






Performance Improvement of Cream Churning Using a Topologically Modified Dual Möbius Strip Working Body

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Abstract. A theoretical and computational study of cream churning with a Möbius-strip impeller revealed stable 3D vortex circulation combining active, intermediate, and stagnant zones. Increasing impeller speed and strip width enhances turbulence and mixing efficiency.

Keywords: Möbius strip, vortex flows, velocity vector, turbulent circulation, flow structures.

1 Introduction

The churning of cream is a complex physico-chemical process involving the mechanical disruption of fat globule membranes, their subsequent coalescence, and the formation of the spatial structure of butter. The efficiency of this process is largely determined by the design of the working element, its operational parameters, and the hydrodynamic conditions generated within the processing volume.

According to theoretical studies [1-5], the efficiency of cream churning is governed by three key factors: active foam formation, turbulent flow development, and the manifestation of cavitation effects. Modern designs of churning mechanisms must therefore promote these phenomena to facilitate the formation of a stable and uniform butter grain structure.

In recent years, increasing attention has been devoted to the development of non-conventional geometries of working elements aimed at enhancing churning intensity and improving flow hydrodynamics. The most notable designs include membrane-based, rotary-blade, eccentric, and reciprocating mechanisms [6–11]. Despite specific technological advances—such as reduced energy consumption, minimized fat losses, and shorter processing duration—most existing systems still suffer from several limitations, including uneven energy distribution, formation of stagnant (“dead”) zones, increased mechanical wear, and operational complexity.

To overcome these shortcomings, it is essential to introduce working elements with innovative geometries capable of generating three-dimensional vortex motion, ensuring uniform mixing, and maintaining active interaction with the processed medium under optimal turbulence and foaming conditions.

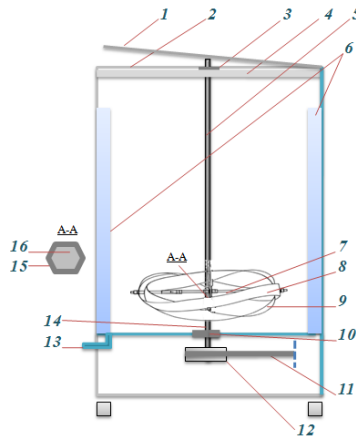
A particularly promising direction is the use of working elements based on the double Möbius strip configuration, which, due to its geometric asymmetry and continuous surface topology, promotes the formation of complex three-dimensional vortex structures. This unique flow pattern enhances the mixing intensity, shortens the churning time, and improves the overall efficiency of butter formation.

The *objective of this study* is to analyze the hydrodynamic characteristics of flow behavior occurring during the churning of cream using a working element designed as a double Möbius strip, and to theoretically substantiate the patterns of vortex structure formation and mixing zones within the working volume of the apparatus.

2 Materials and Methods

2.1 Experimental Apparatus

The working mechanism (see **Error! Reference source not found.**), consists of two metallic strips — the **outer** (8) and **inner** (9) bands — each twisted by 180° , forming Möbius strips with opposite twist directions. Both strips rotate synchronously in the same direction and are rigidly fixed to the connecting rod (7), which is attached to the vertical drive shaft (5). The upper end of the shaft is mounted on a self-aligning bearing (3) with a rotating support, ensuring stable axial rotation within the bracket (4), while the lower part is connected to the **intermediate shaft** (14) through bushings (15, 16) and the sealing assembly (10) located at the bottom of the working vessel (2). The intermediate shaft carries a driven pulley (12) connected by a V- belt (11) to an electric motor (not shown in the diagram). The vessel (2) is provided with a cover (1), baffles (6) along the internal wall, and a drain outlet (13) at the bottom. This configuration ensures stable rotation, self-centering alignment, and effective mixing during the churning process.



1 — cover; 2 — working vessel; 3 — self-aligning bearing with rotating support; 4 — mounting bracket; 5 — drive shaft; 6 — baffles; 7 — connecting rod; 8 — outer strip; 9 — inner strip; 10 — sealing assembly; 11 — V-belt; 12 — driven pulley; 13 — drain pipe; 14 — intermediate shaft; 15, 16 — bushings.

Fig. 1. Batch-type butter churn equipped with a double Möbius-strip impeller.

2.2 Hydrodynamic Characteristics of the Flow

During operation, the rotational motion of the double Möbius strips generates a complex three-dimensional vortex flow characterized by the following features:

- centrifugal forces displace the cream from the central zone toward the periphery of the vessel;
- oppositely twisted Möbius strips create shear layers and localized turbulence zones, enhancing mixing and fat globule coalescence;
- the continuous, asymmetric surface of the strips induces helical and recirculating trajectories of cream particles, promoting homogeneous distribution of energy and momentum throughout the working volume.

2.3 Methodology

The study was based on a theoretical and computational analysis of the hydrodynamic processes occurring within the working volume of the churn. The following key parameters were determined:

- Centrifugal velocity v (m/s):

$$v = \omega(r + kb) \tag{1}$$

$$\omega = 2\pi n/60 \tag{2}$$

where ω - angular velocity, (rad/s); n - rotational speed, (rad); r - strip radius, (m); b - strip width (m); k - correction factor accounting for the velocity variation across the strip width, $k = 0, 5$.

The average resultant centrifugal velocity $v_{avg.}$ (m/s) for the dual-strip impeller was determined as:

$$v_{avg.} = \frac{v_1 + v_2}{2} \tag{3}$$

- Centrifugal force $F(N)$:

$$F = \rho V \frac{\omega^2 (r+kb)^3 - r^3}{3} \tag{4}$$

where ρ – density of cream, (kg/m^3); V – ovolume of cream, (m^3).

This equation considers the mass distribution of the liquid along the strip width from radius r to $r + b$. Since each fluid particle experiences a different moment arm relative to the axis of rotation, the integration over the strip width introduces the term $\frac{(r+kb)^3 - r^3}{3}$, which represents the averaged moment of inertia of the liquid layer.

The generalized mean centrifugal force $F_{avg.}$ (N) for the two strips is:

$$F_{avg.} = \frac{F_1 + F_2}{2} \tag{5}$$

- To characterize the flow field, the horizontal U (m/s) and vertical V (m/s) velocity components were calculated using the following relationships:

$$U = -v_{avg.} \cdot \cos\left(\theta \cdot \frac{\pi}{180}\right) \tag{6}$$

$$V = v_{avg.} \cdot \sin\left(\theta \cdot \frac{\pi}{180}\right) \tag{7}$$

Where θ – angular coordinate ($^\circ$), defining the local position of a flow particle along the rotation plane.

The results of theoretical calculations were supported by numerical integration and graphical visualization of velocity fields and force distributions. These analyses enabled the identification of active, intermediate, and stagnant zones within the working

volume, as well as the assessment of the influence of geometric and kinematic parameters on the intensity of vortex motion.

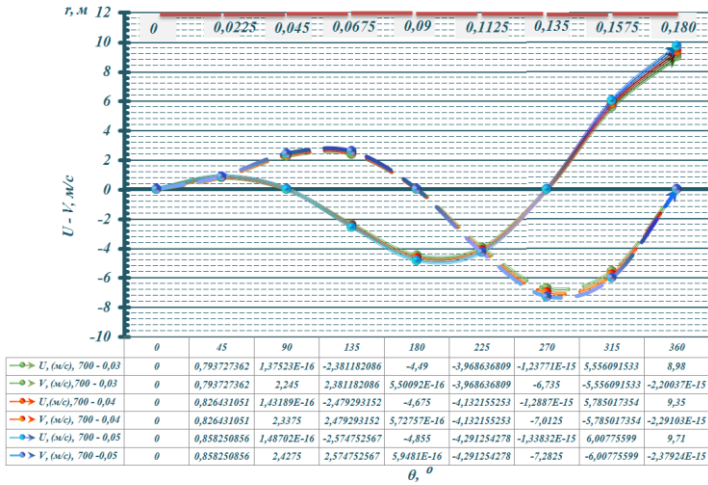
3 Results and Discussion

During the churning of cream and the mixing of the mass, vortex flows are formed within the vessel as a result of the action of the centrifugal velocity v_{avg} (m/s) and centrifugal force F_{avg} (N). These flows include zones of acceleration (velocity increase), turbulence, and stagnation, and are generally characterized by a high degree of instability.

The nature and direction of the flow under these conditions are described by the velocity vector, which consists of two mutually perpendicular components: the horizontal component U (m/s), reflecting the velocity and direction of motion along the radius of rotation, and the vertical component V (m/s), reflecting the velocity and direction of motion upward or downward. These components play an important role in the study of vortex motion, as they determine the direction and nature of the flow. The more pronounced the fluctuations of these components, the higher the level of turbulence, which contributes to the accelerated formation of butter grains.

To investigate the direction and characteristics of the vortex flow generated by the churning mechanism, the components of the flow velocity vector were determined under various operating conditions. Based on these data, graphs were constructed to illustrate the motion of the vortex flow velocity components (see Error! Reference source not found.2).

By identifying the rotation frequencies n (rpm) and ribbon widths b (m), at which the velocity vector components reach their maximum values, it becomes possible to determine the optimal design parameters that ensure more intensive mixing, as well as to identify regions of vortex flow acceleration and stagnant zones.



a)

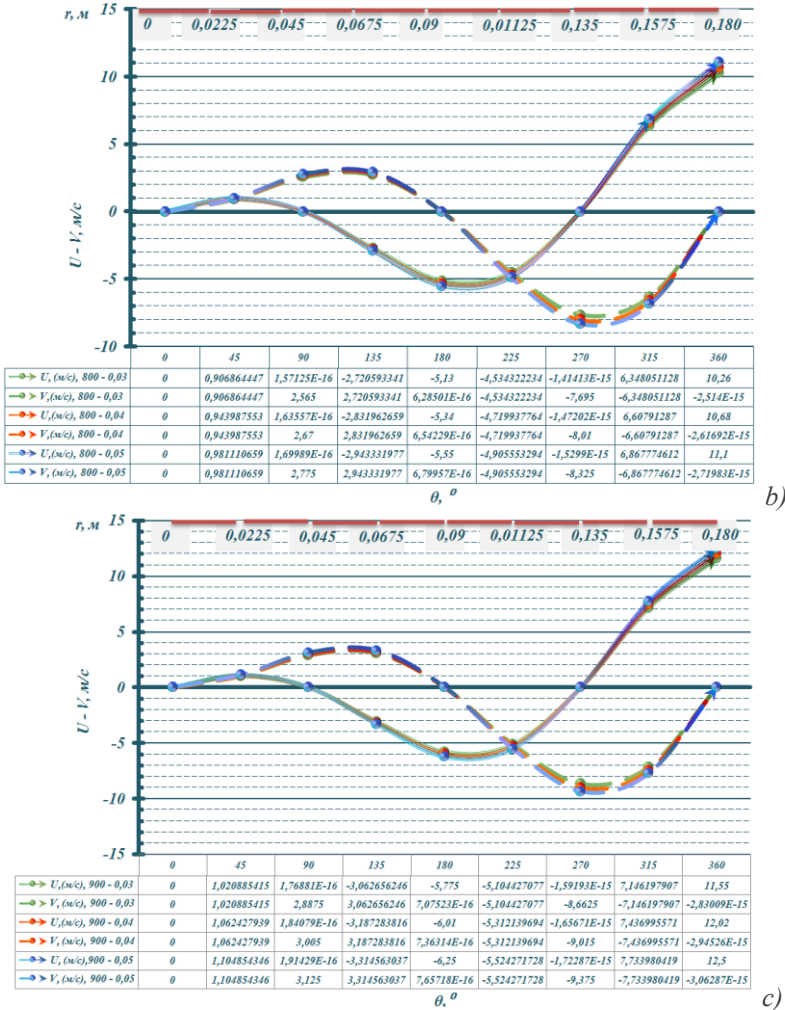


Fig. 2. Flow characteristics of the horizontal (U) and vertical (V) components of the velocity vector of the cream's vortex motion in the container depending on n , (rpm) and b , (m):
 a - $n = 700 \text{ rpm}$; b - $n = 800 \text{ rpm}$; c - $n = 900 \text{ rpm}$)

For all operating parameters ($n = 700 \rightarrow 900 \text{ rpm}$ and $b = 0.03 \rightarrow 0.05 \text{ m}$), the horizontal (U) and vertical (V) components of the velocity vector vary sinusoidally, which is attributed to the geometric configuration of the Möbius strip.

The upper limits of both velocity components were observed at the following locations:

- for the horizontal component U — at the outer boundary of the peripheral zone, corresponding to $r = 0.180 \text{ m}$. and $\theta = 360^\circ$, which marks the completion of a full rotation;

- for the vertical component V — at the beginning of the peripheral zone along the left radial direction, corresponding to $r = 0.135 \text{ m}$. and $\theta = 270^\circ$.

The lower limits ($\mathbf{0} \rightarrow \mathbf{1} \text{ m/s}$) of both components were recorded in the following regions:

- for the horizontal component \mathbf{U} — within the central zone, from $\mathbf{r} = \mathbf{0.0} \rightarrow \mathbf{0.045} \text{ m}$. and $\theta = \mathbf{0} \rightarrow \mathbf{90}^\circ$, corresponding to the initial position along the right radius of rotation, as well as at $\mathbf{r} = \mathbf{0.135} \text{ m}$. and $\theta = \mathbf{270}^\circ$, located at the onset of the peripheral zone on the left radius;
- for the vertical component \mathbf{V} — in the central region at $\mathbf{r} = \mathbf{0} \rightarrow \mathbf{0.0225} \text{ m}$. and $\theta = \mathbf{0} \rightarrow \mathbf{45}^\circ$ (initial phase of rotation), in the intermediate–transitional zone at $\mathbf{r} = \mathbf{0.09} \text{ m}$. and $\theta = \mathbf{180}^\circ$ (bottom point of the circular trajectory), and at $\mathbf{r} = \mathbf{0.180} \text{ m}$. and $\theta = \mathbf{360}^\circ$, which corresponds to the termination of a full revolution at the end of the peripheral zone.

These results demonstrate that the velocity distribution exhibits pronounced spatial periodicity and symmetry relative to the rotational axis, confirming the structured nature of vortex formation and flow modulation induced by the Möbius-type churning mechanism.

Thus, the horizontal component \mathbf{U} of the velocity vector reaches its maximum values at the periphery and its minimum values at the center, whereas the vertical component \mathbf{V} demonstrates a more complex spatial distribution with extrema at specific locations. Under these conditions, the central zone ($\mathbf{r} < \mathbf{0.045} \text{ m}$) is dominated by a laminar flow regime, while the peripheral region ($\mathbf{r} > \mathbf{0.135} \text{ m}$) exhibits a transition to fully developed turbulence.

The direction of the horizontal component \mathbf{U} is initially oriented from the center toward the vessel wall (to the right), denoted graphically as “ $+\mathbf{U}$ ” along the positive axis. This behavior is observed throughout the central zone ($\mathbf{r} = \mathbf{0} \rightarrow \mathbf{0.045} \text{ m}$, small radius, $\theta = \mathbf{0} \rightarrow \mathbf{90}^\circ$), where the influence of the centrifugal force is weak and the corresponding velocity remains low. This region can therefore be considered a quasi-stagnant initiation zone of vortex generation, where the formation of primary flow instabilities begins.

At rotational frequencies of $\mathbf{n} = \mathbf{700} - \mathbf{800} \text{ rpm}$, the value of the horizontal component \mathbf{U} in the central zone varies within $\mathbf{0} \leq \mathbf{U} < \mathbf{1} \text{ m/s}$, while at $\mathbf{n} = \mathbf{900} \text{ rpm}$, its magnitude in the core region ($\mathbf{r} = \mathbf{0.0225} \text{ m}$; $\theta = \mathbf{45}^\circ$) exceeds $\mathbf{U} \geq \mathbf{1} \text{ m/s}$. The growth of \mathbf{U} near the geometric center ($\mathbf{r} = \mathbf{0.0225} \text{ m}$) corresponds to the transitional angular region ($\theta = \mathbf{45}^\circ$), where the centrifugal effect gradually extends to adjacent zones. Beyond this point, the velocity decreases again, forming a zone of velocity increment — a quasi-stationary circular flow encompassing the central part of the system, the outer surface of the turbulent vortex core, and the entire contact region of the churning mechanism (see **Error! Reference source not found.2**).

In the intermediate (transition) zone ($\mathbf{r} = \mathbf{0.045} \rightarrow \mathbf{0.135} \text{ m}$, mean radius, $\theta = \mathbf{90} \rightarrow \mathbf{270}^\circ$), the horizontal velocity component \mathbf{U} exhibits a parabolic distribution and is directed toward the center (to the left), denoted as “ $-\mathbf{U}$ ” along the negative axis. In this region, the main circulation of the cream mass occurs, accompanied by localized velocity fluctuations caused by the complex geometry of the Möbius strip. Up to the midpoint of the zone ($\mathbf{r} = \mathbf{0.09} \text{ m}$; $\theta = \mathbf{180}^\circ$), the flow accelerates toward the center, after which the velocity gradually decreases to near-zero values at the outer boundary of the intermediate zone. This area can be characterized as a dynamic circular transition region with an increasing circumferential velocity.

Here, the flow direction reverses due to the interaction of opposing streams, generating pronounced fluctuations in velocity — both decreases and subsequent surges. After this reversal, the flow reorients outward (to the right), from the center toward the vessel walls and bottom, again denoted as “+ U ” along the positive axis. Entering the peripheral zone ($r = 0.135 \rightarrow 0.180 \text{ m}$, outer radius, $\theta = 270 \rightarrow 360^\circ$), the flow reaches the maximum centrifugal force and the most intense interaction between opposing streams. As a result, a nonuniform velocity profile forms along the width of the Möbius strip.

This peripheral zone represents the dynamic circular flow region, where the horizontal component U increases sharply, reaching its maximum at the end of the periphery ($r = 0.180 \text{ m}$; $\theta = 360^\circ$). At this stage, the flow impinges on the vessel wall, creating the highest velocity gradients and generating stable vortex bands and nucleation centers for butter grain formation. This area can therefore be identified as the boundary sub-layer zone (see **Error! Reference source not found.2**), crucial for the development of sustained turbulent circulation and phase transformation processes during churning.

The direction of the vertical component of the velocity vector V is initially oriented upward, which on the graph is denoted as “+ V ” along the positive direction of the axis. In this direction, the increase in velocity first occurs in the central zone ($r = 0 \rightarrow 0.045 \text{ m}$, small radius, $\theta = 0 \rightarrow 90^\circ$), where the influence of the centrifugal force is relatively weak and the primary vortex sources are formed. The velocity growth continues toward the central part of the intermediate zone ($r = 0.045 \rightarrow 0.135 \text{ m}$, mean radius, $\theta = 90 \rightarrow 270^\circ$), where, due to the increasing circumferential velocity and the geometric features of the Möbius strip, local fluctuations in the flow velocity are observed. In this region, the flow gradually transitions from a quasi-stationary circular regime to a dynamic one. The vertical velocity component V continues to increase until the point $r = 0.0675 \text{ m}$; $\theta = 135^\circ$, where it reaches its maximum value as a result of a smooth redistribution of centrifugal influence across different regions of the working volume. Subsequently, in the central point of the transitional zone — at the lower position of the trajectory ($r = 0.09 \text{ m}$; $\theta = 180^\circ$) — the velocity decreases to values approaching zero. At this point, the convergence of opposing flow streams leads to a change in flow direction, causing an abrupt drop followed by a secondary rise in velocity (see **Error! Reference source not found.2**).

From the central point of the intermediate zone ($r = 0.09 \text{ m}$; $\theta = 180^\circ$) to the end of the peripheral zone ($r = 0.135 \rightarrow 0.180 \text{ m}$, outer radius, $\theta = 270 \rightarrow 360^\circ$), the vertical velocity component V acquires a parabolic distribution and is directed downward, toward the bottom of the vessel. On the graph, this is denoted as “- V ” along the negative direction of the axis. In this region, the vertical velocity reaches its peak value at the beginning of the peripheral zone — at the point located along the left radius of rotation ($r = 0.135 \text{ m}$; $\theta = 270^\circ$) — and subsequently decreases toward zero near the completion of a full revolution ($r = 0.180 \text{ m}$; $\theta = 360^\circ$) at the end of the peripheral zone (see **Error! Reference source not found.2**).

The pronounced oscillations in the vertical velocity component V observed in both the upward and downward flow directions are primarily caused by two types of interactions:

- in the first case, they arise at the interface between the central zone ($r = 0 \rightarrow 0.045 \text{ m}$, small radius, $\theta = 0 \rightarrow 90^\circ$), where vortex sources form at near-zero velocities, and the intermediate zone ($r = 0.045 \rightarrow 0.135 \text{ m}$, mean radius, $\theta = 90 \rightarrow 270^\circ$), where the main circulation develops with increasing tangential velocity.
- in the second case, oscillations are associated with the boundary between the intermediate zone ($r = 0.045 \rightarrow 0.135 \text{ m}$, mean radius, $\theta = 90 \rightarrow 270^\circ$) — also characterized by strong circulation — and the peripheral zone ($r = 0.135 \rightarrow 0.180 \text{ m}$, outer radius, $\theta = 270 \rightarrow 360^\circ$), where both centrifugal velocity and force reach their maximum levels. In both cases, these oscillations result from the interaction of two counter-directed flow streams.

Across all operating conditions of the mixing mechanism ($n = 700 \rightarrow 900 \text{ rpm}$, $b = 0.03 \rightarrow 0.05 \text{ m}$), the observed conditions $U > 0$, $V > 0$ within the central zone ($r = 0 \rightarrow 0.045 \text{ m}$, small radius $\theta = 0 \rightarrow 90^\circ$) indicate a rapid upward flow directed from the bottom toward the vessel walls. The transition of the velocity vector toward $U < 0$ and $V > 0$ within the central region of the intermediate zone ($r = 0.045 \rightarrow 0.135 \text{ m}$, mean radius, $\theta = 90 \rightarrow 270^\circ$) corresponds to a rising diagonal motion of the vortex directed toward the center. Within the same zone ($r = 0.045 \rightarrow 0.135 \text{ m}$, $\theta = 90 \rightarrow 270^\circ$), where $U < 0$, $V < 0$ and $V > U$, the flow transitions to a more intense downward motion toward the center. Finally, in the terminal part of the peripheral zone ($r = 0.135 \rightarrow 0.180 \text{ m}$, outer radius, $\theta = 270 \rightarrow 360^\circ$), the conditions $U > 0$ and $V < 0$ correspond to an arched descending flow from the center toward the vessel walls — representing a segment of the helical vortex structure.

4 Conclusion

The theoretical and computational-graphical investigations have revealed the fundamental regularities governing the formation of vortex flows during cream churning with a dual Möbius strip-type working element. It was established that the interaction between the horizontal (U) and vertical (V) components of the velocity vector defines the spatial structure of the flow, encompassing laminar, transitional, and turbulent zones. In the central region ($r < 0.045 \text{ m}$), low-velocity areas are formed where the initial generation of vortical structures occurs, initiating the mixing of the cream mass. The intermediate zone ($r = 0.045 \rightarrow 0.135 \text{ m}$) is characterized by intensive circulation, alternating flow directions, and local velocity fluctuations, which enhance turbulence and accelerate the coalescence of fat globules. At the periphery ($r > 0.135 \text{ m}$), the centrifugal velocity and force reach their maximum values, leading to the formation of stable vortex bands, intensified mass exchange, and active disruption of fat globule membranes.

The geometric asymmetry of the Möbius strip induces the development of complex three-dimensional spiral and asymmetric vortex structures, ensuring a more uniform energy distribution within the working volume and enhancing hydrodynamic mixing efficiency. The observed oscillations of the velocity vector components (U , V) result from the interaction of counter flows and directional changes, promoting a stable turbulent regime and improving the structural quality of the butter grains.

Thus, the use of a dual Möbius strip as the working element of the butter churn provides:

- enhanced mixing intensity and reduced churning time;
- uniform distribution of kinetic energy and elimination of stagnant zones;
- stable three-dimensional vortex structures that promote the efficient formation of butter granules.

The findings possess both scientific and practical significance for optimizing the parameters of small-capacity batch butter production systems, predicting hydrodynamic flow characteristics, and advancing technologies for efficient butter manufacturing.

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