



How Modern Portfolio Theory Helps Investors Manage Risk in Volatile Markets

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Abstract. Value-at-Risk (VaR) is a widely used risk management tool in financial markets to quantify potential portfolio losses at a given confidence level. Traditional VaR estimation methods, such as Variance-Covariance, Historical Simulation, and Monte Carlo Simulation, are not efficient when markets exhibit fat tails, volatility clustering, or structural breaks leading to continuous evolution in VaR estimation. This study adopted a mixed-methods design, combining a critical literature review on the transformation in Modern Portfolio Theory (MPT) with quantitative empirical analysis implemented in R. The reviewed empirical studies demonstrate a shift in MPT modelling algorithms toward adaptive ML and hybrid models, which improve VaR estimation, highlighting the growing need to address dynamic market risks in the stock market. Empirically, the study investigates four national equity indices: S&P 500 (United States), FTSE 100 (United Kingdom), Shanghai Composite (China), and Nikkei 225 (Japan). Daily data from January 2022 to October 2025 were analyzed. Risk exposures were assessed using 95% VaR, CVaR, HVaR, and Monte Carlo CVaR. CVaR captured large tail risks of 2.0–3.1%. Monte Carlo CVaR produced far smaller values, ranging from approximately 0.03% to 0.06% losses. FTSE appeared most stable, while Nikkei 225 and S&P 500 were highly vulnerable. Shanghai Composite displayed episodic but severe volatility. Overall, the findings confirm that modern downside risk metrics and adaptive estimation techniques strengthen MPT frameworks, enabling investors to mitigate losses and allocate capital prudently.

Keywords: Modern Portfolio Theory, Value-at-Risk, Volatile Markets.

1 Introduction

Financial markets in the last five years have been marked by high levels of uncertainty and turbulence arising from global shocks such as the COVID-19 pandemic, inflationary pressures, supply-chain disruptions due to geopolitical conflicts, and emerging cryptocurrency markets such as Bitcoin. Risks in one market are transmitted to others due to the dynamic connectedness of stock markets. These increased market risks and volatilities make investment decisions more complex, thereby increasing the risk associated with investing in a single stock or sector. Volatility is used to define the size of return fluctuation over time. According to standard deviation estimates, Value at Risk

(VaR), or statistical model estimation such as Generalized Autoregressive Conditional Heteroskedasticity (GARCH) [1,2]. Volatility in the market has been one of the significant fields of concern in portfolio management under evolving market risks. Organizations are able to build immunity against unexpected financial shocks through the use of digital adaptive risk management assessment models and implementing strategic planning and market diversification. Markowitz provides Modern Portfolio Theory (MPT) which is one of the foundational theories that is still developing to enhance improvement in the determination of risk in the stock market [3].

According to MPT, the portfolio with the least risk in providing the same returns is known as the most optimal. Its diversification strategy entails combining assets with low or negative correlations to minimize risk of systematic losses [4]. MPT has continuously evolved. Scholars focus on improving estimation of the risk in volatile market environments [5, 6]. The MPT traditionally used standard deviation as a measure of total volatility. However, VaR is a more effective approach for estimating downside risk to allow investors to gauge potential losses over a given period.

Traditional standard deviation measures total volatility (both upside and downside). However, VaR focuses on the potential for losses, aligning with investor preferences for avoiding negative outcomes. Yet, traditional VaR estimates, such as weight regularization using lasso and ridge constraints, and tail risk measures, including Conditional Value at Risk (CVaR), are often sensitive to shocks, leading to concentrated, unstable, and inefficient weights (Boudt et al., 2022). To address this, competing VaR estimation algorithms such as Hierarchical Risk Parity have been suggested [7]. Recent empirical studies suggest robust optimization approaches such as non-linear shrinkage estimators, Bayesian models like Black-Litterman [5], regime-switching models [8], and incorporating hybrid deep learning models such as XGBoost [9] to obtain stable and more reliable risk-adjusted returns. The advanced frameworks respond to time-varying volatility and structural market breaks, hence improving risk evaluations.

The directions of research demonstrate that while the efficient frontier remains the guiding principle, the operationalization of MPT is evolving to handle non-stationarity and extreme events. Apart from an extensive literature review on recent innovations in MPT, this study conducts an empirical investigation of four representative national equity indices: the S&P 500 for the United States, the FTSE 100 for the United Kingdom, the Shanghai Composite (000001.SS) for China, and the Nikkei 225 for Japan. The study period, spanning January 2022 to October 2025, captures significant market volatility and recovery episodes. National indices are used as representatives of general market trends. The reviewed empirical studies further demonstrated that investors can leverage machine learning and hybrid models to improve VaR estimations. The shift in MPT modelling algorithms toward adaptive systems reflects the growing need to capture dynamic market risks. The empirical results show that different indices carry distinct risk exposures, with N225 and GSPC exhibiting the greatest susceptibility to extreme downside losses, while FTSE remains the most stable. Such evidence underscores the limitations of traditional optimization that relies heavily on unstable return and covariance estimates, often leading to extreme portfolio weights. The results of downside-oriented metrics such as VaR, CVaR, Historical VaR, and Monte Carlo CVaR provide a more reliable assessment of extreme loss risks in diversified portfolios.

Generally, the research suggests that investors should cautiously make optimal capital allocation by balancing return and volatility using innovations in MPT modelling.

2 Literature Review

Empirical investigations of mean–variance portfolio implementations over the last decade have emphasized the fragility of the naïve M–V optimization and have evaluated remedies that improve risk minimization in stressed markets. The VaR has long been a widely applied downside risk measure in finance, capturing the potential extreme downside risk percentage loss under adverse market conditions. However, VaR has important theoretical limitations, particularly its failure to satisfy the subadditivity property required of coherent risk measures. Empirically, Panda and Deb show that VaR models are unable to capture significant downside volatility risks of alternative investment funds and recommend the adoption of more effective risk management strategies [10]. To address this, other innovations such as Historical VaR (HVaR) and Monte Carlo VaR (MCVaR) methods, and CVaR, also known as Expected Shortfall minimization, have been introduced as improved tail risk statistics. The innovations offer superior methodological advantages compared to the traditional standard VaR.

The HVaR uses actual historical return distributions, capturing real market patterns without assuming normality. Conversely, the MCVaR simulates a large number of possible scenarios, allowing flexibility in modelling non-linear risks and complex return dynamics. The CVaR quantifies the average loss of the worst-case scenarios above a specified threshold and efficiently evaluates volatile assets by accounting for extreme losses; hence, it is widely used in portfolio management, finance, and machine learning for optimization and risk assessment. Studies are also exploring different VaR-based optimization variants that could be effective when modeling return distributions under extreme rare market conditions, such as the Covid-19 pandemic, geopolitical conflicts, and negative news.

Kauhanen compared HVaR and MCVaR in modelling the volatility of the S&P 500, STOXX Europe 600, and MSCI Emerging Markets indices during the COVID-19 pandemic [6]. The research determined that the standard VaR could not detect the 2020 risk better than volatility, whereas fewer violations as well as stress test stability were achieved by HVaR and MCVaR. Hence, in crisis-driven scenarios that are non-compliant with standard regulations, VaR underestimates risk. Simulated applications of VaR, however, quantify downside risk a great deal more sturdily. In the same vein, empirical evidence in Panda and Deb attests to such a critique through VaR's failure to estimate tail risks in alternative investment funds, where CVaR stands as a more reliable measure [10]. The study that we discussed says that most of them have come to the same conclusion which is: All VaR and even CvaR versions can be used as a good tool of risk management even in times of volatile markets.

Increasingly, researchers are applying robust estimation techniques to reduce parameter uncertainty and construct more robust portfolios. Nocera modified the Black–Litterman model by deriving prior implied returns without imposing any CAPM assumptions and benchmarks [11]. The resulting portfolios were more flexible due to the

Bayesian methodology used in the analysis and also exhibited improved Sharpe ratios. Marakbi used the sample covariance with the Gerber Statistic estimator that resulted in tighter risk-adjusted returns and drawdowns, especially during recessionary periods [12]. Batrancea et al. took the mean–variance objective to formulate the Distributional Robust Multivariate Stochastic Cone Order model, which uses the Wasserstein ambiguity sets for modeling multivariate tail risk [13]. A tested strategy employed in Borsa Istanbul equities improves their respective Sharpe ratios and reduces drawdowns against aggressive benchmarks. Huang et al. suggested a hybrid Tail Mean–Variance approach suggested that combines plug-in estimation with the $1/N$ rule and their approach was found to provide better downside protection than standard plug-in approaches [14]. Bui and Wu found that robust mean–variance optimization using genetic algorithms outperformed global indices by producing higher returns and stability with shorter computation time [15].

Minimum-risk portfolios also prevailed in the diversification-focused strategies in recent studies. Lindell and Hultqvist (2025) proved that minimum-risk portfolios outperformed risk parity/equally weighted allocations in Sweden due to having higher Sharpe ratio and Sortino ratio for the minimum-risk portfolio [16]. Deković and Šimović state that the use of cluster gains of portfolios with S&P 500 data results in lower-volatility portfolios due to hierarchical risk parity [17]. In this case, weight assignment depends on the utilization of the clusters. In that sense, use of the simple $1/N$ rule sometimes works better on a risk-adjusted return basis. Generally, these studies show the variety of robust estimation and allocation methods developed in order to maximize the portfolio efficiency under uncertainty.

Volatility was also estimated by recent research based on ML and combined models. Combination of Black-Litterman model, CEEMDAN-GRU and XGBoost performed better than the Markowitz, minimum-variance, equally-weighted and risk-parity approaches in return forecasting of ten-country exchange-traded funds based on one study by Barua and Sharma [9]. Black litterman captures investor sentiment; cepidangan engineered and GRU non-linear dynamic system; XGBoost model time-varying volatility (XGBoost). This coming together generates improved risk-adjusted return predictions than all other approaches. Bui and Wu also point out that Deep Reinforcement Learning (DRL) is more flexible and adaptive to learn than the less flexible adaptive dynamic of the MPT approaches like Maximum Sharpe and Minimum Variance [15].

After the same treatment, Wysocki and Sakowski established deep learning algorithms to covariance matrix estimation with the LSTM recurrent neural networks [18]. They have applied the probabilistic models DeepVAR and GPVAR for multivariate forecasting a day ahead. The LSTM-RNNs, based on the information ratios and the annualized returns, gave the most promising results. It suggests that deep learning models are not only able to capture long-term dependencies but also bring about better results. Therefore, they increase the efficacy of portfolio covariances and variance in sophisticated markets such as stocks and cryptocurrencies. Collectively, the reports show how machine and deep learning techniques are increasing empirical portfolio optimization and reacting to dynamic but unstable and uncertain market performance such as with stocks or crypto.

The literature shows a clear shift from traditional mean–variance optimization (MVO) toward more robust, data-driven, and hybrid frameworks. While MVO established the foundation for balancing risk and return, its dependence on unstable mean and covariance estimates often produced extreme weights. Estimation refinements such as shrinkage, factor models, and covariance regularization improved stability and reduced turnover. In parallel, downside-aware measures VaR, CVaR, HVaR, and MCVaR (emerged to better capture fat-tailed risks, while alternative strategies like risk parity and diversification-ratio maximization emphasized structural diversification over return forecasts).

More recent advances focus on regime-aware and machine-learning hybrids. Time-varying volatility models (e.g., GARCH, FIGARCH) allow portfolios to adapt to changing conditions, while ML tools such as XGBoost, deep networks, and ensembles enhance predictive power for returns and risk. Hybrid frameworks that embed ML forecasts within equilibrium models (e.g., Black–Litterman with CEEMDAN-GRU or XGBoost inputs) have outperformed traditional Markowitz and risk-parity allocations. Overall, portfolio research has evolved from rigid optimization to adaptive and predictive systems that strengthen risk control and return potential.

3 Data and Methods

3.1 Research Design

This study employed a mixed-methods approach, comprising a critical literature review with quantitative empirical analysis implemented in R. A critical literature review of studies was conducted to capture methodological diversity and the evolution of MPT. Innovative empirical models reflecting how portfolio optimization has evolved in response to market complexity and uncertainty, and their usefulness are deduced. Only peer-reviewed journal articles and authoritative textbooks were considered. The period of coverage was not restricted to reflect the trajectory of portfolio optimization over the past two decades. The review provided the conceptual foundation for understanding the evolution of portfolio optimization, whereas the empirical component illustrates widely used risk metrics across major global equity indices.

3.2 Data Sources and Preparation

Daily adjusted closing prices were collected from Yahoo Finance using the ``quantmod`` package for four representative equity markets: the S&P 500 (`^GSPC`), the FTSE 100 (`^FTSE`), the Shanghai Composite (000001.SS), and the Nikkei 225 (`^N225`) [19]. The study window covered January 1, 2022, to October 1, 2025, to capture recent market trends. Adjusted prices were merged into a single dataset and converted into continuously compounded log returns. Missing values were imputed using the last observation carried forward (LOCF) imputation technique.

3.3 Risk Metrics and Estimation Procedures

To minimize complexity, empirical analysis focused on comparing traditional and robust downside-aware risk measures. VaR represented the baseline, computed using the historical quantile method at the 95% confidence level. The CvaR/Expected Shortfall was estimated as the mean of returns below the historical VaR threshold. To contrast parametric assumptions, Historical VaR under the Gaussian distribution (HVaR) was also calculated. Finally, MC-CvaR was implemented by resampling return series through bootstrap simulations to approximate tail risk. Line charts of log returns were used to visualize the temporal evolution of asset performance, capturing periods of high and low volatility. Complementary bar plots displayed the estimated risk measures (VaR, CVaR, HVaR, MC-CVaR) side by side, providing a clear comparison of traditional and robust downside-risk assessments. The analysis and visualizations were implemented in R using packages such as `'quantmod'`, `'Performance Analytics'`, `'ggplot2'`, `VaR ()` and `ES ()` functions [19].

4 Empirical Results

4.1 Log-return Trends

Figure 1 shows the daily log-return trends of the national stock indices from January 2022 to July 2025, highlighting patterns of volatility and market movements over the period.

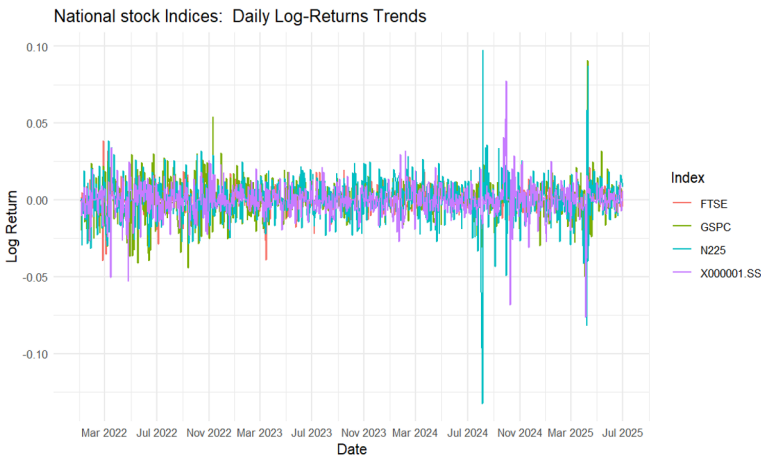


Fig. 1. National stock Indices: Daily Log>Returns Trends from January 2022 to July 2025.

4.2 Risk Metrics

Table 1 presents the descriptive and risk metrics for the four indices, including mean, standard deviation, skewness, kurtosis, and downside risk measures, computed at the 95% confidence level to denote the potential loss that is expected to be exceeded only

5% of the time. Figure 2 illustrates the comparative 95% VaR and CVaR for the four indices.

Table 1. Descriptive Statistics and Downside Risk Measures

Index	Mean	SD	Skewness	Kurtosis	VaR 95	CVaR 95	HVaR 95	MC CVaR 95
GSPC	0.0003	0.0115	0.0854	9.3295	-0.0179	-0.0270	-0.0186	-0.0004
FTSE	0.0002	0.0081	-0.7750	8.5780	-0.0127	-0.0207	-0.0132	-0.0003
X000001.SS	-0.0001	0.0102	-0.3609	14.1758	-0.0149	-0.0241	-0.0168	-0.0006
N225	0.0004	0.0136	-0.7530	19.1916	-0.0191	-0.0309	-0.0220	-0.0003

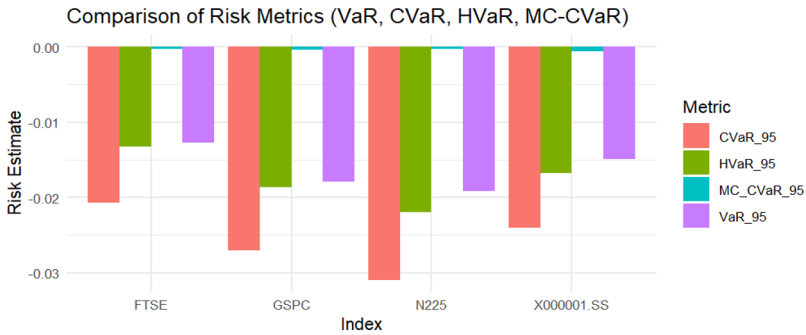


Fig. 2. Comparison of 95% VaR, CvaR, HvaR, and MC-CVAAr

The four indices exhibit distinct risk profiles according to both descriptive and downside risk metrics. N225 (Nikkei 225) emerges as the riskiest market, with the highest volatility (SD = 0.0136), largest VaR_95 (-0.01914) and CVaR_95 (-0.03093), pronounced negative skewness (-0.7530), and extremely high kurtosis (19.19). These measures indicate frequent moderate gains but occasional extreme losses, making N225 highly susceptible to severe downside events. GSPC (S&P 500) follows closely, showing substantial tail risk (VaR_95 = -0.01791, CVaR_95 = -0.02701), moderate volatility (SD = 0.0115), slightly positive skew (0.0854), and high kurtosis (9.33), reflecting relatively symmetric returns but fat-tailed behavior. X000001.SS (Shanghai Composite) presents moderate risk, with lower volatility (SD = 0.0102) and smaller VaR_95 (-0.01486) and CVaR_95 (-0.02408), though its high kurtosis (14.18) and negative skew (-0.361) suggest occasional extreme losses.

FTSE, in contrast, is the least risky among the four indices. It exhibits the lowest volatility (SD = 0.0081), smallest extreme losses (VaR_95 = -0.01271, CVaR_95 = -0.02071), and moderately high negative skew (-0.775) with kurtosis of 8.58. This indicates that while downside risk is somewhat asymmetric, potential losses are less severe compared to the other markets. Across all indices, CVaR_95 consistently exceeds VaR_95, capturing more extreme losses, and MC_CVaR_95 provides minor variations reflecting simulation-based tail risks, demonstrating that multiple metrics are needed for a comprehensive risk assessment.

The comparison of risk measures across the four indices reveals that the selection of metrics yields consistent asset ranks based on risks, albeit with varying degrees of risk

levels. Traditional VaR and its parametric version (HVaR) are broadly consistent, indicating moderate losses ranging from -1.3% to -1.9% across the indices, with the Nikkei (N225) showing the highest downside risk. The CVaR estimates come out with a higher risk of losses, capturing tail risk of $2.0\text{--}3.1\%$. Hence, they give a more negative picture. Monte Carlo CVaR gives extremely tiny values, showing a much smaller risk compared to the other two methods. This disparity means that traditional and parametric methods are equivalent though CVaR is a legitimate extension that captures fat-tailed losses. The Monte Carlo bootstrap is off the line with the rest, and indications of true downside exposures will be less precise. Overall, one can see from the above that the risk measures are highly sensitive to which measure one employs, highlighting the choice of technique in portfolio risk measurement.

Market choice according to an investment perspective is risk-level contingent. Risk-averse investors prioritizing stability and lower potential losses would likely prefer FTSE, which offers the most consistent exposure. Those willing to accept higher volatility for greater potential returns might consider GSPC or N225, despite their larger downside risk. X000001.SS occupies an intermediate position, providing moderate risk with occasional tail events. Overall, integrating both descriptive statistics and downside measures provides a more complete view of market behavior, highlighting that FTSE is the safest, N225 the riskiest, and GSPC and Shanghai offer moderate alternatives depending on investor preferences.

4.3 Portfolio Specification

A portfolio specification was created using the Portfolio Analytics framework, incorporating constraints such as full investment, long-only positions, and box limits to prevent over-concentration in a single asset. The optimization objectives included maximizing expected return while minimizing overall volatility and tail risk (CVaR at 95% confidence level), thus reflecting a hybrid, downside-aware allocation strategy. DEoptim was employed as a global optimization algorithm to efficiently explore the solution space and identify weights that satisfy all constraints while optimizing the defined risk-return criteria.

The optimized portfolio weights for the four selected stock indices, obtained using a downside-aware portfolio optimization approach, are summarized in Table 2.

Table 2. Optimized Portfolio Weights for Selected National Stock Indices

Index	GSPC	FTSE	X000001.SS	N225
Weight	0.350	0.496	0.074	0.080

The optimized portfolio resulting from this procedure allocated 35% to GSPC, 49.6% to FTSE, 7.4% to Shanghai, and 8% to N225. Based on the descriptive statistics, the FTSE and GSPC exhibited lower volatility based on SD and moderate tail risk based on skewness than X000001.SS and N225 (See Table 2). Thus, FTSE and GSPC get the highest weights since they are less risky. Conversely, X000001.SS and N225 showed higher potential losses based on mean and fatter tails, hence, they get low weights. While FTSE (Mean = 0.0002) had a lower mean return than GSPC (Mean = 0.0003),

and is perceived as less risky ($SD = 0.0081$) than GSPC ($SD = 0.012$), it is worth noting that these differences are relatively minor. As a result, an investor should apportion a higher proportion of investments to FTSE (49.6%) than GSPC (35%). Besides, X000001.SS gets a lower allocation (7.4%) than N225 (8.0%) since it is less risky ($SD = 0.0102$) than N225 ($SD = 0.0136$) despite recording an average loss over the study period (Mean = -0.001) than N225, which recorded average positive returns (Mean = 0.0004). Thus, the allocation helps cushion investors from mistreating gains without regard for losses. Generally, model assistant portfolio allocations in diversification help investors mitigate or avoid extreme losses.

4.4 Performance Evaluation

The summary statistics of the optimized portfolio weighted using the results in Table 3 are summarized in Table 3 alongside the performance metrics comprising the Sharpe ratio, and 95% VaR & CVaR.

Table 3. Performance Metrics of the Optimized Portfolio

Metric	Value
Mean Return	0.0002
Standard Deviation	0.0071
Annualized Sharpe Ratio (Rf = 0%)	0.3687
VaR (95%)	-0.0114
CVaR (95%)	-0.0175

Generally, the diversified and optimized portfolio achieves a positive mean return with lower volatility than all the individual indices. The annualized Sharpe ratio is positive (0.369), suggesting that the weighted portfolio earned 0.3687 units of excess return (over the risk-free rate of 0%) for each unit of total volatility [20]. The VaR of 1.14% indicates the maximum expected loss on the diversified portfolio in one day will not exceed 1.14% of the portfolio value, with only a 5% chance of experiencing worse losses. The 95% CvaR of -0.0175 means that if losses exceed the VaR threshold, the average expected loss in the worst 5% of cases would be about 1.75% of portfolio value. The results show that weighting a portfolio yields lower volatility and minimizes exposure to the potential extreme losses.

4.5 Simulated Portfolio Risk-Return Distribution with Optimized Portfolio

Figure 3 is a scatter plot of 5,000 randomly simulated portfolio allocations constructed from the four selected indices: GSPC, FTSE, Shanghai Composite, and N225. Each point represents a portfolio combination, with the x-axis showing portfolio standard deviation (risk) and the y-axis showing expected return. The color gradient reflects the Sharpe ratio, where red indicates higher risk-adjusted performance and blue indicates lower.

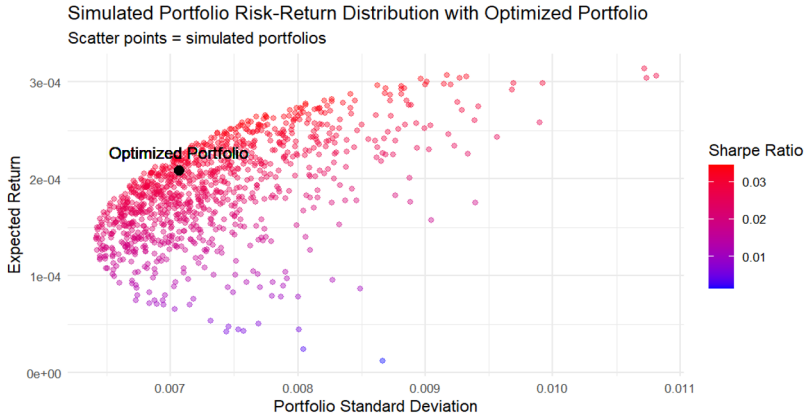


Fig. 3. Simulated Portfolio Risk-Return Distribution with Optimized Portfolio

The plot shows a positive risk-return relationship, with higher expected returns generally associated with higher volatility. Most portfolios cluster around the optimal positive expected returns (Mean = 0.000208, SD = 0.00707), indicating that diversification across the four indices effectively reduces extreme risk. Thus, the diversification achieves a favorable balance of relatively moderate risk with higher expected return compared to the majority of random portfolios. The high Sharpe ratio of the optimal portfolio also suggests that this allocation is efficient in terms of risk-adjusted performance.

5 Conclusion

This study employed a mixed-methods approach, combining a critical literature review and empirical analysis, to examine how Modern Portfolio Theory supports investors in managing risk in volatile markets. The empirical results show that different indices carry distinct risk exposures, with N225 and GSPC exhibiting the greatest susceptibility to extreme downside losses, while FTSE remains the most stable. Such evidence underscores the limitations of traditional optimization that relies heavily on unstable return and covariance estimates, often leading to extreme portfolio weights. The results of the downside-oriented metrics such as VaR, CVaR, Historical VaR, and Monte Carlo CVaR provide an assessment of extreme loss risks of a diversified portfolio. The reviewed empirical studies demonstrated that investors can leverage ML and hybrid models to improve VaR estimations. The shift in MPT modelling algorithms toward adaptive systems shows the growing need to address dynamic market risks in the stock market.

This study provides meaningful insights for portfolio management practice. Investors should cautiously balance risk by balancing return and volatility to mitigate losses. The innovation in MPT modelling helps investors reduce exposure to sharp market declines and allocate capital more cautiously across assets.

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