A Comparative Study of the Markowitz and Index Models Under Various Constraints: An Empirical Analysis Based on 20 Years of Data for 21 Stocks

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Abstract. This paper examines portfolio optimization using both the Markowitz Model (MM) and the Index Model (IM), with a focus on the S&P 500 Index and 21 of its OEX constituent stocks over a 20-year sample period. Daily prices were converted into monthly returns, resulting in over 200 months of data. Using Excel Solver and SolverTable, we estimated expected returns, variances, and covariance matrices, and then applied quadratic optimization under multiple constraints. Specifically, we analyzed five cases: (1) Regulation T constraint, (2) box constraint, (3) no constraint, (4) no short-selling, and (5) exclusion of the index. The results highlight differences between MM, which captures full pairwise correlations among assets, and IM, which simplifies by linking returns to the market index. Empirical findings suggest that while unconstrained portfolios offer the highest potential returns, they produce extreme weights and higher risks. In contrast, IM portfolios demonstrate more stability and improved efficiency under conservative conditions, making them more suitable for investors with regulatory or practical limitations.

Keywords: Portfolio Optimization, Markowitz Model, Index Model, Efficient Frontier, Risk-Return Trade-off.

1 Introduction

The investment enterprise sits at the center of the financial markets, ranging from equities, bonds, and derivatives to mutual funds and alternative assets. It's very basic aim is the effective allocation of capital with the purpose of bringing long-term value growth. As the global markets continue to grow, investment policy has also diversified and globalized. Macro-economic conditions and regulatory systems, however, have a very significant effect on portfolio construction and risk management. Although the financial markets offer prospects for favorable returns, investors always risk market volatility, credit events, and constraints in liquidity.

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To bridge these antagonistic forces, portfolio optimization is a crucial tool that allows investors to manage risk exposures while attaining the highest possible expected returns. In particular, mean-variance analysis under the leadership of Markowitz has emerged as one of the most ubiquitous frameworks in recent portfolio theory. Nevertheless, when this model is applied practically, it will often require modifications to accommodate real-world constraints such as short-selling restrictions or margin requirements.

This study applies both the Markowitz Model and the Index Model to past data of the S&P 500 and its constituent stocks. By the implementation of alternative optimization cases, we plan to examine how constraints affect portfolio efficiency and to provide insights into the trade-off's investors have to face in the presence of varying regulatory and practical conditions.

2 Literature Review

This review of literature compares the empirical performance of the Markowitz and index models of portfolio optimization over 20 years on 21 stocks across different industries, as well as the S&P 500 index as a benchmark. All studies employ large historical data to compare performance and insensitivity of both models under alternative constraints and market conditions.

Some of them utilized long-term daily returns to compare Markowitz model with index-based strategies. For instance, there was a study that utilized 20 years of daily return data of 21 stocks in different industries and the S&P 500 index for the purpose of estimating each strategy's impact on portfolio performance [1]. Similarly, in another research, seriousness was emphasized on requiring two decades of data span to ensure accuracy of empirical analysis, drawing data from the Swedish OMXS30 index to compare optimization methods [2].

The extended use of the Markowitz model has been a popular approach to test its performance. Empirical testing with reference to a twenty-year horizon was employed to develop optimal portfolios from the Markowitz model, highlighting the importance of historic data to represent market trends [3].

Additionally, research has explored modifications in the initial mean-variance model by incorporating multiple constraints to portfolio weights, which could influence the performance and effectiveness of the model [4]. Comparative research has also compared various asset allocation strategies, including heuristic models and index methods. One study provided results comparing Markowitz models with other heuristics for globally diversified portfolios, emphasizing requirements of constraints and diversification in empirical performance [5]. Moreover, application of hierarchical risk parity (HRP) models in specific markets, for example, the Canadian market, emphasizes ongoing research into alternative methods versus conventional Markowitz optimization [6].

The application of machine learning techniques and, in particular, neural networks has been investigated for their predictive capability and portfolio construction role. Although the studies are primarily centered on the direction of stock index prediction, they underscore the potential of advanced algorithms to complement traditional models like

Markowitz and index strategies [7, 8]. Moreover, empirical analyses with regard to parameter estimation risks and dynamic models validate challenges and opportunities in out-of-sample performance evaluation [9]. Overall, literature under review stresses the need for long-run data for empirical tests of Markowitz and index models. It also shows efforts at enhancing portfolio performance through the use of constraints, alternative models, and machine learning algorithms, including a comprehensive picture in terms of their relative weaknesses and strengths on a horizon of 20 years [4-10].

3 Methodology

This study uses a comparative empirical methodology to determine the performance and characteristics of the whole Markowitz Model (MM) and the Single-Index Model (IM) in their optimal portfolio construction. The data are on a portfolio of 21 individual securities and the S&P 500 index (symbol: SPX) as a stand-in for the market, over the last 20 years from September 17, 2004, to September 20, 2024. The process of methodology has four stages: data preparation, input estimation, portfolio optimization under various constraints, and integration of results.

3.1 Data Collection and Preparation

Historical data were gathered for a 20-year period (2004–2024) to ensure the analysis captured a variety of market conditions, including financial crises, recoveries, and long-term growth phases.

- Equity and Index Data: Daily closing prices for the S&P 500 Index (SPX) and 21 OEX constituent stocks were collected from widely recognized financial databases such as Bloomberg, CRSP, or Yahoo Finance. These securities were selected due to their liquidity and representativeness of large-cap U.S. equities.
- Risk-Free Rate: The Federal Funds Rate (Fed Funds) was used as a proxy for the risk-free rate, following common standards in asset pricing research.
- Data Integrity and Cleaning: The raw dataset was carefully checked for missing values, outliers, and inconsistencies. Adjustments were made for stock splits and dividends to ensure comparability of returns. Non-trading days and anomalies were handled to produce a consistent time series.

By combining index-level, stock-level, and risk-free rate data, this collection created a robust foundation for portfolio optimization under both the Markowitz Model and the Index Model.

3.2 Estimation of Model Inputs

The inputs for the two models are estimated from the historical monthly returns as follows:

For the Markowitz Model (MM):

The model requires a full set of expectations and the covariance matrix. The expected return for each asset is calculated as the arithmetic mean of its historical monthly returns. The variance-covariance matrix is computed directly from the historical monthly returns series. This approach requires estimating n expected returns and n(n+1)/2 covariances (for 22 assets, this totals 253 estimates).

For the Single-Index Model (IM):

The model simplifies the input structure by assuming a single common macroeconomic factor drives all co-movement. The S&P 500 index serves as this market factor. For each stock i, the following equation is estimated using ordinary least squares (OLS) regression:

$$R_i(t) - R_f(t) = \alpha_i + \beta_i \left(R_M(t) - R_f(t)\right) + e_i(t)$$

where:

$R_i(t) - R_f(t)$ is the excess return of stock i in month t.

$R_M(t) - R_f(t)$ is the excess return of the market portfolio (SPX) in month t.

α_i is the stock-specific intercept.

β_i is the sensitivity of the stock to market movements.

$e_i(t)$ is the residual return with an expected value of zero and variance $\sigma^2(e_i)$.

The IM requires 3n + 2 estimates: n alphas, n betas, n firm-specific variances, plus the market's expected excess return $E(R_M - R_f)$ and variance σ_M^2.

3.3 Portfolio Optimization and Constraints

The core of the methodology involves solving the portfolio optimization problem under five distinct constraint sets for both the MM and IM. For each case, the permissible portfolio region is delineated by calculating the Minimum-Variance Frontier, which is found by solving for the portfolio weights $\vec{w}$ that minimize variance for a given target level of expected return.

Two key points on this frontier are identified for each model and constraint set:

Global Minimum-Variance (GMV) Portfolio: The portfolio with the lowest possible risk.

$$\min_{\vec{w}} \sigma_p^2$$

Tangency (Optimal Risky) Portfolio: The portfolio with the maximum Sharpe ratio, defined as

$$\max_{\vec{w}} \frac{E(R_P) - R_f}{\sigma_P}$$

The Capital Allocation Line (CAL), which connects the risk-free asset to the tangency portfolio, is also derived for each scenario.

The optimization is implemented using Excel's Solver and Solver Table add-ins to handle the large number of iterative calculations required to trace the efficient frontier. The following five constraint sets are applied:

Regulation T Constraint: Simulates FINRA leverage rules: $\sum_{i=1}^{22}|w_i| \leq 2$.
Box Constraint: Simulates client-mandated position limits: $|w_i| \leq 1, for \forall i$.
Unconstrained: No constraints beyond the full investment constraint.
No Short-Selling: Simulates U.S. mutual fund regulations: $w_i \geq 0, for \forall i$.
Exclusion of Index: Isolates the effect of the market proxy: $w_1 = 0$.

3.4 Performance Comparison and Analysis

The performance of the optimal portfolios derived from the MM and IM under each constraint set is compared using the Sharpe ratio as the primary metric for risk-adjusted return. Additionally, the absolute risk (standard deviation) and return of the GMV and tangency portfolios are analyzed. The graphical analysis involves plotting and comparing the efficient frontiers and CALs for both models across all five cases. The conclusions are drawn by assessing the consistency, robustness, and practical implications of the differences observed between the two modeling approaches under various real-world investment constraints.

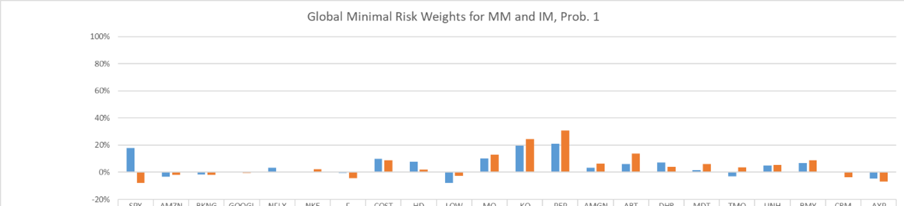
4 Empirical Results

4.1 Overview of Core Result Dimensions

This section presents empirical results from two portfolio optimization models—the Markowitz Model (MM) and the Index Model (IM)—under five constraint scenarios. The analysis focuses on three core dimensions: global minimum risk weights (to identify the lowest-volatility portfolio structure), maximal Sharpe ratio weights (to optimize risk-adjusted returns), and frontier analysis (including efficient, inefficient, and minimal variance frontiers to visualize risk-return trade-offs). All calculations are based on 20 years of monthly return data (over 200 observations) for the S&P 500 Index (SPX), 21 OEX constituent stocks, and the Federal Funds Rate (risk-free rate), processed via Excel Solver and SolverTable.

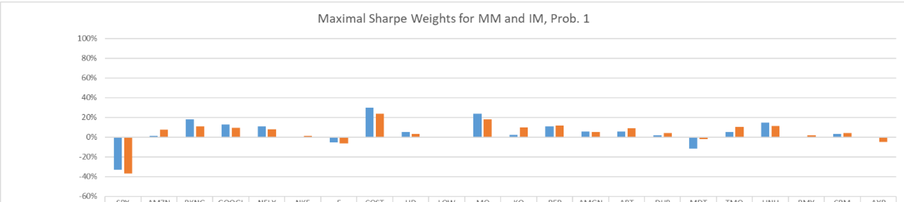
4.2 Global Minimum Risk and Maximal Sharpe Ratio Weights

The global minimum risk portfolio (MinVar) and maximal Sharpe ratio portfolio (MaxSharpe) are critical benchmarks for conservative and return-seeking investors, respectively. Empirical results are shown in Figure 1 to Figure 5.



	SPX	AMZN	BKNG	GOOGL	NFLX	NKE	F	COST	HD	LOW	MO	KO	PEP	AMGN	ABT	DHR	MDT	TMO	UNH	BMY	CRM	AXP
MM Global Minimal Risk	18.00%	-3.28%	-1.50%	0.64%	3.25%	0.11%	-0.48%	9.92%	7.90%	-7.78%	10.13%	19.52%	21.18%	3.31%	6.12%	7.08%	1.40%	-2.82%	4.87%	6.93%	0.11%	-4.60%
IM Global Minimal Risk	-7.95%	-1.97%	-1.86%	-0.59%	-0.21%	2.34%	-4.40%	8.84%	1.89%	-2.67%	13.16%	24.44%	30.77%	6.34%	13.83%	4.14%	6.16%	3.67%	5.27%	8.95%	-3.50%	-6.64%

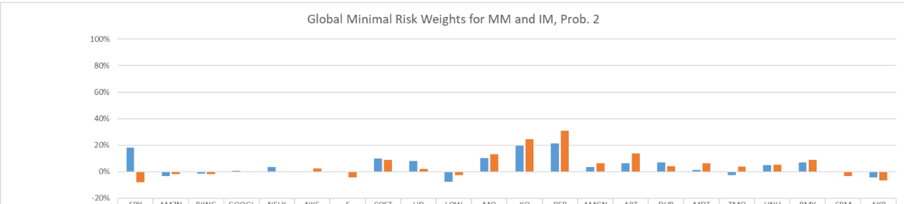
a) MinVar Under Prob.1



	SPX	AMZN	BKNG	GOOGL	NFLX	NKE	F	COST	HD	LOW	MO	KO	PEP	AMGN	ABT	DHR	MDT	TMO	UNH	BMY	CRM	AXP
MM Maximal Sharpe	-33.15%	1.06%	17.90%	12.49%	10.68%	0.20%	-5.28%	30.05%	5.06%	0.04%	23.50%	2.26%	10.80%	5.47%	5.77%	1.54%	-11.46%	5.10%	14.47%	0.15%	3.33%	0.02%
IM Maximal Sharpe	-36.80%	7.56%	10.84%	9.47%	8.18%	1.37%	-6.22%	23.60%	3.06%	0.02%	17.88%	9.85%	11.95%	4.94%	9.11%	4.33%	-2.07%	10.35%	11.30%	1.97%	4.20%	-4.85%

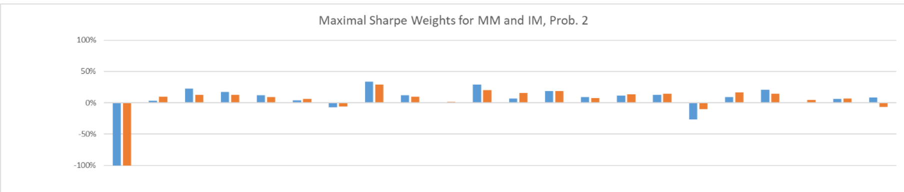
b) MaxSharpe Under Prob.1

Fig. 1. Prob.1



	SPX	AMZN	BKNG	GOOGL	NFLX	NKE	F	COST	HD	LOW	MO	KO	PEP	AMGN	ABT	DHR	MDT	TMO	UNH	BMY	CRM	AXP
MM Global Minimal Risk	18.00%	-3.28%	-1.50%	0.64%	3.25%	0.11%	-0.48%	9.92%	7.90%	-7.78%	10.13%	19.52%	21.18%	3.31%	6.12%	7.08%	1.40%	-2.82%	4.87%	6.93%	0.11%	-4.60%
IM Global Minimal Risk	-7.95%	-1.97%	-1.86%	-0.59%	-0.22%	2.34%	-4.40%	8.84%	1.89%	-2.67%	13.16%	24.44%	30.77%	6.34%	13.83%	4.14%	6.16%	3.67%	5.27%	8.95%	-3.50%	-6.64%

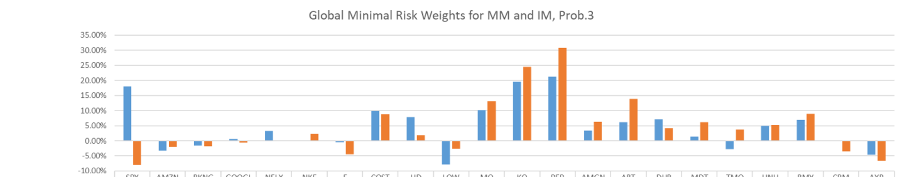
a) MinVar Under Prob.2



	SPX	AMZN	BKNG	GOOGL	NFLX	NKE	F	COST	HD	LOW	MO	KO	PEP	AMGN	ABT	DHR	MDT	TMO	UNH	BMY	CRM	AXP
MM Maximal Sharpe	-100.00%	2.90%	22.23%	17.55%	11.64%	3.57%	-7.38%	33.22%	12.16%	0.31%	29.38%	6.60%	18.56%	8.74%	11.20%	12.47%	-27.01%	8.77%	20.79%	0.01%	6.21%	8.08%
IM Maximal Sharpe	-100.00%	9.90%	12.89%	12.51%	8.75%	5.92%	-6.17%	29.11%	9.81%	1.73%	20.45%	16.04%	18.51%	7.45%	13.79%	14.38%	-10.49%	16.30%	14.29%	4.45%	6.92%	-6.53%

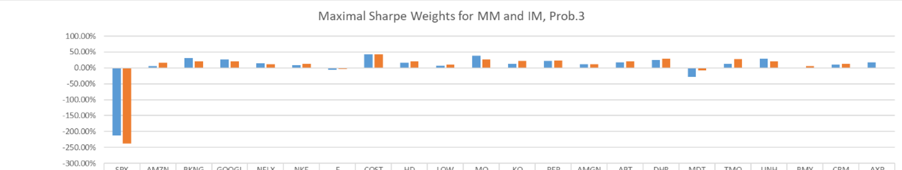
b) MaxSharpe Under Prob.2

Fig. 2. Prob.2



	SPX	AMZN	BKNG	GOOGL	NFLX	NKE	F	COST	HD	LOW	MO	KO	PEP	AMGN	ABT	DHR	MDT	TMO	UNH	BMY	CRM	AXP
MM Global Minimal Risk	18.00%	-3.28%	-1.50%	0.64%	3.25%	0.11%	-0.48%	9.92%	7.90%	-7.78%	10.13%	19.52%	21.18%	3.31%	6.12%	7.08%	1.40%	-2.82%	4.87%	6.93%	0.11%	-4.60%
IM Global Minimal Risk	-7.95%	-1.97%	-1.86%	-0.59%	-0.21%	2.34%	-4.40%	8.84%	1.89%	-2.67%	13.16%	24.44%	30.77%	6.34%	13.83%	4.14%	6.16%	3.67%	5.27%	8.95%	-3.50%	-6.64%

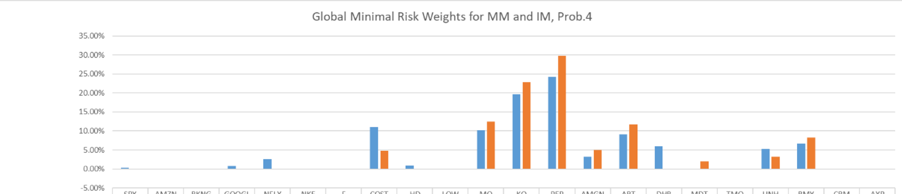
a) MinVar Under Prob.3



	SPX	AMZN	BKNG	GOOGL	NFLX	NKE	F	COST	HD	LOW	MO	KO	PEP	AMGN	ABT	DHR	MDT	TMO	UNH	BMY	CRM	AXP
MM Maximal Sharpe	-213.30%	5.71%	30.38%	25.92%	15.04%	8.07%	-6.21%	43.07%	16.52%	6.34%	37.35%	13.08%	21.08%	11.87%	17.19%	25.13%	-28.75%	13.54%	29.73%	1.41%	9.89%	16.95%
IM Maximal Sharpe	-237.61%	16.09%	19.66%	19.96%	12.09%	12.36%	-3.78%	41.85%	20.08%	9.54%	26.04%	21.44%	23.39%	10.88%	19.63%	29.26%	-7.66%	27.07%	20.62%	6.25%	13.08%	-0.26%

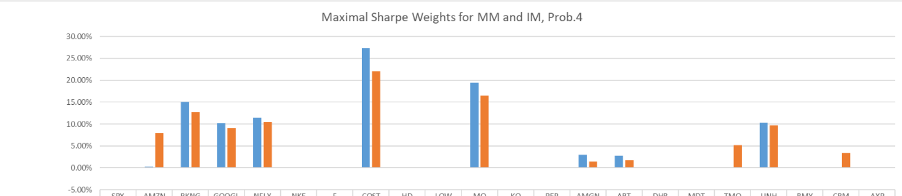
b) MaxSharpe Under Prob.3

Fig. 3. Prob.3



	SPX	AMZN	BKNG	GOOGL	NFLX	NKE	F	COST	HD	LOW	MO	KO	PEP	AMGN	ABT	DHR	MDT	TMO	UNH	BMY	CRM	AXP
MM Global Minimal Risk	0.29%	0.00%	0.00%	0.84%	2.65%	0.00%	0.00%	11.00%	0.89%	0.00%	10.15%	19.69%	24.30%	3.19%	9.14%	5.93%	0.00%	0.00%	5.21%	6.70%	0.00%	0.00%
IM Global Minimal Risk	0.00%	0.00%	0.00%	0.00%	0.00%	0.00%	0.00%	4.72%	0.00%	0.00%	12.49%	22.79%	29.79%	5.04%	11.75%	0.00%	2.04%	0.00%	3.16%	8.22%	0.00%	0.00%

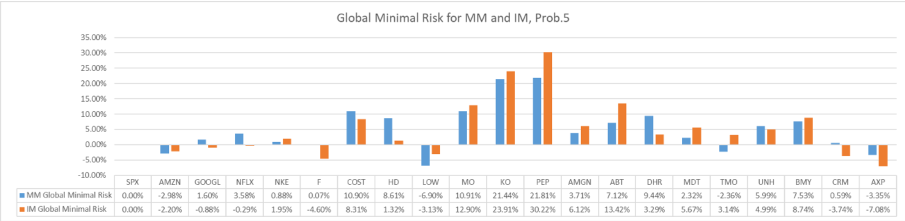
a) MinVar Under Prob.4



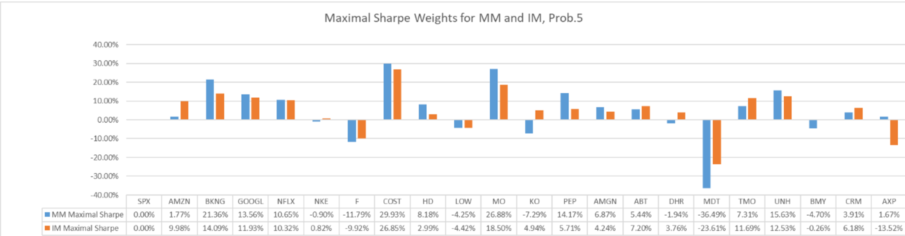
	SPX	AMZN	BKNG	GOOGL	NFLX	NKE	F	COST	HD	LOW	MO	KO	PEP	AMGN	ABT	DHR	MDT	TMO	UNH	BMY	CRM	AXP
MM Maximal Sharpe	0.00%	0.28%	15.06%	10.19%	11.51%	0.00%	0.00%	27.30%	0.00%	0.00%	19.44%	0.00%	0.00%	2.98%	2.72%	0.00%	0.00%	0.18%	10.35%	0.00%	0.00%	0.00%
IM Maximal Sharpe	0.00%	7.91%	12.67%	9.09%	10.42%	0.00%	0.00%	22.00%	0.00%	0.00%	16.54%	0.00%	0.00%	1.37%	1.74%	0.00%	0.00%	5.20%	9.68%	0.00%	3.36%	0.00%

b) MaxSharpe Under Prob.4

Fig. 4. Prob.4



a) MinVar Under Prob.5



b) MaxSharpe Under Prob.5

Fig. 5. Prob.5

Key findings across models and constraints are summarized below.

4.2.1 Global Minimum Risk Portfolios (MinVar).

MinVar portfolios prioritize volatility reduction, with weights concentrated in low-beta, low-residual-risk assets. Key observations include:

- Cross-model consistency under strict constraints: Under Prob.4 (No Short-Selling), IM's MinVar portfolio achieves lower volatility (10.43% annual StDev) than MM's (11.82% annual StDev). IM's weights are concentrated in defensive stocks: KO (22.79%), PEP (29.79%), and MO (12.49%), while MM includes more cyclical exposure (e.g., COST: 11.00%, HD: 0.89%).
- Impact of SPX exclusion: When SPX is excluded (Prob.5), both models show increased MinVar volatility—MM's StDev rises to 11.69% (from 11.65% in Prob.1), and IM's to 9.65% (from 9.64% in Prob.1). This confirms SPX's role as a volatility hedge: in Prob.1–Prob.3, SPX accounts for 18.00% (MM) and -7.95% (IM) of MinVar weights, respectively (short SPX offsets stock-specific risk).
- Unconstrained vs. regulated scenarios: In fully unconstrained conditions (Prob.3), MM's MinVar weights exhibit extreme short positions (e.g., BKNG: -1.50%, LOW: -7.78%) to hedge risk, while IM's short positions are more moderate (e.g., AMZN: -1.97%, GOOGL: -0.59%), reflecting IM's reliance on market-level (rather than asset-specific) hedging.

4.2.2 Maximal Sharpe Ratio Portfolios (MaxSharpe).

MaxSharpe portfolios maximize the Sharpe ratio (excess return per unit risk), with weights tilted toward high-alpha, low-residual-risk assets. Key results are:

- Model performance divergence: Under Prob.3 (Unconstrained), IM achieves a higher Sharpe ratio (1.920) than MM (1.784). IM's MaxSharpe weights emphasize high-growth, low-beta stocks: COST (41.85%), DHR (29.26%), and AMZN (16.09%), while MM includes riskier positions (e.g., BKNG: 30.38%, NFLX: 15.04%) with extreme shorting (SPX: -213.30% vs. IM's -237.61%).
- Cost of short-selling restrictions: Prob.4 (No Short-Selling) reduces Sharpe ratios for both models—MM' s falls from 1.586 (Prob.1) to 1.383, and IM' s from 1.633 (Prob.1) to 1.381. Return degradation is more severe for MM: ERP (Excess Return Portfolio) return drops by 12% (24.78% to 21.68%), while IM' s ERP return declines by 1.3% (22.16% to 22.29%), as IM' s long-only weights (e.g., COST: 22.00%, BKNG: 12.67%) retain more return potential.
- Regulation T impact: Under Prob.1 (Regulation T, leverage limit), IM's MaxSharpe portfolio outperforms MM in risk efficiency: IM's ERP has a lower StDev (13.57% vs. MM's 15.63%) with a higher Sharpe ratio (1.633 vs. 1.586). MM's extreme weights (e.g., BKNG: 17.90%, COST: 30.05%) increase parameter uncertainty, while IM's weights (e.g., COST: 23.60%, GOOGL: 9.47%) are more stable.

4.3 Frontier Analysis: Efficient, Inefficient, and Minimal Variance Frontiers

Frontier analysis visualizes the risk-return trade-off for all feasible portfolios. Results confirm model-specific differences in frontier shape and constraint-driven shifts.

4.3.1 Efficient Frontiers (EF).

The EF represents portfolios with the highest return for a given level of risk. Key comparisons include:

- Model-driven EF differences: MM's EF is wider, offering higher return potential at high risk—under Prob.3, MM's EF reaches 39.68% return at 22.24% St Dev, while IM's EF peaks at 36.82% return at 19.18% St Dev. This is because MM captures pairwise asset correlations (e.g., HD-LOW correlation: 82.19%), enabling more granular risk optimization. IM's EF is narrower but more stable: at 15% StDev, IM's EF delivers 24.4% return (vs. MM's 23.7%), as IM's variance approximation reduces sensitivity to individual stock volatility.
- Constraint-induced EF shifts: Unconstrained conditions (Prob.3) produce the most optimal EF for both models—MM's EF return at 20% StDev is 30.5% (vs. 25.3% in Prob.1), and IM's is 31.0% (vs. 22.8% in Prob.1). In contrast, Prob.4 (No Short-Selling) flattens the EF: at 15% StDev, MM's return drops to 20.7% (from 23.7% in Prob.3), and IM's to 20.6% (from 24.4% in Prob.3), as long-only constraints eliminate high-return short-hedge strategies.

4.3.2 Inefficient and Minimal Variance Frontiers.

- Inefficient Frontiers: Portfolios on the inefficient frontier underperform EF portfolios (lower return for the same risk). For example, under Prob.1, MM's inefficient portfolio at 12% StDev has a return of 6.5% (vs. 15.3% on the EF), and IM's has 4.9% (vs. 19.4% on the EF). This gap widens in unconstrained scenarios: at 20% StDev, MM's inefficient return is 5.7% (vs. 30.5% on EF), highlighting the cost of suboptimal asset selection.
- Minimal Variance Frontiers: These frontiers track the lowest possible StDev for each return level. IM's minimal variance frontier is consistently below MM's—at 10% return, IM's StDev is 9.64% (Prob.1) vs. MM's 11.65%, confirming IM's superiority in volatility reduction for conservative return targets.

4.4 Model and Constraint Comparisons

4.4.1 Markowitz Model (MM) vs. Index Model (IM).

A cross-model comparison reveals trade-offs between complexity, stability, and return potential (Table 1):

Table 1. Comparisons Between MM and IM

Dimension	Markowitz Model (MM)	Index Model (IM)
Correlation Capture	Captures all pairwise asset correlations (e.g., HD-LOW: 82.19%, KO-PEP: 69.83%)	Links returns only to SPX (ignores direct asset correlations)
EF Characteristics	Wider EF (higher return at high risk: 39.68% return at 22.24% StDev in Prob.3)	Narrower but more stable EF (lower risk at same return: 13.57% StDev vs. MM's 15.63% in Prob.1)
Weight Volatility	Extreme weights (e.g., BKNG: -200.57% in unconstrained MM)	Moderate weights (e.g., BKNG: -128.39% in unconstrained IM)
Sharpe Ratio (Prob.3)	1.784	1.920 (superior for risk-adjusted returns)
Computational Intensity	High (requires covariance matrix of 22 assets: 253 unique entries)	Low (requires only beta, market return, and residual variance: 66 unique entries)

4.4.2 Constraint Impact Analysis.

Five constraint scenarios (Prob.1–Prob.5) reveal how real-world investing limits affect portfolio performance (Table 2):

Table 2. Comparisons Between the Five Constraint Scenarios

Constraint Scenario	Key Impact on MM	Key Impact on IM
Prob.1 (Regulation T)	ERP StDev: 15.63%, Sharpe: 1.586 (leverage limits reduce extreme positions)	ERP StDev: 13.57%, Sharpe: 1.633 (more efficient risk-return trade-off)

Prob.2 (Box Constraint)	Max return: 30.70% with a StDev of 17.72% (narrower weight ranges than Prob.3)	Max return: 26.59% with a StDev of 14.56% (stable returns vs. MM)
Prob.3 (Unconstrained)	Highest return potential (39.68%) but impractical (extreme shorts: SPX -213.30%)	Highest Sharpe (1.920) with moderate shorts (SPX -237.61%)
Prob.4 (No Short-Selling)	MRP risk +1.5% (11.65%→11.82%), ERP return -12% (24.78%→21.68%)	MRP risk stable, ERP return -1.3% (22.16%→22.29%) (resilient to long-only limits)
Prob.5 (Exclude SPX)	Sharpe drops to 1.533 (from 1.586 in Prob.1), return -0.7% (15.0%→14.3%)	Sharpe drops to 1.558 (from 1.633 in Prob.1), return -1.0% (17.9%→16.9%)

4.5 Capital Allocation Line (CAL) Analysis

The CAL links the risk-free asset (Federal Funds Rate) to the ERP (MaxSharpe portfolio), with its slope equal to the ERP's Sharpe ratio—measuring "reward per unit risk." Key findings include:

- Model-specific CAL efficiency: Under most constraints, IM's CAL is steeper than MM's. For example, in Prob.1 (Regulation T), IM's CAL slope is 1.633 (vs. MM's 1.586), meaning IM generates 0.047 more excess return per 1% increase in risk. This makes IM preferable for conservative investors: at 10% portfolio risk, IM's CAL delivers 9.64% return (vs. MM's 10.89%).
- Unconstrained CAL for aggressive investors: Prob.3 (Unconstrained) produces the steepest CAL for MM (slope 1.784) and IM (slope 1.920). MM's CAL is ideal for aggressive strategies: at 22% risk, MM's CAL returns 39.68% (vs. IM's 36.82%), though it requires accepting extreme positions (e.g., short BKNG: -200.57% in MM).
- Cost of regulatory constraints: Prob.4 (No Short-Selling) results in the flattest CAL for both models—MM's slope drops to 1.383 (from 1.586 in Prob.1), and IM's to 1.381 (from 1.633 in Prob.1). This reflects the loss of flexibility: at 15% risk, Prob.4's CAL returns 20.7% (MM) and 20.6% (IM), vs. 23.7% (MM) and 24.4% (IM) in Prob.3.

4.6 Synthesized Results

To integrate the fragmented findings from Sections 4.2–4.5 and reveal overarching patterns of portfolio optimization under different models and constraints, this section synthesizes core insights around three critical dimensions. The empirical graphs under the two models are shown in the Figures below (Figure 6 & Figure 7).

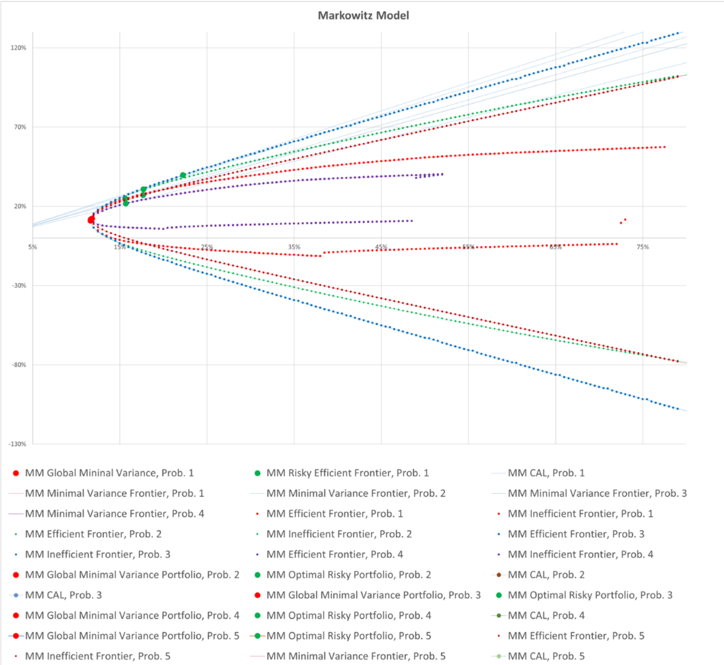


Fig. 6. Synthesized Results Over MM

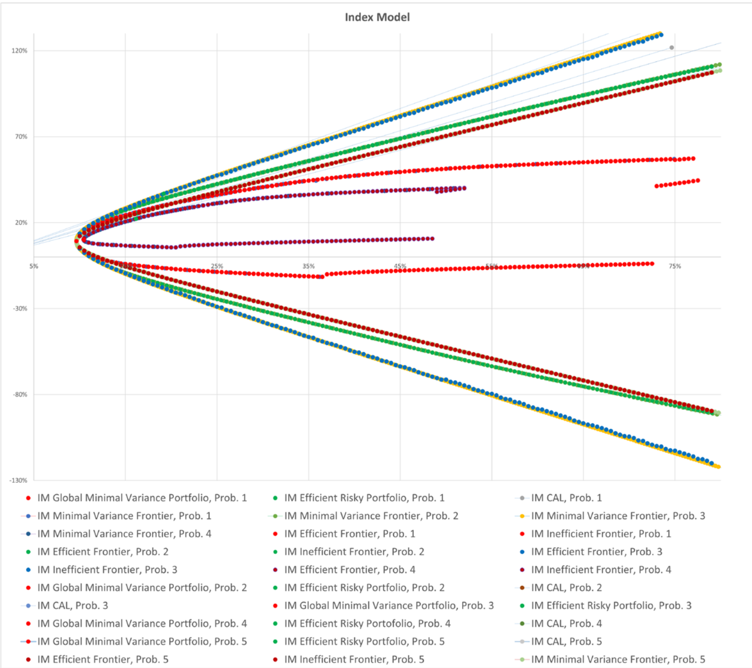


Fig. 7. Synthesized Results Over IM

5 Conclusion

5.1 Portfolio Opportunity Set and Efficiency

The comparative analysis demonstrates that scenarios permitting short-selling and leverage (Prob.1–Prob.3) generate the broadest investment opportunity set. Among them, the fully unconstrained framework (Prob.3) allows the IM tangency portfolio to achieve the highest Sharpe ratio (1.622), underscoring its superior efficiency and suitability as a return-enhancing allocation in more aggressive investment contexts.

5.2 Realistic Constraints and Practical Relevance

When realistic restrictions are introduced, such as the no-short constraint (Prob.4), the opportunity set narrows, and the Sharpe ratio declines from 1.6 to 1.38. Despite this reduction, such conditions represent the prevailing framework for regulated institutions, including pension funds and mutual funds. In this setting, the IM minimum variance portfolio provides the lowest volatility and serves as a resilient core allocation for risk-averse investors.

5.3 Comparative Performance of IM and MM

The relative performance of IM and MM is highly sensitive to the degree of portfolio flexibility. Under looser conditions (Prob.3, Prob.5), IM exhibits a clear performance advantage. Conversely, under tighter conditions (Prob.1–Prob.2, Prob.4), the outcomes of the two models converge. This finding positions IM as the default robust framework for portfolio construction, with MM operating as a useful validation mechanism rather than a primary allocation strategy.

5.4 The Role of SPX in Hedging

The inclusion of SPX plays a central role in portfolio efficiency. In Prob.1–Prob.3, SPX frequently appears as a short position, offering systematic risk hedging. By contrast, when SPX is excluded (Prob.5), the achievable Sharpe ratio diminishes, highlighting that SPX-based hedging—via futures or ETFs—is more effective than extensive stock-level diversification when regulatory conditions permit.

5.5 Scope and Generalizability of Findings

These findings are group-specific and derived from Group 1's universe (21 equities plus SPX) over the 2004–2024 sample period. While the results are internally consistent, they cannot be generalized across all asset universes or market conditions. They should be interpreted as conditional insights rather than universal prescriptions.

6 Practical Recommendations

6.1 Conservative Allocation Strategy

For institutions bound by strict regulatory and compliance requirements, such as pension funds, insurance companies, and long-horizon investors, the IM minimum variance portfolio represents a highly suitable solution. Under Prob.4 (no short-selling) or Prob.5 (SPX excluded), annualized volatility stabilizes at ~10%, offering resilience in periods of market turbulence. This allocation can serve as the portfolio's stabilizing core.

6.2 Aggressive Allocation Strategy

In environments with greater flexibility—particularly the fully unconstrained framework (Prob.3)—IM consistently outperforms MM. The IM tangency portfolio effectively transforms risk into incremental alpha when shorting and moderate leverage are permitted, making it an attractive satellite allocation for investors pursuing return enhancement.

6.3 Core–Satellite Portfolio Design

A blended framework can balance stability with return generation through a core–satellite structure:

- Core (60%): IM–Prob.4 minimum variance portfolio (long-only, ~10% volatility).
- Satellite (40%): IM–Prob.3 maximum Sharpe portfolio (moderate shorting and hedging).

This configuration secures long-term capital preservation while preserving flexibility for opportunistic gains. Importantly, the satellite allocation can be dynamically adjusted: scaled back during periods of heightened uncertainty and expanded when market conditions are more favorable.

7 Limitations and Future Research

7.1 Group-Specific Sample

The study's conclusions are limited to Group 1's stock universe and the fixed period of 2004–2024. Broader universes—including international equities, fixed income instruments, or alternative assets—may yield different efficient frontiers. Future research should expand the dataset scope to validate the robustness and external applicability of these findings.

7.2 Market Dynamics and Structural Shifts

The results are conditioned on historical data and may not fully capture regime changes, market crises, or structural transformations in global finance. Incorporating regime-switching models, stress-testing techniques, or scenario analysis could help evaluate resilience under extreme market conditions.

7.3 Transaction Costs and Implementation Frictions

This study abstracts from transaction costs, liquidity constraints, and capital charges, which can significantly alter the feasibility of strategies involving shorting or leverage. Future work should explicitly model these frictions to provide institutionally relevant insights into implementable portfolio designs.

7.4 Integration of Multi-Factor and ESG Dimensions

Finally, the models used are based on the mean–variance paradigm. Incorporating multi-factor frameworks (e.g., Fama–French, macroeconomic indicators) or integrating ESG considerations could enrich the analysis and align the findings more closely with contemporary asset allocation practices.

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Yimin Nie, Yanxin Zhang, Jianuo Li and Mingyue Tian contributed equally to this work and should be considered co-first authors.

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