



Teaching Research on Enhancing High School Students' Logical Reasoning Competency Based on Python

A Case Study of "Binomial Distribution"

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Abstract. Based on the requirements of the "General High School Mathematics Curriculum Standards (2017 Edition, 2020 Revision)", this paper addresses the common teaching issue of "emphasizing formula memorization over essential understanding" in the high school binomial distribution curriculum. It constructs a teaching plan integrated with Python visualization technology. By using Python to plot distribution graphs under different parameters (n, p), the patterns, maximum value characteristics, and the trend towards continuity under large sample sizes of the binomial distribution are visually presented. This effectively overcomes teaching difficulties such as judging the applicability conditions of the binomial distribution and abstracting real-world problems into models.

Keywords: Logical Reasoning Competency, Binomial Distribution, Python

1 Introduction

Logical reasoning, as a key component of mathematical core competencies, serves as the cornerstone for students to understand abstract concepts, establish knowledge connections, and engage in rational judgment. The General High School Mathematics Curriculum Standards (2017 Edition, 2020 Revision) explicitly defines it as the competency to derive other propositions from given facts and statements in accordance with established rules¹.

Xu Jianxiang explored strategies for cultivating high school students' logical reasoning skills, emphasizing the enhancement of students' reasoning abilities through deep engagement with textbooks, the creation of contextualized scenarios, and the construction of evaluation systems to meet the demands of college entrance examinations². Zhang Jinlan proposed that high school mathematics problem-solving instruction should adopt a "problem-driven, thinking-progressive, feedback-optimized" model, systematically fostering students' logical reasoning skills through scenario design, guided thinking, and process refinement³. Chen Hongbo emphasized the need for systematic training, critical thinking, and well-designed tasks to help students construct logical frameworks, enhance problem-solving and innovative abilities, and optimize

teaching through contextualization and analogy to promote cognitive development⁴. Mandasari pointed out that consciously creating diverse and authentic problem situations in mathematics education can stimulate students' deep thinking and promote the development of logical reasoning and creative thinking when solving complex problems⁵.

The binomial distribution, as a core model in the "Probability and Statistics" theme, addresses the common issue in traditional mathematics teaching where "formula memorization often overshadows conceptual understanding"⁶. This instructional design takes a series of contextualized scenarios, such as "basketball shooting accuracy" and "coin tossing," as the logical starting point, guiding students to identify the characteristics of n independent repeated trials from specific phenomena. The lesson will deeply integrate Python programming and visualization technologies, utilizing simulated experiments and dynamic graphing to make abstract distribution patterns visually accessible. This approach not only deepens students' understanding of the model's essence but also cultivates their logical reasoning ability to identify patterns in data and express real-world problems in mathematical language⁷.

2 The Instructional Design

2.1 Teaching Process

1. Create Situations, Intuitive Perception.

Teaching Activity 1 :The teacher describes the scenario and asks: Our school basketball team is having free throw practice today (Figure 1). Each person takes 10 shots as one set. Xiao Wang is our team's star player. In his first set today, he made 5 out of 10 shots. If we view this set of shots as one trial, what are the characteristics of this trial? Students think, discuss, and answer.



Fig. 1. Basketball Shooting Practice Photo.

Teaching Activity 2 : The teacher asks student Xiao Li to toss a one-yuan coin 6 times at the operating desk. The results of heads and tails obtained are shown in Figure

2. If we view these 6 coin tosses as one trial, what are the characteristics of this trial? Students think, discuss, and answer.



Fig. 2. Outcomes of 6 Coin Tosses (Heads/Tails)

Teaching Activity 3 :The teacher asks student Xiao Zhang to roll a die 7 times at the operating desk. The results obtained are shown in Figure 3. If we view these 7 die rolls as one trial, what are the characteristics of this trial? Students think, discuss, and answer.



Fig. 3. Outcomes of 7 Dice Rolls

Teaching Activity 4 :The teacher asks: Can we abstract the similarities and differences between these three examples?

Design Intention :(1) Starting from familiar sports activities and life games, creating authentic problem situations effectively stimulates learning interest; (2) Through three different examples, students are led to think about their similarities and differences, enhancing their logical reasoning skills, and leading to the following questions through inquiry.

2. Introduce the Topic, Establish Concepts.

Teaching Activity 1: The teacher asks: Taking the shooting experiment as an example, does Xiao Wang making the first shot affect the probability of making the second shot? Student answers: No effect. The teacher continues: Is the probability of Xiao Wang making or missing each shot independent of the others? Student answers: Yes. The teacher asks again: Can these 10 shots be viewed as 10 independent trials? Student answers: Yes. The teacher asks: Similarly, are the outcomes of tossing coins and rolling dice also multiple independent trials? Student answers: Yes. The teacher gives the definition of n independent trials. The teacher emphasizes: Trials must maintain consistent conditions.

Design Intention :(1) Guide students to think step by step, enhancing their logical reasoning ability and cultivating their logical reasoning competency; (2) Make students clearly understand the definition of n independent trials: conducting n trials under the same conditions, where the results of each trial do not affect each other.

Teaching Activity 2 :The teacher asks: Taking the shooting experiment as an example, how many outcomes are possible for each of Xiao Wang's shots? What are they?

Student answers: Two, make a shot and miss a shot. The teacher continues: How many outcomes are possible for each of Xiao Li's coin tosses? What are they? Student answers: Two, heads and tails. The teacher summarizes: Both the shooting and coin tossing problems involve conducting multiple independent trials, where each trial has only two outcomes. This is a Bernoulli trial. The teacher asks: Using event A to represent "tossing a coin once results in heads", what is $P(A)$? Student answers: $1/2$. The teacher asks: Based on a large amount of data from past free throw practice, Xiao Wang's shooting frequency is around 0.6. Using event B to represent "Xiao Wang makes one shot", what is $P(B)$? Student answers: 0.6. The teacher continues: What principle is this based on? Student answers: The statistical definition of probability. Based on the above analysis, the specific definition of a Bernoulli trial is given.

Design Intention : (1) Through examples, guide students to clearly understand the definition of an n -fold Bernoulli trial: Under completely identical conditions, conducting n independent trials, where each trial has only two outcomes A and not- $A - \bar{A}$, and $P(A) = p$. Such a trial is called a Bernoulli trial or Bernoulli scheme. (2) Make students clearly understand the two conditions of a Bernoulli trial: each trial has only two outcomes A and \bar{A} ; independently repeated n times.

Teaching Activity 3: The teacher asks: Let's look at example 3 again. Is it a Bernoulli trial? Why? Students think, discuss, and answer: It is not a Bernoulli trial, because each experiment has 6 possible outcomes, which does not meet the condition of a Bernoulli trial."

Design Intention: Deepen students' understanding of the Bernoulli scheme through a counterexample.

3. Group Cooperation, Derive the Formula.

Teaching Activity 1 : The teacher asks: Taking coin tossing as an example, If we require tossing the coin 6 times and getting heads 4 times", how many different sequences are there in total? how many different sequences are there in total? Students think and answer. The teacher shows two of these sequences (Figure 4 and Figure 5). The teacher continues: How many such sequences are there in total? Student answers: C_6^4 . The teacher asks: What is the relationship between Situation 1 and Situation 2? Can they both happen in one single trial? Student answers: They cannot happen at the same time; they are mutually exclusive. The teacher probes deeper: What is the relationship among all these C_6^4 sequences? Student answers: They are all mutually exclusive. The teacher summarizes: According to the finite additivity of probability, there is a sum of C_6^4 probabilities.



Fig. 4. Case 1



Fig. 5. Case 2

Teaching Activity 2: The teacher asks: In Situation 1, what is the relationship between the outcome (heads/tails) of each coin? Student answers: Independent of each other. The teacher further asks: What is the probability of the event "Heads, Heads, Heads, Heads, Tails, Tails" in Situation 1? Student answers: $\frac{1}{2} \times \frac{1}{2} \times \frac{1}{2} \times \frac{1}{2} \times (1 - \frac{1}{2}) \times (1 - \frac{1}{2}) = (\frac{1}{2})^4 (1 - \frac{1}{2})^2$. The teacher shows Figure 6. The teacher then asks: What is the probability of Situation 2? Is it the same as Situation 1? Student answers: Yes. The teacher shows Figure 7. The teacher continues: Are the probabilities for all these C_6^4 sequences the same? Students think and answer: Yes.



Fig. 6. Probability of Case 1

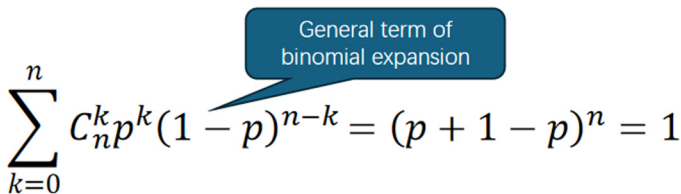


Fig. 7. Probability of Case 2

Teaching Activity 3: The teacher asks: Since these C_6^4 sequences are mutually exclusive, and the probability of each sequence is $(\frac{1}{2})^4 (1 - \frac{1}{2})^2$, then what is the probability of the event "tossing 6 times, getting heads 4 times"? Students think and answer: $C_6^4 (\frac{1}{2})^4 (1 - \frac{1}{2})^2$. The teacher further asks: Still using coin tossing as an example, let "tossing n times, getting heads k times" as event B, then what is P(B)? Students form groups of four and discuss, then answer: $C_n^k (\frac{1}{2})^k (1 - \frac{1}{2})^{n-k}$. The teacher asks: In the shooting problem, Xiao Wang's probability of making each shot is 0.6. What is the probability of making exactly 5 shots out of 10? Students discuss and answer: $C_{10}^5 (0.6)^5 (1 - 0.6)^{10-5}$. The teacher asks: Going further, if another player's probability of making each shot is p, what is the probability of making exactly k shots out of n?

Students discuss and answer: $C_n^k(p)^k(1-p)^{n-k}$. The teacher asks: What values can k take? Students think and answer: $k = 0, 1, 2, \dots, n$.

Teaching Activity 4: The teacher asks: In an n -fold Bernoulli trial, if event A occurs with probability p each time, what is the probability that A occurs exactly k times? First, let's answer: Who is the random variable X ? Student answers: The number of times event A occurs. The teacher asks: What values can the random variable X take? Students think and answer: $0, 1, 2, \dots, n$. The teacher asks: What is the probability of taking these values? Let's use $X = k, k = 0, 1, \dots, n$, as an example. Students think and answer: $P(X = k) = C_n^k p^k (1-p)^{n-k}, k = 0, 1, \dots, n$. The teacher asks: Is the above a distribution list? First, is $C_n^k p^k (1-p)^{n-k}$ non-negative? Students think and answer: Yes. The teacher asks: Is $\sum_{k=0}^n C_n^k p^k (1-p)^{n-k}$ equal to 1? Students think and derive manually. The teacher shows (Figure 8).



$$\sum_{k=0}^n C_n^k p^k (1-p)^{n-k} = (p + 1 - p)^n = 1$$

Fig. 8. Demonstration of Probability Summation

The teacher summarizes: $P(X = k) = C_n^k p^k (1-p)^{n-k}, k = 0, 1, \dots, n$ is a distribution list, and it is called the binomial distribution. Then gives the definition of the binomial distribution.

Design Intention: (1) Improve students' logical thinking ability to analyze and solve problems through progressive questioning, developing their logical reasoning literacy. (2) Emphasize that to solve practical problems, one must first clearly identify the random variable. (3) Give the definition of the binomial distribution: If the random variable X represents the number of times event A occurs in n -fold Bernoulli trials, where the probability of event A occurring each time is p , then the distribution list of X is $P(X = k) = C_n^k p^k (1-p)^{n-k}, k = 0, 1, 2, \dots, n$. We say X follows a binomial distribution with parameters n and p , denoted as $X \sim B(n, p)$. (4) Make students clearly understand the meaning of the parameters: n represents the number of trials; p represents the probability of A occurring in each trial.

4. Explore Parameters, Grasp Patterns.

Teaching Activity 1: The teacher asks: Can we use a tool to display the distribution list of the binomial distribution graphically? Taking $n = 20, p = 0.2$ as an example, draw the distribution list graph. Students try to draw by hand. The teacher shows the result plotted by Python (Figure 9).

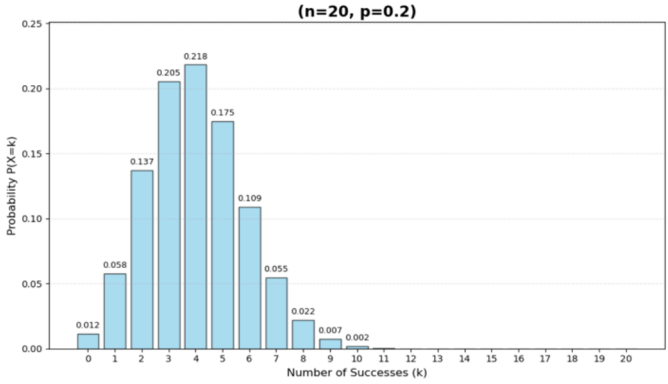


Fig. 9. Bar Chart of the Probability Distribution for $n = 20, p = 0.2$

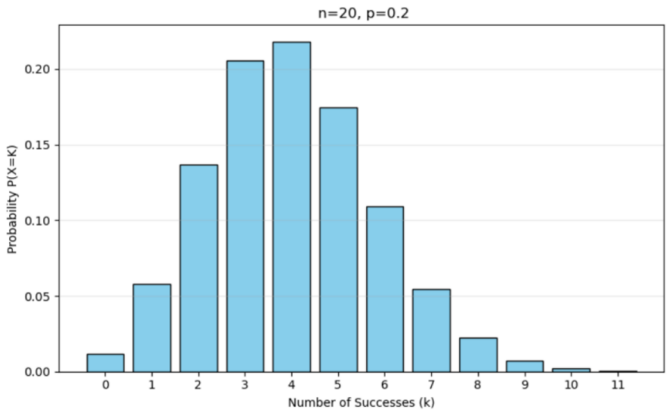


Fig. 10. Bar Chart of the Probability Distribution for $n = 20, p = 0.5$

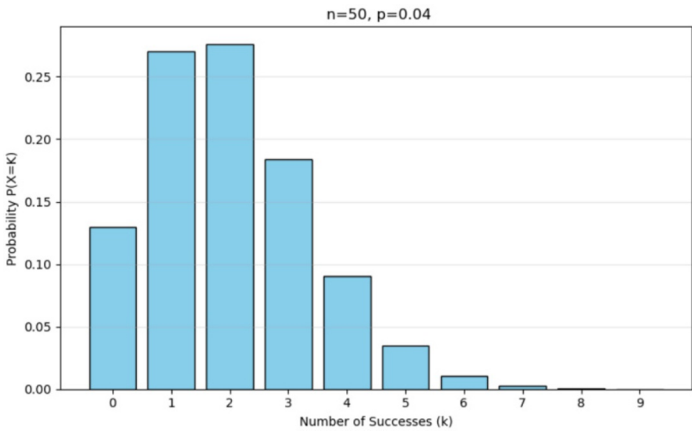


Fig. 11. Bar Chart of the Probability Distribution for $n = 50, p = 0.04$

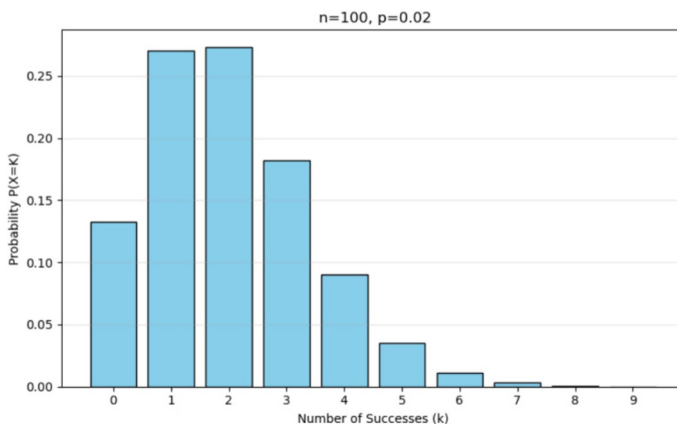


Fig. 12. Bar Chart of the Probability Distribution for $n = 100, p = 0.02$

The teacher continues using Python software to show the distribution list graphs for $n = 20, p = 0.5$; $n = 50, p = 0.04$; and $n = 100, p = 0.02$ (Figures 10-12). The teacher asks: What conclusions can be drawn from observing Figures 9-12? Students discuss and answer.

Design Intention :(1) Let students experience firsthand the difficulty of manually drawing distribution list graphs, naturally leading to the convenience of Python plotting. (2) Through visually observing distribution graphs with different parameters, allow students to independently discover that the distribution shape varies with different parameters. (3) Let students independently discover that the binomial distribution graph always increases monotonically to a maximum point, then decreases monotonically. (4) Make students clearly understand that the maximum is always attained near $n \times p$. (5) Focus on cultivating students' logical reasoning literacy in discovering patterns from specific phenomena, formulating hypotheses, and verification.

Teaching Activity 2: The teacher continues to show: the distribution list graph for $n = 1000, p = 0.4$ (Figure 13). The teacher asks: What conclusions can be drawn from observing Figure 13? What is the difference between Figure 13 and Figures 9-12? Students discuss and answer.

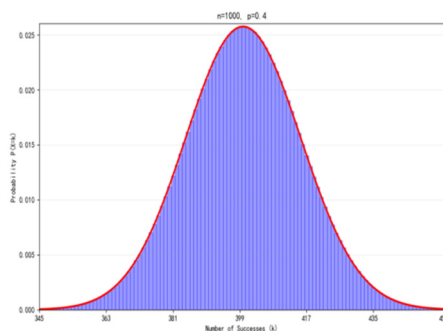


Fig. 13. Bar Chart of the Probability Distribution for $n = 1000, p = 0.4$



Fig. 14. Photo of a Chime Bell Set

Design Intention : (1) Deepen students' understanding of the three properties: the distribution shape varies with different parameters; the distribution graph always increases then decreases; the maximum point is around $n \times p$. (2) Use Python to plot the graph for $n = 1000, p = 0.4$, guiding students to observe the trend towards continuity and symmetry under large sample conditions. (3) Through questions, guide students to compare the discrete distribution characteristics for small n , and discover the pattern where the graph tends to become a smooth "bell shape", similar to the bell in Figure 14, paving the way for subsequent learning of the normal distribution. (4) Use information technology to demonstrate the transition from discrete probability points to a continuous curve, cultivating students' visual imagination and logical reasoning literacy.

3 Consolidation Exercises, Application Enhancement

Example 1 Which of the following random variables follows a binomial distribution?
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- A. The number of heads when tossing a fair coin 10 times.
- B. The number of spades when drawing 5 cards from a deck without replacement.
- C. The number of times a red ball is drawn when drawing with replacement 5 times from a bag containing red, white, and blue balls.
- D. The height distribution of students in a class.

Design Intention: Test students' accurate understanding of the definition and applicable conditions of the binomial distribution. By setting positive and negative examples, strengthen students' grasp of the two key premises: "n independent repeated trials" and "each trial has only two outcomes".

Example 2 :After chickens are vaccinated with a certain vaccine, 80% will not be infected by a specific virus. If 5 chickens are vaccinated, find:

- ① The probability that no chicken gets infected;
- ② The probability that exactly 1 chicken gets infected.

Design Intention :Cultivate students' ability to abstract a real-world problem (vaccine efficacy in chickens) into a probability model $X \sim B(5, 0.2)$. Through specific calculations, consolidate students' understanding of the parameters in the binomial distribution model $P(X = k) = C_n^k p^k (1 - p)^{n-k}, k = 0, 1, 2, \dots, n$.

Example 3 :During the school club activity day, a "mystery box prize station" is set up. The probability that each mystery box contains a "limited commemorative badge" is 0.4. A student buys 10 boxes at once. Use the binomial distribution $X \sim B(10, 0.4)$ to describe the probability distribution of "the number of limited badges obtained". Guess at what number of badges obtained the probability of this distribution will be maximum?

Design Intention :Using the familiar life scenario of "mystery box draws" as a context, to connect abstract binomial distribution problems with real-life experience. By calculating $n \times p$ to guess the maximum point, let students begin to test the core rule: "the maximum probability of the binomial distribution occurs around $n \times p$ " and strengthen their understanding of this conclusion.

4 Summary

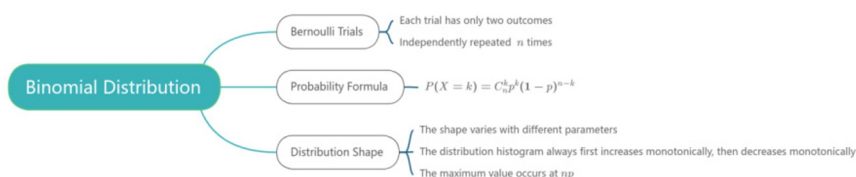


Fig. 15. Class Summary Diagram

Design Intention: Through Figure 15, help students form an overall cognitive schema about the binomial distribution, rather than memorizing isolated formulas.

5 Assigning Homework

- (1) Required exercises: Problems 1 and 2 on textbook page P76.
- (2) Optional exercises: Problems 8 and 9 on textbook page P77.

Design Intention :Consolidate core knowledge through post-class homework assignments. Setting required and optional exercises allows students of different ability levels to gain something from the homework.

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