



Regime-Dependent Volatility Dynamics: Evidence from Time-Series Analysis

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Abstract. This paper examines the volatility hedging role of precious metals against U.S. stock market risk from a regime-based perspective. While a large literature documents the safe-haven properties of precious metals—particularly gold—based on price or return behavior during periods of market stress, far less is known about their effectiveness in hedging market volatility itself. This distinction is crucial, as price-level diversification does not necessarily imply a reduction in market uncertainty. Using volatility measures constructed from daily price data, the study investigates the relationship between precious metals volatility and stock market volatility across different market regimes defined by the periods before, during, and after the global financial crisis. The empirical results reveal pronounced regime dependence and asset heterogeneity in volatility hedging effectiveness. Gold, despite exhibiting price-level divergence from equity markets consistent with a safe-haven role, tends to co-move with stock market volatility across most regimes, indicating that its widely documented hedging properties do not extend to volatility risk. In contrast, other precious metals display more heterogeneous and regime-dependent volatility relationships, with limited evidence of volatility hedging emerging only under specific market conditions. Overall, the findings help reconcile conflicting evidence in the literature by highlighting the conceptual difference between price-based safe-haven behavior and volatility-based hedging, and they underscore the importance of explicitly accounting for market regimes when evaluating the risk management role of precious metals.

Keywords: Volatility modeling; Regime-dependent analysis; Time-series methods; Risk measurement; Applied statistics

1 Introduction

Periods of elevated stock market volatility are associated with heightened uncertainty and increased risk exposure for investors. Identifying assets that can mitigate market risk is therefore a central issue in financial economics and risk management. Precious metals, particularly gold, are widely regarded as hedging or safe-haven assets, especially during episodes of financial stress. However, empirical evidence on their effectiveness remains mixed.

A key reason for these conflicting findings lies in how hedging effectiveness is defined. Much of the existing literature evaluates the role of precious metals based on price-level or return-based relationships, documenting instances in which gold prices diverge from equity prices during market downturns. At the same time, other studies find that precious metals fail to provide reliable protection when risk is measured in terms of volatility. This apparent inconsistency suggests that price-level diversification and volatility hedging capture different dimensions of risk mitigation that are often conflated.

From a risk management perspective, this distinction is critical. An asset may preserve portfolio value while still amplifying uncertainty if its volatility rises alongside market volatility. Moreover, financial markets exhibit pronounced regime shifts, and hedging relationships may vary substantially across tranquil and turbulent periods. Ignoring such regime dependence may obscure important variation in hedging effectiveness.

This paper examines whether precious metals hedge U.S. stock market volatility and how volatility-based hedging differs from their widely documented price-level safe-haven behavior across market regimes. Using volatility measures constructed from daily price data, the analysis investigates the relationship between precious metals volatility and stock market volatility before, during, and after the global financial crisis.

2 Literature Review

A large body of literature examines the role of precious metals as hedging or safe-haven assets against stock market risk. Early studies, primarily focused on gold, distinguish between hedging and safe-haven properties and document that gold prices may diverge from equity prices during periods of financial stress [1], [2]. This framework has been widely adopted to evaluate the protective role of precious metals in return or price space.

More recent research emphasizes that hedging effectiveness is highly sensitive to how risk is defined. While much of the early literature relies on return-based measures, an increasing number of studies highlight that stock market volatility represents a distinct and economically relevant dimension of risk. Volatility-based analyses show that assets capable of preserving portfolio value may still fail to stabilize market uncertainty, underscoring the conceptual difference between price-level diversification and volatility hedging [3].

In recent years, the literature has increasingly shifted toward analyzing volatility spillovers and cross-market connectedness between precious metals and equity markets. Using realized volatility measures, time–frequency methods, and time-varying parameter models, recent studies document that volatility interactions between gold and stock markets intensify during periods of heightened uncertainty, such as the COVID-19 crisis and the post-pandemic period [4]–[6]. These findings suggest that precious metals are often embedded within broader volatility transmission networks rather than acting as isolated hedging instruments.

Extending beyond gold, recent empirical evidence reveals substantial heterogeneity across precious metals. Studies show that silver, platinum, and palladium exhibit asymmetric and time-varying volatility relationships with equity markets, indicating that volatility hedging effectiveness is neither uniform across assets nor stable over time [7], [8]. In addition, some existing literature emphasizes the importance of regime dependence in volatility dynamics and risk transmission. Recent regime-based studies demonstrate that volatility correlations and spillover mechanisms differ markedly across tranquil and turbulent market states. Ignoring such regime shifts may lead to misleading inference, as full-sample estimates implicitly average across heterogeneous market conditions [9], [10].

Apart from these progresses, two important gaps remain. First, much of the volatility spillover literature focuses on connectedness or transmission intensity rather than explicitly evaluating volatility hedging effectiveness. Second, relatively few studies jointly integrate volatility-based risk measures with regime-dependent analysis when assessing the hedging role of precious metals [11], [12]. This paper addresses these gaps by distinguishing between price-based safe-haven behavior and volatility-based hedging, and by providing a regime-dependent assessment of volatility interactions between precious metals and the U.S. stock market.

3 Data and Volatility Construction

Our analysis utilizes daily price series for selected precious metals and the U.S. equity market to examine volatility-based hedging behavior under different market conditions. The dataset covers gold, silver, platinum, and palladium prices, together with the S&P 500 index, over the period from January 2005 to July 2014. This time span encompasses a wide range of market environments, including episodes of heightened uncertainty (the 2008 global financial crisis), which is essential for assessing regime-dependent hedging effects.

To quantify market risk, daily price observations are transformed into continuously compounded returns. Volatility is measured using a realized volatility framework constructed at the monthly frequency. Specifically, monthly volatility is computed from the dispersion of daily returns within each month and scaled to an annualized measure. This transformation allows the analysis to focus on fluctuations in market risk rather than price levels and facilitates comparisons of volatility dynamics across assets and market regimes.

The conversion from daily returns to monthly volatility results in a reduced number of observations, yielding 114 monthly volatility estimates. While this aggregation reduces the sample size, it provides smoother and more robust measures of risk that are well suited for regime-based analysis, where excessive high-frequency noise may obscure underlying patterns in hedging relationships.

Summary statistics for the volatility measures are presented in Table 1. The reported statistics reveal pronounced differences in the magnitude and dispersion of volatility across precious metals and the equity market. Such heterogeneity highlights the importance of allowing hedging effectiveness to vary across assets and market conditions, rather than assuming uniform behavior over time.

Table 1. Summary Statistics of Monthly Volatilities

<i>N=114</i>	Mean	Median	Standard Deviation	Minimum	Maximum
Gold	0.19	0.17	0.08	0.07	0.50
Silver	0.34	0.30	0.15	0.11	0.87
Platinum	0.21	0.19	0.10	0.09	0.69
Palladium	0.33	0.28	0.18	0.11	1.46
Copper	0.29	0.25	0.14	0.09	0.99
Oil	0.26	0.23	0.22	0.08	2.16
Stock	0.17	0.14	0.12	0.06	0.80

4 Methodology

4.1 Volatility-Based Regression Framework

This study adopts a first-difference regression framework to examine the volatility hedging role of precious metals against stock market risk as volatility series exhibit strong persistence and serial correlation in levels. By modeling changes in volatility rather than volatility levels, the first-difference approach mitigates spurious regression concerns and provides a more appropriate representation of short-term movements in market uncertainty.

The baseline hedging relationship is specified in following formula:

$$\Delta \sigma_{sp} = \beta_1 \Delta \sigma_g + \beta_2 \Delta \sigma_s + \beta_3 \Delta \sigma_{pt} + \beta_4 \Delta \sigma_{pa} + \beta_5 \Delta \sigma_{cu} + \beta_6 \Delta \sigma_{oil} + \Delta u \quad (1)$$

where σ_{sp} denotes stock market volatility, and σ_g , σ_s , σ_{pt} , σ_{pa} , σ_{cu} , σ_{oil} represent the volatilities of gold, silver, platinum, palladium, copper, oil respectively. All volatility measures are transformed into first difference estimators from the traditional OLS regression model.

The first-difference regression model offers two advantages for the present analysis. First, by modeling changes in volatility, it reduces the influence of strong persistence in volatility series and avoids spurious inference driven by highly autocorrelated levels. Second, the specification aligns naturally with the concept of volatility hedging, as it

evaluates whether shifts in uncertainty in one market are associated with contemporaneous shifts in uncertainty in another.

4.2 Market Regime Definition

Financial markets exhibit pronounced regime shifts, particularly around episodes of elevated uncertainty. To examine whether volatility hedging effects vary across market conditions, this study adopts a regime-based framework that distinguishes between different phases of market volatility.

Market regimes are defined with reference to the global financial crisis, dividing the sample into pre-crisis (January 2005 to June 2007), crisis (July 2007 to December 2009), and post-crisis (January 2010 to July 2014) periods. This classification reflects economically meaningful differences in the behavior of market uncertainty, investor sentiment, and risk transmission mechanisms. Rather than assuming a constant volatility environment, the regime definition allows changes in market uncertainty to be interpreted within distinct structural contexts.

Importantly, the regime classification is applied to the first-differenced volatility series. This ensures that regime comparisons are based on changes in volatility dynamics rather than on differences in long-run volatility levels across periods. By focusing on first differences, the analysis evaluates whether shocks to market uncertainty in precious metals and equity markets interact differently across regimes, consistent with the notion that hedging effectiveness may depend on prevailing market conditions.

4.3 Regime-Dependent Hedging Analysis

To assess regime-dependent volatility hedging, the first-difference hedging regression is estimated separately for each market regime. Specifically, the baseline specification relates changes in stock market volatility to contemporaneous changes in precious metals volatility within each regime. This subsample estimation approach allows the hedging coefficient to vary across market states without imposing restrictive parametric assumptions.

Under this framework, effective volatility hedging is identified by a negative and statistically significant relationship between changes in precious metals volatility and changes in stock market volatility. Positive coefficients indicate volatility co-movement, implying that shocks to uncertainty in precious metals coincide with increases in equity market volatility. Insignificant estimates suggest weak or unstable hedging effects.

Estimating the model in first differences across regimes provides a transparent assessment of whether volatility hedging properties strengthen, weaken, or reverse during periods of financial stress. By applying the same first-difference specification consistently across regimes and across precious metals, the analysis facilitates direct comparison of regime dependence and cross-asset heterogeneity in volatility hedging effectiveness.

4.4 Interpretation and Scope

It is important to emphasize that the methodology focuses exclusively on volatility dynamics and does not assess price- or return-based hedging performance. As such, the analysis does not aim to evaluate portfolio returns or investment profitability. Instead, it isolates the role of precious metals in mitigating market uncertainty, thereby complementing existing studies that emphasize price-level safe-haven behavior.

This distinction ensures that the empirical findings can be interpreted within a coherent risk management framework, in which volatility is treated as a primary measure of market risk.

5 Empirical Results

This section presents the empirical findings on the hedging effects of precious metals against U.S. stock market volatility. All results discussed are drawn directly from the first-difference regression analyses. The emphasis here is on interpreting the empirical evidence from a hedging and market-regime based perspective.

5.1 Full-Sample Hedging Effects

The full-sample hedging regression results are reported in Table 2, providing a benchmark assessment of average volatility linkages over the entire sample period.

Table 2. First Difference Regression Output

	Coefficient	t-Stat	P-value	Key Stats
Gold D1.	0.331	2.570	0.012**	F (6, 94) = 8.670 Prob > F = 0.000 R-squared = 0.356 Adj R-squared = 0.315 Root MSE = 0.069
Silver D1.	0.005	0.070	0.942	
Platinum D1.	0.042	0.420	0.676	
Palladium D1.	-0.005	-0.150	0.881	
Copper D1.	0.216	3.480	0.001***	
Oil D1.	0.086	2.940	0.004***	
_cons	0.000	0.060	0.956	

The estimates indicate little evidence of volatility hedging when relationships are averaged over the entire sample period. Changes in gold volatility are positively and significantly associated with changes in stock market volatility, implying volatility co-movement rather than hedging. In contrast, the coefficients for silver, platinum, and palladium are statistically insignificant, suggesting weak or unstable volatility linkages.

Among non-precious commodities, copper and crude oil exhibit positive and significant coefficients, indicating that volatility shocks in industrial commodities tend to coincide with increases in stock market volatility. Overall, the full-sample results

show that precious metals do not provide consistent volatility hedging benefits, motivating a regime-based analysis to assess whether hedging effects emerge under specific market conditions.

5.2 Regime-Dependent Hedging Performance

The regime-based results (Table 3) reveal substantial variation in volatility hedging effectiveness across market conditions. When the sample is divided into pre-crisis, crisis, and post-crisis periods, volatility linkages between precious metals and the stock market differ markedly in both magnitude and direction.

For gold, the estimated coefficients are positive before and after the financial crisis and statistically significant in both regimes, indicating volatility co-movement rather than hedging. During the crisis period, the coefficient remains positive but becomes statistically insignificant, suggesting an unstable relationship between gold volatility and stock market volatility under extreme market stress. Overall, the results indicate that gold does not provide effective volatility hedging in any regime, despite changes in market conditions.

Silver exhibits a distinct regime-dependent pattern. While no significant volatility relationship is observed before or during the crisis, the coefficient becomes negative and statistically significant in the post-crisis period. This finding suggests that silver provides volatility hedging benefits only under specific market conditions, highlighting the importance of accounting for regime dependence when evaluating hedging effectiveness.

In contrast, platinum and palladium display weak and inconsistent volatility relationships across all regimes, with no robust evidence of volatility hedging. These results reflect the limited role of these metals in mitigating market uncertainty. Among industrial commodities, copper and crude oil show positive and significant coefficients in certain regimes, indicating that volatility shocks in these markets tend to coincide with increases in stock market volatility rather than providing hedging benefits.

Taken together, the regime-based results demonstrate that volatility hedging by precious metals is neither uniform nor stable over time. Hedging effectiveness depends critically on market conditions and varies substantially across assets, reinforcing the limitations of full-sample analysis and the importance of regime-based approaches.

Table 3. Coefficient Analysis of Regression Model in Different Market Regimes

Commodity	Before Financial Crisis	During Financial Crisis	After Financial Crisis
Gold	0.569**	0.424	0.377**
Silver	0.007	0.301	-0.257***
Platinum	0.074	-0.174	0.334*
Palladium	-0.090	-0.064	-0.007
Copper	-0.129	0.266**	-0.079
Crude Oil	-0.020	0.094**	0.491***

5.3 Volatility Hedging vs. Price Safe Haven: The Case of Gold

Gold illustrates the distinction between price-based safe-haven behavior and volatility hedging. Price-level evidence (Fig.1) shows that gold prices often diverge from equity prices during periods of market stress, particularly during the financial crisis, supporting its role as a safe-haven asset in return space.

However, the volatility-based results (Table 3) convey a different message. As shown in Table 3, gold volatility is positively associated with stock market volatility before and after the crisis, and remains unstable during the crisis period. This indicates volatility co-movement rather than hedging, implying that increases in market uncertainty are accompanied by higher uncertainty in the gold market

This contrast highlights that price protection does not necessarily translate into risk stabilization. While gold may preserve portfolio value during equity market downturns, it does not consistently reduce volatility. Distinguishing between price-level and volatility-based analyses is therefore essential for accurately assessing the risk management role of precious metals.

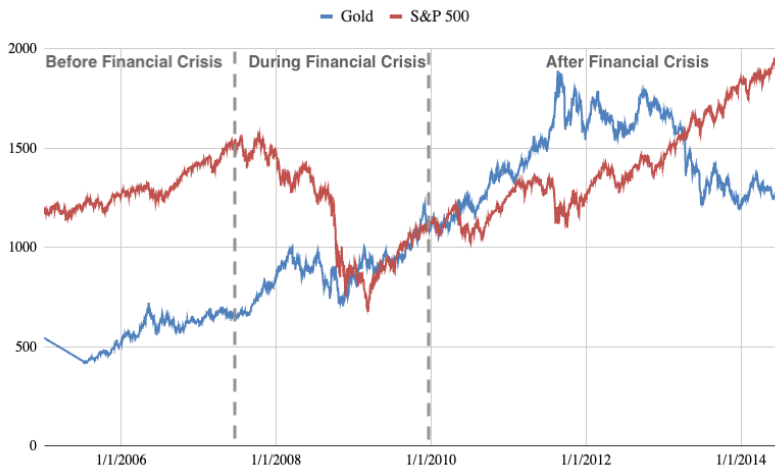


Fig. 1. Daily Prices (\$) of Gold vs. S&P 500 Index

6 Discussion

The empirical results provide clear evidence that the hedging role of precious metals depends critically on how risk is defined and on prevailing market conditions. While gold is widely regarded as a safe-haven asset based on price or return behavior, the volatility-based analysis reveals a fundamentally different pattern. Across most market regimes, changes in gold volatility tend to move in the same direction as changes in stock market volatility, indicating volatility co-movement rather than effective volatility hedging.

This distinction helps reconcile conflicting findings in the existing literature. Studies emphasizing price-level or return-based measures often conclude that gold offers protection during periods of market stress, whereas volatility-based results suggest limited risk-reduction benefits. The present analysis shows that these conclusions are not contradictory but instead reflect different dimensions of risk mitigation. Gold may preserve portfolio value when equity prices decline, yet it does not consistently stabilize market uncertainty.

The regime-based results further highlight the importance of market conditions in shaping hedging effectiveness. Volatility relationships between precious metals and the stock market vary substantially across pre-crisis, crisis, and post-crisis periods, underscoring the limitations of full-sample analysis. In particular, the absence of robust volatility hedging by gold across regimes suggests that its hedging role is more constrained than commonly perceived, while other metals exhibit heterogeneous and regime-specific behavior.

From a risk management perspective, these findings imply that treating precious metals as universally effective hedging instruments may be misleading. Investors concerned with managing volatility risk should distinguish between assets that hedge returns and those that hedge uncertainty. Incorporating volatility-based and regime-aware analysis provides a more nuanced and accurate assessment of hedging performance.

Several limitations should be acknowledged. The analysis relies on monthly volatility measures, which may smooth short-term dynamics, and the regime classification is based on a crisis-centered partition rather than endogenous regime switching. Future research could explore alternative volatility measures, higher-frequency data, or more flexible regime definitions to further refine the assessment of volatility hedging behavior.

7 Conclusion

This paper provides a regime-based assessment of the volatility hedging role of precious metals against U.S. stock market risk using a volatility-focused framework. By distinguishing between price-level safe-haven behavior and volatility-based hedging, the analysis offers a clearer perspective on how precious metals contribute to risk management under different market conditions.

The empirical results show that gold, despite its widely documented safe-haven role in price or return space, does not provide consistent volatility hedging benefits. Across most market regimes, changes in gold volatility tend to co-move with changes in stock market volatility, indicating limited effectiveness in stabilizing market uncertainty. Other precious metals exhibit heterogeneous and regime-dependent behavior, with evidence of volatility hedging emerging only under specific conditions rather than uniformly over time.

These findings help reconcile conflicting evidence in the literature by highlighting the importance of how risk is defined and measured. Evaluations based solely on price or return behavior may overstate hedging effectiveness when the objective is to manage

volatility rather than preserve returns. Incorporating volatility-based and regime-aware analysis therefore offers a more informative perspective on the risk management role of precious metals.

From a practical perspective, the results imply that precious metals should not be treated as universally effective hedging instruments. Instead, their usefulness in managing volatility risk depends critically on market conditions. Incorporating regime information may therefore improve volatility hedging strategies, particularly during periods of heightened market stress.

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