Risk Measurement of Sovereign Wealth Funds: The Significant Advantages of ES over VaR

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Abstract. The paper focuses on optimizing the risk assessment system of Sovereign Wealth Funds (SWFs) as the core research direction, with an emphasis on the different applications of Value at Risk (VaR) and Expected Shortfall (ES). SWFs with long-term investment undertake economic and diplomatic goals. It still faces multiple risks. Risk management is of significance. By contracting the theories between ES and VaR, the paper find that VaR only estimates the loss threshold at a given confidence level. VaR's limitations include ignoring the extreme losses scale, lacking subadditivity, and being unreliable under market pressure. ES can overcome the limitations to reflect extreme risk more comprehensively, and the optimization results are more stable. Moreover, the article sets a framework for the application in SWFs to indicate that ES is superior to VaR, considering the debt crisis, fragmented world, and other extreme scenarios. The paper also shows some weaknesses of ES to improve the risk assessment method in SWEs.

Keywords: Sovereign Wealth Funds (SWFs), Value at Risk (VaR), Expected Shortfall (ES).

1 Introduction

1.1 Research Background

Sovereign Wealth Funds (SWFs) adhere to a long-term investment-oriented philosophy. They are the key tool for achieving the microeconomic goal (e.g., long-term wealth reserve) and diplomatic strategy (e.g., enhancing the national image). Moreover, the investment of SWFs exhibits the large scale, diversification and high risks. It is significant for SWFs to manage risks[1]. Before 2011, the Basel Committee on Banking Supervision adopted Value at Risk VaR as the leading risk assessment indicator. However, in 2001, the Basel Committee on Banking Supervision indicated that VaR is insufficient in assessing the portfolios' tail risk and liquidity risk, and VaR is non-subadditive. Basel III indicates that Expected Shortfall (ES) can serve as a key risk assessment indicator to replace VaR. While addressing the shortcomings of VaR, ES can leverage its own advantages[2]. By now, most research concentrates on ES applied to banks and investment organizations, with a lack of research for SWFs.

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1.2 Research Content

Based on the important role of SWFs in the global macroeconomy and the lack of research on SWFs by ES, the research focuses on SWF risk assessment. It mainly involves studying the literature and comparing the principles and drawbacks of VaR and ES. By drawing on the Government Pension Fund Global (GFPG) stress test, the study shows that ES is superior to VaR in SWF risk assessment.

1.3 Research Significance

Through the exploration of the principles of ES and VaR, the study finds that VaR only focuses on the probability of a certain threshold, and ES quantifies the expected value of losses that exceed the VaR threshold. Using ES SWFs can better capture unpredictable tail risks such as extreme climate events and financial crises. From the derivation processes of VaR and ES, the study find that VaR is non- subadditive. However, ES can make up for the weakness. ES can reflect the average reduction in the extreme losses of portfolios after adding a certain type of asset. It provides a scientific basis for long-term asset allocation across markets and asset classes, which is significant for SWFs.

2 Literature Review

2.1 The Risk Structure

As an important management tool for national wealth, SWFs always carriy multiple risks such as liquidity risk, currency risk, credit risk and exchange risk. All of the risks are caused by the market environment in which sovereign funds engage in cross-border investment (such as macroeconomic fluctuations and monetary shocks)[3]. Besides, in the context of deepening globalization and financial integration, the international finance landscape is undergoing rapid changes. The frequency and scope of extreme market events (e.g., financial crisis and geopolitical conflict) are on the rise. Only sovereign funds that establish a comprehensive and forward-looking risk management system, especially those that identify, assess and predict risks effectively, can avoid unhedged risk exposure and maintain the stability of the investment portfolio value. It provides strong protection for national finance and foreign exchange reserves and avoids chain reactions.

In 2016, Public Investment Fund (PIF) invested 45 billion in SoftBank Vision Fund, accounting for 45% of the total size of SoftBank Vision Fund. PIF has become the largest limited partner of SoftBank Vision Fund. However, SoftBank Vision Fund invests in many of American technology unicorn companies (e.g., Uber and WeWork). The companies are affected by stock market volatility, economic uncertainty, and regulatory pressure, leading to a significant drop in their valuations. It hinders the financial performance of SoftBank Vision Fund. Besides, the valuation correction of U.S. tech stocks and overall downward pressure on the market have imposed a heavy burden on the tech companies held by SoftBank Vision Fund. This has caused Silver Vision

Fund's investment losses to continue to expand, and the stock price of SoftBank Group has declined[4]. It leads to PIF's loss. PIF's 2022 financial statement showed an overall loss of USD 14.72 billion[5].

2.2 The Principle and Limitations of VaR

Principle. In the field of financial risk management, VaR is a crucial tool. Given the confidence levelα and holding periodτ, VaR assess the potential loss X in value of a portfolio. There are three common methods for calculating VaR: Historical Simulation Method, Variance-Covariance Approach and Monte-Carlo Simulation.

Historical Simulation Method is a straightforward and practical approach to risk assessment. It relies on historical return data, which is valuable information for understanding how portfolios performed in the past different market scenarios. The data is utilized to simulate the possible future return distribution. By organizing the simulated returns, people can identify the quantile corresponding to the confidence level(α). The quantile represents VaR.

Variance-Covariance Approach assumes that the asset returns follow a specific distribution like the normal distribution. The first step is to calculate the mean, variance, and covariance of the asset returns. After calculations, the method combines them with the quantile corresponding to the confidence levelα[6].

$$VaR_{\alpha}(X,\tau) = inf\{x: P(X < -x) \le \alpha\} \tag{1}$$

Monte-Carlo Simulation is a complex but flexible approach. It starts with calibrating the parameters of a stochastic process and considering the randomness of market fluctuations. Monte-Carlo Simulation relies on quantities of historical data. Based on the data, it can calibrate the parameters of the stochastic process. Then, it can generate numerous future asset paths using the random processes. Each path represents possible situations of the portfolio value during the holding period. After that, one can analyze the loss distributions constructed from the losses calculated for each generated asset paths. Lastly, ones can get the quantile corresponding to the confidence level α from the loss distributions. The quantile is the VaR.

The Limitations of VaR. While there are lots of methods to calculate VaR, no one can ignore the fact that VaR only focuses on the probability of losses exceeding a certain threshold. It ignores the loss scales, which makes it difficult for VaR to capture the tail effect. It results in an underestimation of potential risks[7].

Most importantly, VaR is non-subadditive. The VaR of a portfolio is greater than the sum of the individual VaRs of the assets in that portfolio. It cannot reflect the real risk condition after diversifying the investment portfolio[8].

VaR is unreliable in an extremely volatile market environment. When prices significantly fluctuate, the risk condition may be underestimated[7]. During the COVID-19 period, the stock market sharply declined. Due to pubic panic selling, the key global indicators like Dow Jones Industrial Average (DJIA) declined dramatically in a few weeks. VaR is useless in the sudden market crash.

The minimization of VaR may involve multiple local minima, resulting in unstable optimization[9].

ES Make Up For the Shortcomings of VaR. ES is subadditive, meaning it is a coherence-satisfying measure. The ES of the portfolio does not exceed the sum of its sub-portfolios. It can show the risk-reducing effect of capital dispersion[8]. SWFs tend to have a wide-ranging investment portfolio spread across different asset classes, regions and industries. When a SWF diversifies the investments into stable domestic bonds and high-growth emerging market stocks, the ES value of the portfolio is lower than the simple sum of the ES values of the bond sub-portfolio and stock sub-portfolio. It shows that ES can effectively demonstrate how a SWF mitigates overall risk through a cross-asset diversification strategy. It can more accurately assess the benefits of diversification.

Both the threshold and the average loss value surpassing the threshold are considered in ES. It can more comprehensively reflect extreme risks.9Geopolitical conflicts, global economic crises, large-scale commodity price fluctuations, and other extreme events have huge impacts on SWFs. ES considers both the threshold and the average loss exceeding the threshold. When SWFs have considerable exposure in international real estate and stock markets, and a global economic recession leads to extreme losses in the markets, ES provides a comprehensive measure of the extent of losses beyond a threshold. It helps risk managers learn more about the potential impacts of extreme risks to make more proactive and comprehensive strategies.

The minimization problem of ES is a convex optimization problem under reasonable assumptions, and it usually has a unique global optimal solution[10]. When SWFs want to achieve specific return targets and minimize risks, the convexity of ES optimization makes sure that SWFs can find the unique global optimum. For example, when a SWF rebalances across different assets like government bonds, foreign stocks and alternative investments, taking ES into optimization flow can more effectively find optimal asset allocation. It helps SWFs align investment strategies with long-term sustainability and risk management objectives. Especially in a complex global financial environment, SWFs can make wiser and better decisions.

3 ES Application Framework

3.1 Risk Quantification

ES indicates the average loss exceeding VaR, given a confidence level α[6].

$$ES_\alpha(X,\tau) = \frac{1}{\alpha}\int_0^\alpha VaR_m(X,\tau)\,dm \tag{2}$$

$$ES_\alpha(X,\tau) \geq VaR_\alpha(X,\tau) \tag{3}$$

Based on the formulas, NBIM mainly adopts historical simulation method and Monte Carlo Simulation Method to calculate ES_α. Historical simulation method simulates the performance of investment portfolio under past situations by collecting historical market data that expresses asset price fluctuation in different conditions. Then calculate ES_α. Monte Carlo Simulation Method sets random process and parameters based on historical data. Then, lots of random sampling simulations will be carried out to generate multiple market scenarios[11]. Finally, calculate ES_α. Besides, NBIM considers debt crisis, fragmented world and other systemic risks into calculations.

3.2 Scenario Design

The paper designs the scenario according to the 2024 Stress Testing published by NBIM.

Debt crisis: Aging populations, climate change, international conflict and expansionary fiscal policy increase debt risk. The shortage of funds and the political uncertainty lead to bond selling and term premium and liquidity premium rises. This results in a shock to the corporate bond market and financial intermediaries. The rise in mortgage rates, the suppression of consumption and investment by credit tightening, the decline in corporate expected cash flows, and political uncertainty push up equity risk premiums. It forms a vicious cycle of debt crisis.

AI correction: Because markets have a high expectation of future revenue growth, the estimated value in AI industries increases rapidly. Inappropriate regulatory intervention, technical barriers, shortage of supply, and other negative factors may prevent investment in AI from gaining return. The expectation of revenue growth may fall back to the level before the boom in AI. This probably causes expected a permanent downturn in cash flows and a rise in equity premium. However, due to the rise in demand for government bonds, the term premium declines.

Fragmented world: It assumes that the world has been divided into multiple dependent economic blocks. Deterioration of the geopolitical environment causes an increase in tariffs, enhancement of regulation and strict restrictions on foreign investment. Uncertainty and economic cooperation the reduction slow down global economic. It exacerbates market volatility and expected cash flows permanently decline. Moreover, the increase in nearshoring and friendshoring pushes up inflation in major economies[12].

3.3 Innovation Point

Taking debt crisis, fragmented world and other extreme conditions into evaluation to extreme risks can improve the assessment of the tail risk. This breaks through the limitation to Var thresholds. The asset allocation of SWFs spans the entire globe. Taking debt crisis, fragmented world into account helps SWFs accurately identify high-risk exposure so that they can mitigate extreme impacts by adjusting asset duration and adding hedging tools (e.g., currency swaps). It can avoid the passive situation where the VaR threshold is not triggered.

ES combines the practical asset allocation of a fund with macro-geopolitical scenarios. This satisfies the axioms of coherent risk measures and supports the adequate diversification of portfolio risk. The investment portfolios of SWFs always cover diverse global asset classes and complex industry ecosystems. Its risk structure is highly correlated and non-linear. ES can objectively reflect the effects of risk hedging and diversification in cross-asset and cross-reginal allocation. It helps SWFs avoid excessive risk aversion and underestimation of risks due to distorted risk measurement. Moreover, decisions made by SWFs involve multi-level and multi-dimensional considerations of risks. The subadditivity of ES ensures that the risk measurement results at different levels and dimensions are inherently.

4 Demonstration of Effects

4.1 Data Presentation

Based on the 2024 GPFG stress test data of NBIM, this part demonstrates the superiority of ES in the risk measurement of SWFs from multiple dimensions.

The following figures (Figure 1 to Figure 4) show the ES values of the actual portfolio as at 31 December, denominated in JPY, CHF, USD, EUR, GBP, NOK, CAD, and AUD at different confidence levels (90%, 95%, 97.5%, 99%). It can be seen from the charts that as the confidence level increases, the ES values generally show an upward trend. Moreover, the report indicates that GPFG is sensitive to the choice of report currency. When the Norwegian Krone does not depreciate, the ES values of all currencies rise significantly. The higher the confidence level (from 90% to 99%), the greater the increase in the ES value. It breaks through the VaR limitation of the Single Confidence Level Threshold. ES can quantify the scale of extreme events and reveal the severity of exchange-rate and tail risks through comparison at multiple confidence levels[12].

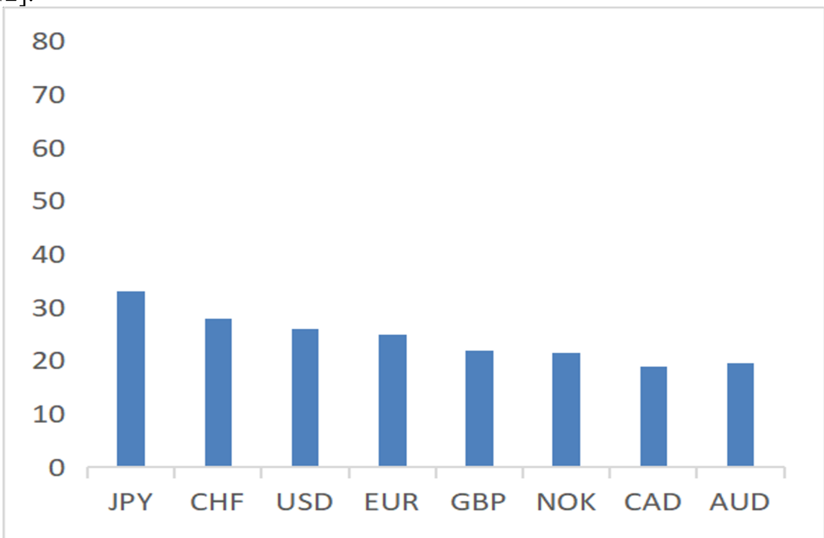

Fig. 1. Expected shortfall of actual portfolio as at 31 December 2024. Confidence level 90%. Percent.

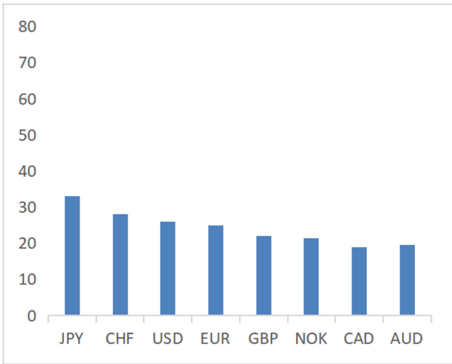

Fig. 2. Expected shortfall of actual portfolio as at 31 December 2024. Confidence level 95%. Percent.

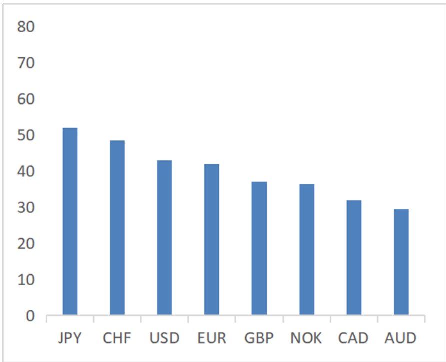

Fig. 3. Expected shortfall of actual portfolio as at 31 December 2024. Confidence level 97.5%. Percent.

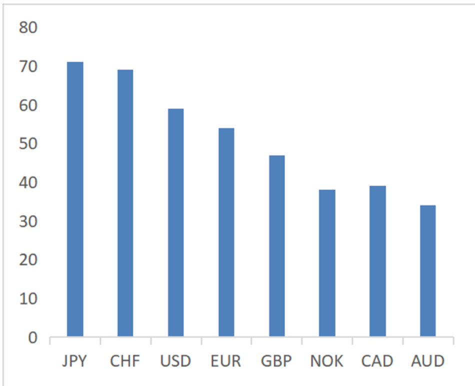

Fig. 4. Expected shortfall of actual portfolio as at 31 December 2024. Confidence level 99%. Percent.

NBIM designs the prospective scenarios (debt crisis, AI correction and fragmented world) to estimate ES. The report displays scenarios, and the absolute loss is 355billion-the total market value of GPFG. Table 1 presents the exposure, shock, and impact of different asset classes in local currency. It can be seen in Table 1 that the loss rate ranges from 18% to49% in extreme scenarios, and the absolute loss is 355billion-789.5billion kroner. Equities, fixed income, and tangible assets are impacted with varying degrees. ES can quantify the overall impact of the extreme events on the total value of the portfolio by 35% to 40%. However, VaR only offers a static loss threshold. By contrast, VaR only provides a static threshold. It cannot present the comprehensive impacts of risks on the portfolio, which fails to support the implementation of a layered approach to managing risks.

Table 1. Hypothetical scenario impact for GPFG portfolio as at 31 December 2024

	Value	AI Correction	Debt Crisis	Fragmented World	AI Correction	Debt Crisis	Fragmented World
Equities in local currency							
Developed markets-small cap	1041	-35	-55	-54	-360	-571	-558
Developed markets-large cap	11194	-31	-49	-47	-3426	-5433	-5306
Emerging and Frontier markets	1499	-15	-38	-41	-217	-565	-614
Total in local currency	13735	-29	-48	-47	-4004	-6570	-6478
	Value	AI Correction	Debt Crisis	Fragmented World	AI Correction	Debt Crisis	Fragmented World
Fixed income in local currency							
Developed markets-short term treasuries	958	1	0	0	9	1	-5
Developed markets-long term treasuries	2352	12	-25	-5	284	-592	-112
Developed markets-government related	493	11	-21	-5	56	-105	-23
Developed markets-corporates	1561	10	-17	-6	158	-266	-91
Total in local currency	5460	9	-18	-4	510	-968	-232
Real Assets in local currency							
Listed real estate	356	-12	-66	-43	-41	-234	-153
Unlisted real estate	395	-4	-30	-19	-15	-119	-74
Unlisted infrastructure	48	2	-11	-2	1	-5	-1
Total in local currency	800	-7	-45	-28	-56	-358	-228
Total in local currency	19755	-18	-40	-35	-3550	-7895	-6938

Small cap and large cap are based on benchmark definitions. Long term treasuries include maturities of 3 years or more. Corporates include securitized bonds. Unlisted real estate shows gross asset value for exposure and listed real estate only includes equity exposure. Derivatives are mapped to the relevant asset class. The totals include Cash and FX.

4.2 Weakness of ES

While ES is superior to VaR in SWFs, there are still some defects. First, ES is not elicitable. While ES provides a coherent measure to reflect tail risk, ES backtests often require full distributional assumptions or multiple procedures to introduce additional estimation error[13].

Second, different methods used in ES evaluations have apparent deviations. For example, the Gaussian model undervalues ES. This causes the phenomenon of actual loss surpassing the estimated ES value. Contrary to the Gaussian model, the stable Pareto model tends to overvalue. It perhaps results in capital waste[14].

Third, the ES calculation process is based on historical data. While ES carefully considers the upper tail risk of loss, the uncertainty and systemic risks brought by extreme risk (like climate change) are far beyond the scope that historical data can reflect[15].

Finally, the ES calculation relies on the average value of tail loss. It needs a larger sample size and more complex estimation methods, especially at a high confidence level. It sets a higher demand for data quality and sample size[16].

5 Conclusion

By systematically analyzing relative academic literature and cases, the study finds that ES can better show the tail risks of SWFs so that it can better understand SWFs' stabilizing role in the global macroeconomy and achieve their sustainable operation. Besides, ES shows obvious advantages in the mitigating effect of diversification on extreme risks. While ES is superior to VaR, it still has some shortcomings, such as a complex backtesting process, a large estimation with different methods, reliance on historical data, and so on. Other models need to be combined to play the role of ES better and improve the abilities of ES in risk assessment.

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