}\right) + \beta_2 SMB_t + \beta_3 HML_t + \epsilon_{it} \tag{12}$$

$$\begin{gathered} ER_{AAPL} = 5.44\% + 1.21 \times (9.3\% - 5.44\%) + 2.39 \times (-2.44\%) \\ -0.9 \times (-3.67\%) \approx 7.57\% \end{gathered} \tag{13}$$

Comparing with the expected return of CAPM model (10.1%), there was a slight difference in the results calculated from the Fama-French 3-Factor Model. The actual year-to-date return for Apple is approximately -6.95%. Even though both methods did not provide the exact value for expected returns, the Fama-French 3-factor model showed a closer result for the actual value. Therefore, based on the comparison, Fama-French model proved to be a better estimation in asset returns.

4 Discussion

The Fama-French three-factor model represents an essential improvement over CAPM. By taking size and value effects into account, it addresses the limitations of single-factor models. Relatively, the 3-factor model provides more accurate results than CAPM models, but neither can fully represent the real asset returns.

The key limitation lies in the assumption of universal applicability, as the same factors cannot consistently explain investment returns across different markets or periods. Empirical evidence indicates that anomalies shift across regions, time periods, and economic environments. In developed markets, the size premium may diminish, while liquidity or momentum risk may dominate in emerging markets. The value premium also varies with economic cycles and industry conditions. These discrepancies reveal the limitations of the model's static design.

Furthermore, the three-factor model did not concern about other well-documented anomalies like momentum, profitability, and investment activity. Consequently, there will be a significant portion of return variation unexplained. Even though the three-factor model is superior to the CAPM, it fails to address coverage gaps.

These shortcomings indicate that the three-factor model should be viewed as a flexible foundation instead of a definitive solution. More similar models have been developed by extending the number of variables. Extended versions like the Carhart four-factor model or the Fama-French five-factor model capture broader drivers of returns by increasing dimensions.

The Fama-French three-factor model is an advancement in asset pricing theory, but it is not the endpoint. It expands our understanding of expected returns, though it is not a conclusive measure. It helps researchers and practitioners adapt models to different markets and changes in financial systems.

5 Conclusion

The limitations of CAPM demonstrate the inadequacy of relying on a single-factor framework to explain the complexity of asset returns. Although the model remains foundational in finance, its restrictive assumptions and a single beta coefficient led to its weak explanatory power in real markets. The application to Apple Inc. further reveals the inability of CAPM to incorporate firm-specific and behavioral influences, underscoring the need for more comprehensive approaches.

The Fama-French three-factor model marks meaningful progress by including size and value effects, which reduces some of the gaps left by CAPM. However, its static design and limited scope mean that it cannot consistently capture anomalies across markets or economic cycles.

Thus, future research should pursue several more directions. First, models need to incorporate additional risk dimensions—including momentum, profitability, investment activity, and liquidity risk—to expand explanatory coverage. Second, regional and market-specific adaptations should also be prioritized, since factor relevance varies significantly between developed and emerging markets, as they have different industry conditions. Third, researchers should explore dynamic, time-varying factor structures that can better reflect shifting macroeconomic conditions and investor behavior. Finally, considering behavioral and technological drivers, such as investor sentiment and the influence of innovation in firms like Apple, may provide a more holistic understanding of expected returns.

In conclusion, while the Fama-French model improves upon CAPM, the future of asset pricing lies in flexible, multidimensional, and context-sensitive frameworks that evolve in step with global financial markets.

References

1. Perold, A.F.: The Capital Asset Pricing Model. Journal of Economic Perspectives 18(3), 3–24 (2004).
2. Vidal-García, J., Vidal, M.: A Review of Capital Asset Pricing Models in Emerging Markets. (2024).
3. Capital Asset Pricing Model (CAPM): Definition, Formula, and Assumptions. Investopedia. https://www.investopedia.com/terms/c/capm.asp, last accessed 2025/09/23.
4. Cordeiro da Cunha Araújo, R., André Veras Machado, M.: Book-to-Market Ratio, return on equity and Brazilian Stock Returns. RAUSP Management Journal 53(3), 324–344 (2018).
5. Liu, P.: Multi-Period Portfolio Allocation: A One-Shot Stochastic Optimization Approach. (2025).
6. Wang, T.: Bear Beta or Speculative Beta?—Reconciling the Evidence on Downside Risk Premium. Review of Finance 27(1), 325–367 (2022).
7. Fama, E.F., French, K.R.: The Cross-Section of Expected Stock Returns. The Journal of Finance 47(2), 427–465 (1992).
8. Xiao, B.: The Size Effect and the Value Effect in the American Stock Market. International Journal of Financial Research 14(1), 41 (2023).
9. Rutley, D.L., Banhazi, T.M., Pitchford, W.S.: Benefits of multi-factorial data modelling. In: Banhazi, T., Saunders, C. (eds.) CONFERENCE 2007, SEAg, vol. 1, pp. 63–68. Adelaide, Australia (2007).
10. Fama and French three factor model definition: Formula and interpretation. Investopedia. https://www.investopedia.com/terms/f/famaandfrenchthreefactormodel.asp, last accessed 2025/09/23.
11. Womack, K.L., Zhang, Y.: Understanding Risk and Return, the CAPM, and the Fama-French Three-Factor Model. (2003).
12. Small Caps Rebound Is Finally Coming - Time To Buy IWM. Seeking Alpha. https://seekingalpha.com/article/4716882-small-caps-rebound-is-finally-coming-time-to-buy-iwm, last accessed 2025/08/21.
13. Kenneth R. French - Data Library. Dartmouth.edu. https://mba.tuck.dartmouth.edu/pages/faculty/ken.french/data_library.html, last accessed 2025/09/23.